TWO-BOOK SERIES

STRAYER-UPTON ARITHMETICS

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SECOND BOOK

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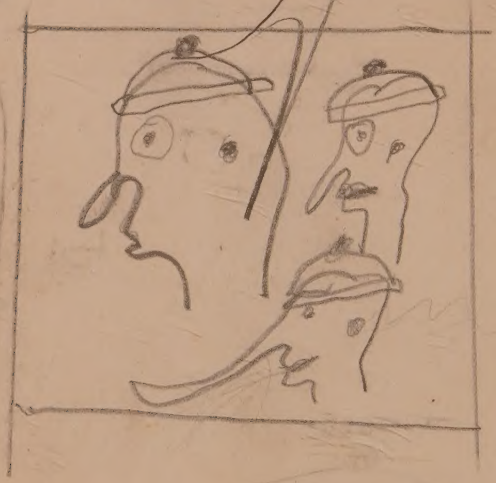

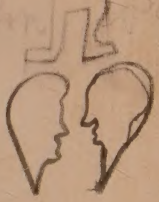

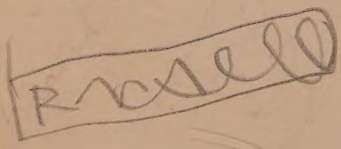

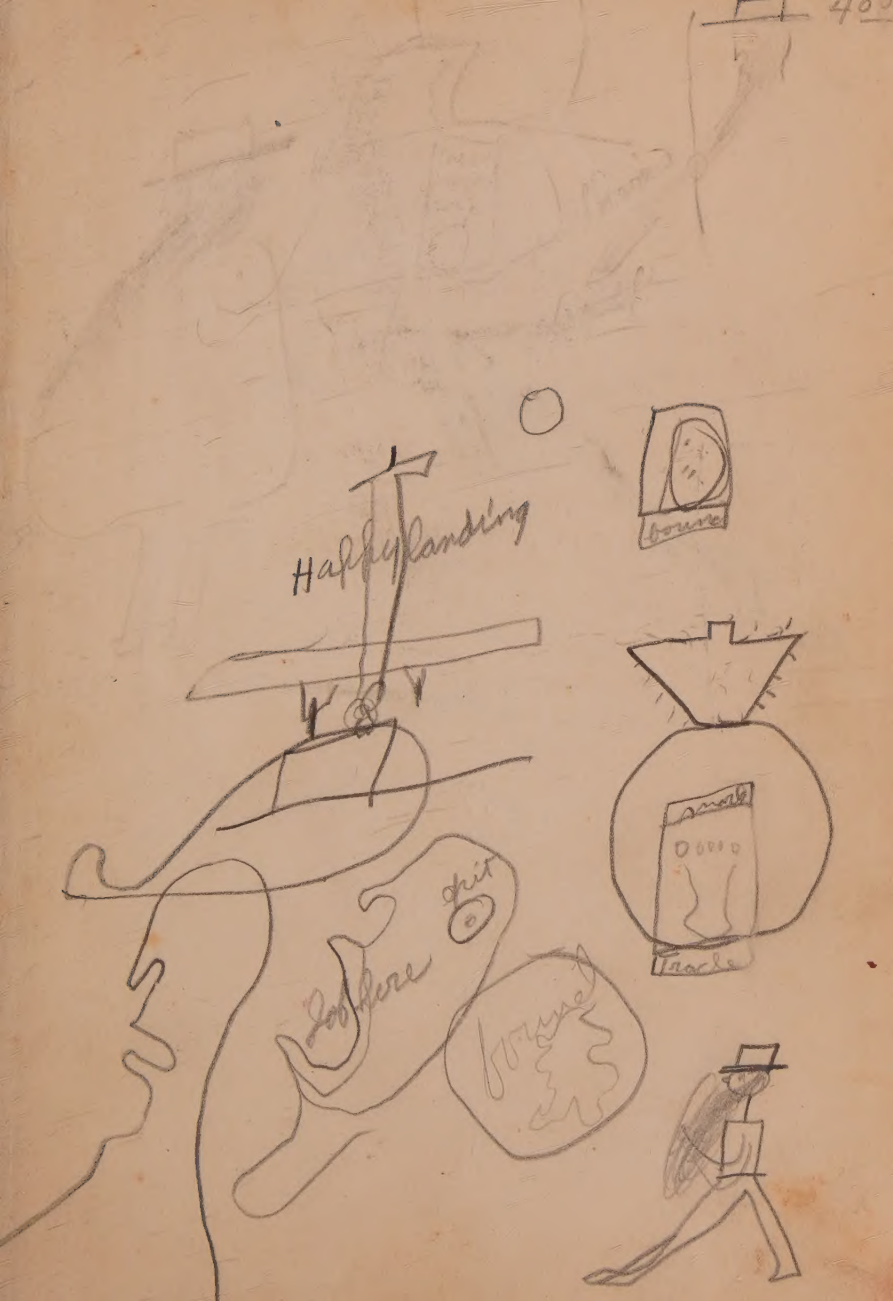
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TWO-BOOK SERIES

STRAYER-UPTON ARITHMETICS

SECOND BOOK

BY

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AMERICAN BOOK COMPANY
NEW YORK CINCINNATI CHICAGO
BOSTON ATLANTA

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STRAYER-UPTON ARITHMETICS, SECOND BOOK

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PREFACE

THIS series of books aims to give pupils the ability to compute easily and accurately, and to enable them to interpret and solve the quantitative situations which they will meet in life. In the achievement of this aim, these books incorporate the most valuable findings of modern experimentation in the teaching of arithmetic, including the results of important researches conducted by the authors. These books conform to the best modern courses of study and present only those methods and materials which have been thoroughly tested in the classroom by many experienced teachers. In no place do they introduce untried theories.

The Second Book of the two-book series is in harmony with the recommendations of the National Committee on Mathematical Requirements and of the Fifth Year-book of the Department of Superintendence.

The following features of this book are especially noteworthy:

1. New points are introduced gradually and are explained as simply as possible. In the treatment of decimals and of percentage, the work has been very carefully graded, so that the pupil is never confronted with more than one new step at a time. Sufficient exercises and practical problems are provided on each topic to give the pupil a complete mastery of it.

2. The individual differences of pupils are amply provided for by the large number of carefully graded abstract exercises, which afford even the less able pupil sufficient drill on any types of work that prove difficult for him.

3. Checking is emphasized throughout the book.

4. Since the decimal is rapidly gaining in importance in

everyday life, often replacing the common fraction, special care has been taken to include a large number of interesting modern problems in decimals in this book.

5. The applications of arithmetic as found in our personal lives and in business and industry are presented in such a way as to make them real and interesting to boys and girls. Special attention is given to such topics as thrift, savings, banking, insurance, purchasing a home, investment, installment buying, and modern business practice.

6. The making of graphs, such as those which appear so frequently in newspapers and magazines, is here developed and graded with unusual care.

7. Form study and geometric constructions are presented through a series of attractive exercises in actual drawing, thus utilizing the pupil's natural interest in constructive work.

8. The formula and the simple equation — two of our most effective mathematical instruments — are here presented as important aids to arithmetic and are applied in mensuration, in the work in proportion, and in indirect problems in percentage, areas, and volumes.

9. To furnish an efficient means of developing the pupil's skill in computation, a series of Improvement Tests is included, embracing 126 tests in whole numbers, fractions, decimals, and percentage. These tests are described on pages 1 to 6.

Chapters I and II of this book cover the work usually given in the sixth grade, Chapters III and IV cover the work of the seventh grade, and Chapters V and VI cover the eighth grade.

George Drayton Strayer
Clifford Brewster Upton

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SECOND BOOK

CHAPTER I

GAINING SKILL IN COMPUTATION

John wishes to improve his record in the standing broad jump. He will improve somewhat if he practices jumping every day; but he will make the fastest progress if he carefully measures the longest distance he jumps each day and writes it down and then tries hard to jump farther the next day. Every good athlete watches his record in this way and always tries to improve it.

In the same way, if you wish to compute as well as or better than any other pupil in your class, you must *watch your record and always try to improve it.*

A good way to improve your skill in computing is by means of Improvement Tests, such as Test 1 A in addition, which is given on page 3. When you try this test, the teacher will time you, telling you when to start and when to stop. At the end of the 4 minutes allowed for the test, the teacher will read the correct answers and you should make a record of your score, using the table on the inside of the back cover of this book to find your score.

When you take the same test again, in a day or two, *do your best to improve your score.* Repeat the test as often as the teacher directs, trying to get all the problems right in the given time.

The teacher will tell you when to try Test 1 B. Keep a record of all your scores on each test, as shown on page 5.

Always try to improve upon your best previous record.

HOW TO TAKE AN IMPROVEMENT TEST

When you are taking an Improvement Test in addition or subtraction, like Test 1 A or 1 B, do not copy the examples. Instead, place the edge of a sheet of paper directly under the first row of examples in the test, and write only the answers along that edge of the paper. Then if there is more than one row of examples on the test, as in Test 1 B, fold these answers under and write the next row of answers along the folded edge.

For an Improvement Test in multiplication or long division, like Test 1 C or 1 D, the examples should be copied on paper before the test begins, with sufficient space between them for working them. The time allowed for the multiplication and long division tests does not include the time required for copying.

Work on each test only the number of minutes permitted for that test, and *stop work immediately at the end of the given time*, even if you are just finishing an example. Otherwise you can not compare your scores on that test from day to day to see whether your work is improving.

If you finish a test before the signal to stop, check as many of the examples as you can in the remaining time. Occasionally the teacher will have you check the entire test, allowing you twice as long for doing and checking all the examples as for only doing them once.

IMPROVEMENT TEST No. 1

Test 1A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
5267	3746	1479	1796	6692
2090	9604	5610	7360	8787
4128	5534	3562	3918	9943
6896	8175	8884	9583	5019
7137	4955	4469	5377	2671
8502	2002	7335	2768	5936

Test 1B — Subtraction (Time 4 min.)

1.	2.	3.	4.	5.
4910	9571	7412	6075	6635
1340	5681	2543	4172	4539

6.	7.	8.	9.	10.
6375	9300	8243	5517	9822
1777	4587	4696	2980	6942

11.	12.	13.	14.	15.
7501	4000	8423	3168	9349
6340	1527	3182	2898	3187

Test 1C — Multiplication (Time 4 min. after copying)

1.	2.	3.	4.	5.
975	769	716	543	674
805	314	195	792	806

Test 1D — Division (Time 4 min. after copying)

1. $67\overline{)49848}$ 2. $48\overline{)40297}$ 3. $23\overline{)11362}$

4. $21\overline{)16926}$ 5. $77\overline{)44671}$

FINDING YOUR SCORE IN AN IMPROVEMENT TEST

This table shows how to find your score on a test.

NUMBER OF EXAMPLES IN TEST	NUMBER OF EXAMPLES RIGHT															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
3	0	3	7	10													
4	0	3	5	8	10												
5	0	2	4	6	8	10											
6	0	2	3	5	7	8	10										
8	0	1	3	4	5	6	8	9	10								
9	0	1	2	3	4	6	7	8	9	10							
10	0	1	2	3	4	5	6	7	8	9	10						
12	0	1	2	3	3	4	5	6	7	8	8	9	10				
A → 15	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10	
16	0	1	1	2	3	3	4	4	5	6	6	7	8	8	9	9	10

B

C

1. Suppose that you take Test 1 B, which contains 15 examples, and get 12 examples right. Look down the first column to the *row* beginning with 15 (shown by Arrow A). Look *along that row* until you reach the *column* with 12 at the top (shown by Arrow B), and you will find the number 8 (shown by Arrow C). This means that your score is 8 on that test.

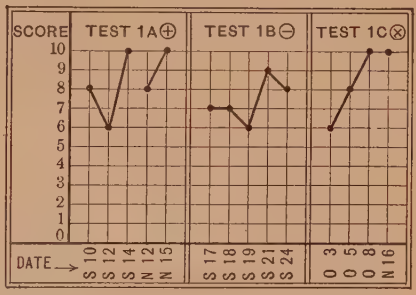
2. In Test 1 A (containing 5 examples), if you get 3 examples right, your score is 6; if you get 5 examples right, your score is 10, or perfect.

3. In finding your score, count only the examples that are entirely right. A partly finished example does not count.

Use the above table to find Frank's scores in these tests:

Number in test	5	8	8	9	10	10	12	12	12	15
Number right	4	5	7	5	7	9	8	10	12	14

KEEPING RECORDS IN IMPROVEMENT TESTS



Here is a picture, or **graph,** of Carl's daily scores. Such a picture is the best way to keep your record.

The dates on which Carl tried each test are given below the bottom line of the graph. S 10 means Sept. 10, which is the first day on which Carl tried Test 1 A. The heavy black dot above S 10, on the horizontal line marked 8, shows that Carl got a score of 8 on Sept. 10. The months are abbreviated as shown at the left.

S = Sept.	F = Feb.
O = Oct.	M = Mar.
N = Nov.	A = Apr.
D = Dec.	My = May
J = Jan.	Ju = June

Reading the Graph of Carl's Record

1. What was Carl's score on Sept. 12? Did he do as well then as on Sept. 10? Did he improve on Sept. 14?

2. How many times did he try Test 1 A before he got a score of 10, or a perfect score?

3. How long after he first got a score of 10 did he try Test 1 A again? What was his score on Nov. 12?

4. Did Carl do as well on Test 1 B as on 1 A? What was his first score on Test 1 B? his lowest score? his last score? Does he need more practice on Test 1 B?

5. Tell his lowest and his highest score on Test 1 C.

6. On which test did Carl improve the most rapidly?

FINDING AND RECORDING SCORES

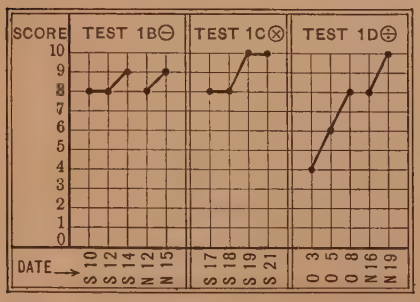
1. How many examples are there in Test 1 B? Anna had 11 examples right in Test 1 B on Sept. 11, 12 examples right on Sept. 12, and 14 right on Sept. 16. Use the table on page 4 to find her score for each day.

Make a graph to show Anna's record on Test 1 B, using the abbreviations on page 5 for the names of the months. Did she improve steadily on this test?

2. On a test of 6 examples, Ben had 5 examples right on Sept. 22, 4 right on Sept. 24, 5 right on Oct. 6, and 6 right on Oct. 8. Find Ben's score on each of these days and make a graph of his record. Does the graph show a steady improvement?

3. On a test of 12 examples, Leon had 8 examples right Nov. 4, 9 right Nov. 5, 9 right Nov. 10, and 12 right Nov. 13. On the same test, Ted had 9 examples right Nov. 4, 7 right Nov. 5, 8 right Nov. 10, and 9 right Nov. 13. Find each boy's scores and make a graph of each record. Which boy's record was better?

4. The first graph at the left shows that on Sept. 10 Elsie scored 8 on Test 1 B. Give her score each of the other times she tried Test 1 B; each time she tried Test 1 C; each time she tried Test 1 D.



On which test did Elsie do the best? On which test did she improve the most? On which does she need more practice?

READING LARGE NUMBERS

In newspapers and magazines you often see very large numbers representing *millions* and *billions*.

1,000,000 is read 1 million and 1,000,000,000 is read 1 billion. Thus, 1 million is another name for 1000 *thousand*, while 1 billion means 1000 *million*.

You have learned to divide large numbers into periods of 3 figures each by commas, beginning at the right. These periods have the following names:

Billions	Millions	Thousands	Units
27	165	023	015

Read this number *27 billion, 165 million, 23 thousand, 15*.

Exercises

These are headlines from newspapers. Read them:

1. Senate approves tax cut of $387,811,000.
2. Rich man leaves $8,500,000 for art.
3. Country used 1,293,184,528 post cards in a year.

Read the populations of these 10 *largest cities in the world:*

4. London	7,480,000	9. Buenos Aires	2,310,000
5. New York	5,620,000	10. Osaka	2,155,000
6. Berlin	4,014,000	11. Moscow	2,018,000
7. Paris	2,871,000	12. Tokyo	1,996,000
8. Chicago	2,702,000	13. Vienna	1,866,000

14. See how many large numbers you can find in newspapers. Bring them to class and read the large numbers.

15. Read several large numbers which you find on the last pages of your geography.

USING ADDITION

1. From a mail-order house Ralph ordered an overcoat for \$11.95, a sled for \$2.95, a set of tools for \$3.50, and a football for \$5.75. He sent \$.52 postage with the order. How much money did he send in all?

2. Dick has a bank account at the Eastern Savings Bank. Last year he deposited \$35.00, \$7.85, \$11.98, \$4.50, and \$15.75. How much did Dick deposit last year?

3. Jane is treasurer of a club at the Trent School. She collected these amounts for a new radio set: \$5.87, \$9.65, \$7.53, \$15.25, \$11.67. How much could they spend for the radio set?

4. May's mother buys the following new furnishings for May's room: a rocking chair for \$5.95, a pair of curtains for \$3.15, a fancy pillow for \$1.25, and a writing desk for \$7.95. How much do these cost?

5. How many miles of railroad are there in New England if there are 2228 mi. in Maine, 1057 mi. in Vermont, 1234 mi. in New Hampshire, 2071 mi. in Massachusetts, 196 mi. in Rhode Island, and 969 mi. in Connecticut?

6. In a recent year the mineral products of California were valued at \$428,175,652, those of Oregon at \$7,364,232, and those of Washington at \$21,159,370. What was the total value of the mineral products of the Pacific States?

7. Find the population of the world if there are 136,000,000 people in North America, 64,000,000 in South America, 142,000,000 in Africa, 921,000,000 in Asia, 476,000,000 in Europe, and 9,000,000 in Australasia.

USING SUBTRACTION

1. The Thorley School has 2519 pupils, of whom 1298 are boys. How many girls attend this school?

2. Grace had $48.14 in the school savings bank. She drew out $3.95 to buy a birthday present for her mother. How much did she still have in the bank?

3. Mr. Root paid $10,575 for a house which he later sold for $14,050. How much had the value of the house increased?

4. David has earned $98.50. He wants to make a trip to California, which will cost him $250. How much more will the trip cost than David has earned?

5. The population of Virginia was 1,225,163 in 1870 and 2,309,187 in 1920. How many inhabitants did Virginia gain during these fifty years?

6. The United States, with its outlying possessions, has about 117,859,495 inhabitants, while France, with her colonies, has only about 96,375,035. How much less is the population of France than that of our country?

7. Recently there were 15,369,404 telephones in the United States and 1,009,258 telephones in Canada. What was the difference between the number of telephones in these countries?

8. One year there were 15,597,628 automobiles in the United States and 154,208 in Germany. How many more cars were there in the United States than in Germany?

9. Ohio had 272,045 farms one year and 256,695 farms ten years later. Did the number of farms in Ohio increase or decrease during these ten years? how much?

USING MULTIPLICATION

1. Frank works in a large wholesale toy store. If he sells 235 toy taxicabs at $1.67 each, how much will the bill for them amount to?

2. The Park School sold 239 tickets to its play at 75¢ each, 165 tickets at 50¢ each, and 210 tickets at 35¢ each. How much money was collected in all from these sales?

3. A school bookstore sold 348 readers at 85¢ each and 415 arithmetics at 75¢ each. How much money was collected from the sale of these books?

4. In one year 8640 electric bulbs were sold to a department store. At an average of 28¢ each, how much was paid for these bulbs?

5. Mr. Wills sold 238 doz. eggs at 45¢ a dozen, 315 doz. at 48¢ a dozen, 214 doz. at 43¢ a dozen, and 237 doz. at 46¢ a dozen. How much did he receive in all?

6. Belle is going to save $1.75 each week out of the money she earns. How much will she have at the end of 28 wk.? 39 wk.? 42 wk.? a year?

7. Elsa's older brother, Dick, earns $18.75 a week in a factory. If he works 51 weeks in a year, how much money does he earn?

8. An airplane can travel 125 mi. an hour. How much farther can you ride in such an airplane in 12 hr. than in a train which goes 35 mi. an hour?

9. An automobile factory produces, on an average, 12,835 passenger cars and 425 delivery cars a week. How many cars in all will it produce in a year?

USING DIVISION

1. It costs an average of $306.60 to feed each lion at the Doho Circus for 1 year (365 days). Find the average cost per day to feed each lion.

2. Jane has $15.05 with which to buy 43 yd. of white lawn for aprons for her sewing class. How much per yard can she pay for the lawn?

3. It cost Mr. Franklin $391.20 to run his car 4890 mi. Find out how much per mile it cost him.

4. It cost Mr. Jenkins, a landlord, $618.00 to heat his apartment house for 125 days during the winter. Find, to the nearest cent, how much per day it cost him.

5. Mrs. Thomas is buying a piano for $350.50. She agrees to pay $62.50 cash and $16 a month till the piano is paid for. How many months will it take her to pay for the piano?

6. Mr. Dale and his family are making an automobile trip of 3180 mi. from New York to San Francisco. They want to make the trip in 30 days. How many miles a day should they average?

7. At a sale Mrs. Peck bought 29 yd. of cretonne for $6.96. How much per yard did she pay?

8. A factory which employs 295 men has a total weekly payroll of $8968. Find the average weekly salary of a workman.

9. Mr. Hart buys a house for $16,000. He pays $3400 cash and agrees to pay the rest in equal monthly installments covering 144 months. How much will he pay in each installment?

FRACTIONS

What You Know About Fractions

You have learned the following things about fractions:

1. A fraction can stand for *a part of one thing.* Thus, $\frac{3}{4}$ of 1 apple is a part of an apple.

2. A fraction can stand for *a part of a group* of things. Thus, $\frac{3}{4}$ of 16 boys, which equals 12 boys, is a part of a group of 16 boys.

3. A fraction can be used to express division. Thus, $\frac{3}{5}$ means $3 \div 5$ and $\frac{17}{4}$ means $17 \div 4$.

In everyday life you often change a half dollar into 5 dimes if 5 dimes are more convenient to use than a half dollar. You know that a half dollar and 5 dimes both have the *same value*, namely 50¢.

In the same way, you may change one fraction into another fraction having the *same value.* Thus, you may change $\frac{1}{2}$ to $\frac{4}{8}$ because $\frac{1}{2}$ is equal to $\frac{4}{8}$. Likewise you may change $\frac{4}{8}$ back to $\frac{1}{2}$ when you find it convenient to do so.

A quick way to change $\frac{1}{2}$ to $\frac{4}{8}$ is to multiply both terms of $\frac{1}{2}$ by 4, as shown in *A*.

$$A.\ \frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

Similarly, to change $\frac{8}{16}$ to $\frac{1}{2}$, you divide both terms of $\frac{8}{16}$ by 8, as shown in *B*.

$$B.\ \frac{8}{16} = \frac{8 \div 8}{16 \div 8} = \frac{1}{2}$$

If you multiply or divide both terms of a fraction by the same number, you get another fraction with the same value.

REDUCING FRACTIONS

1. There are 36 in. in 1 yd. What part of a yard is 1 in.? 5 in.? 9 in.?

2. Allen says that 9 in. is $\frac{1}{4}$ yd. because $\frac{9}{36} = \frac{1}{4}$. Is Allen right? By what number do you divide both terms of $\frac{9}{36}$ to change it to $\frac{1}{4}$?

3. Lucy has 27 in. of ribbon. What part of a yard is that? Then what does $\frac{27}{36}$ equal?

4. There are 1760 yd. in 1 mi. If Bert runs in the 220-yard dash, what part of a mile does he run? What does $\frac{220}{1760}$ equal?

In each fraction tell what number should be put in place of the star so that both fractions will be equal:

5. $\frac{1}{2} = \frac{*}{10}$ **7.** $\frac{2}{3} = \frac{*}{6}$ **9.** $\frac{3}{8} = \frac{*}{32}$ **11.** $\frac{7}{8} = \frac{*}{16}$

6. $\frac{5}{6} = \frac{*}{12}$ **8.** $\frac{3}{4} = \frac{*}{12}$ **10.** $\frac{1}{4} = \frac{*}{32}$ **12.** $\frac{6}{8} = \frac{*}{24}$

Change each fraction as indicated:

13. $\frac{3}{4}$ to 16ths **16.** $\frac{3}{8}$ to 24ths **19.** $\frac{9}{10}$ to 100ths

14. $\frac{1}{2}$ to 12ths **17.** $\frac{1}{4}$ to 60ths **20.** $\frac{3}{5}$ to 100ths

15. $\frac{2}{9}$ to 36ths **18.** $\frac{1}{10}$ to 100ths **21.** $\frac{7}{10}$ to 100ths

Reduce to lowest terms, telling by what number you divide both terms of the fraction:

22. $\frac{8}{36}$ **26.** $\frac{30}{60}$ **30.** $\frac{12}{16}$ **34.** $\frac{24}{36}$ **38.** $\frac{20}{24}$ **42.** $\frac{8}{32}$

23. $\frac{15}{20}$ **27.** $\frac{80}{100}$ **31.** $\frac{20}{60}$ **35.** $\frac{30}{36}$ **39.** $\frac{18}{20}$ **43.** $\frac{36}{60}$

24. $\frac{64}{100}$ **28.** $\frac{40}{50}$ **32.** $\frac{27}{45}$ **36.** $\frac{12}{32}$ **40.** $\frac{25}{75}$ **44.** $\frac{98}{100}$

25. $\frac{18}{36}$ **29.** $\frac{65}{100}$ **33.** $\frac{8}{16}$ **37.** $\frac{48}{50}$ **41.** $\frac{6}{39}$ **45.** $\frac{25}{100}$

46. Count by $\frac{1}{4}$'s to 5, thus: $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, etc.

47. Count by $\frac{1}{8}$'s to 4; by $\frac{1}{16}$'s to 3; by $\frac{1}{6}$'s to 5.

IMPROPER FRACTIONS

Most of the fractions in common use, like $\frac{3}{4}$ and $\frac{2}{3}$, are smaller than 1 whole unit. Such fractions are called **proper fractions.**

There are some fractions, however, like $\frac{4}{4}$ and $\frac{10}{10}$, which are equal to 1 whole unit, and some other fractions, like $\frac{5}{4}$ and $\frac{15}{3}$, which are larger than 1 whole unit. Both these kinds of fractions are called **improper fractions.** An improper fraction is either equal to 1 or greater than 1.

You know that $\frac{6}{1} = 6 \div 1 = 6$. Any whole number may be written in the form of a fraction. Thus, 8 may be written as $\frac{8}{1}$.

Numbers like $5\frac{1}{4}$ and $16\frac{2}{3}$ are called **mixed numbers.**

Which of these are proper fractions? Which are improper fractions?

$\frac{5}{8}$ $\frac{7}{5}$ $\frac{3}{16}$ $\frac{6}{6}$ $\frac{7}{4}$ $\frac{10}{3}$ $\frac{11}{12}$ $\frac{12}{5}$ $\frac{8}{8}$ $\frac{10}{5}$

Reducing Improper Fractions to Mixed Numbers

1. Reduce $\frac{12}{8}$ to a mixed number.

Since $\frac{12}{8}$ means $12 \div 8$, divide 12 by 8, which gives $1\frac{1}{2}$.

To reduce an improper fraction to a whole number or a mixed number, divide the numerator by the denominator.

Reduce to whole numbers or to mixed numbers. Be sure that the fraction in the answer is in its lowest terms:

2. $\frac{15}{8}$	**6.** $\frac{19}{12}$	**10.** $\frac{18}{8}$	**14.** $\frac{15}{5}$	**18.** $\frac{10}{4}$	**22.** $\frac{19}{16}$
3. $\frac{10}{3}$	**7.** $\frac{20}{8}$	**11.** $\frac{25}{12}$	**15.** $\frac{18}{10}$	**19.** $\frac{13}{4}$	**23.** $\frac{26}{16}$
4. $\frac{48}{4}$	**8.** $\frac{41}{5}$	**12.** $\frac{39}{5}$	**16.** $\frac{11}{6}$	**20.** $\frac{17}{2}$	**24.** $\frac{13}{3}$
5. $\frac{37}{5}$	**9.** $\frac{45}{10}$	**13.** $\frac{45}{12}$	**17.** $\frac{55}{15}$	**21.** $\frac{104}{16}$	**25.** $\frac{65}{20}$

ADDING FRACTIONS

1. Alice used $2\frac{1}{2}$ yd. of ribbon on one dress, $1\frac{3}{8}$ yd. on a second dress, and $1\frac{3}{4}$ yd. on a third dress. How much ribbon did she use in all?

You cannot add fractions that have different names, hence you cannot add *halves*, *eighths*, and *fourths*. But you can add $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{3}{4}$ if you first change them all to *eighths*, as shown at the right.

$$2\tfrac{1}{2} = 2\tfrac{4}{8}$$
$$1\tfrac{3}{8} = 1\tfrac{3}{8}$$
$$1\tfrac{3}{4} = 1\tfrac{6}{8}$$
$$5\tfrac{5}{8}$$

The sum of the fractions is $\frac{13}{8}$, which equals $1\frac{5}{8}$. Write $\frac{5}{8}$ and carry 1 to the whole numbers.

You find that Alice used $5\frac{5}{8}$ yd. of ribbon in all.

2. Gus walked $5\frac{1}{4}$ mi. on Thursday, $4\frac{1}{2}$ mi. on Friday, and $7\frac{1}{8}$ mi. on Saturday. How far did he walk in all?

To add fractions having different denominators, first change them to fractions having the same denominator.

Exercises

Add. Check the work by going over it again:

1. $7\frac{1}{2}$	**2.** $5\frac{3}{4}$	**3.** $4\frac{7}{8}$	**4.** $8\frac{1}{3}$	**5.** $9\frac{2}{5}$	**6.** $4\frac{2}{3}$
$4\frac{7}{8}$	$6\frac{7}{8}$	$3\frac{5}{8}$	$4\frac{5}{6}$	$5\frac{1}{2}$	$8\frac{5}{6}$
5	$9\frac{1}{2}$	$9\frac{1}{2}$	$5\frac{1}{2}$	$4\frac{3}{10}$	$9\frac{1}{2}$
7. $8\frac{4}{5}$	**8.** $5\frac{5}{12}$	**9.** $8\frac{3}{8}$	**10.** $9\frac{9}{10}$	**11.** $8\frac{5}{6}$	**12.** $5\frac{5}{16}$
6	$1\frac{1}{2}$	$1\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{2}{3}$	$2\frac{3}{4}$
$9\frac{7}{10}$	$2\frac{2}{3}$	$9\frac{1}{2}$	$4\frac{1}{5}$	$5\frac{5}{12}$	$4\frac{1}{2}$
13. $9\frac{7}{8}$	**14.** $7\frac{2}{3}$	**15.** $2\frac{9}{10}$	**16.** $5\frac{7}{12}$	**17.** $2\frac{1}{2}$	**18.** $1\frac{7}{8}$
$8\frac{1}{16}$	$5\frac{1}{2}$	$9\frac{2}{5}$	5	$3\frac{3}{4}$	$5\frac{7}{8}$
2	$4\frac{5}{12}$	1	$6\frac{1}{2}$	$5\frac{3}{8}$	$2\frac{3}{16}$
$6\frac{1}{2}$	$3\frac{1}{4}$	$8\frac{1}{2}$	$1\frac{1}{12}$	$4\frac{7}{16}$	$3\frac{3}{4}$

JANE SELLS FLOWERS

1. Jane has a flower garden. She sold Mrs. Blake $\frac{5}{6}$ doz. red roses, $1\frac{1}{2}$ doz. white roses, and $1\frac{1}{4}$ doz. pink roses. How many dozen roses did she sell in all?

You must change $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{5}{6}$ to fractions having the same denominator; hence you have to find a number that will exactly contain 2, 4, and 6. Since 12 is the smallest number that will contain each of these numbers, you choose 12 as the least common denominator.

Could you use 24 or 48 as the common denominator? Why is 12 better?

$$\begin{aligned} \tfrac{5}{6} &= \tfrac{10}{12} \\ 1\tfrac{1}{2} &= 1\tfrac{6}{12} \\ 1\tfrac{1}{4} &= 1\tfrac{3}{12} \\ \hline & \quad 3\tfrac{7}{12} \end{aligned}$$

2. Jane sold Mrs. Lee $1\frac{3}{4}$ doz. white asters and $1\frac{2}{3}$ doz. pink asters. How many dozen asters was that?

3. Ada bought $2\frac{1}{2}$ doz. zinnias, $\frac{1}{3}$ doz. dahlias, and $1\frac{3}{4}$ doz. salvias. How many dozen flowers was that?

Practice in Adding Fractions

Add:

1. $2\frac{2}{3}$	**2.** $9\frac{3}{4}$	**3.** $7\frac{1}{2}$	**4.** $8\frac{1}{3}$	**5.** $7\frac{1}{3}$	**6.** $8\frac{1}{2}$
$\frac{1}{4}$	$5\frac{1}{5}$	$4\frac{2}{3}$	$\frac{5}{6}$	$8\frac{3}{4}$	$3\frac{2}{3}$
$6\frac{7}{8}$	$2\frac{3}{10}$	$9\frac{3}{5}$	$1\frac{1}{8}$	$4\frac{5}{6}$	$6\frac{7}{8}$
7. $2\frac{2}{3}$	**8.** $9\frac{1}{2}$	**9.** $7\frac{1}{3}$	**10.** $7\frac{2}{3}$	**11.** $5\frac{1}{8}$	**12.** $3\frac{3}{4}$
$5\frac{1}{4}$	$5\frac{2}{3}$	$9\frac{5}{6}$	$4\frac{1}{6}$	$4\frac{3}{4}$	$8\frac{1}{5}$
$8\frac{2}{3}$	$\frac{7}{8}$	$\frac{5}{8}$	$6\frac{8}{9}$	$2\frac{3}{16}$	$1\frac{3}{10}$
13. $7\frac{3}{4}$	**14.** $9\frac{3}{8}$	**15.** $9\frac{2}{3}$	**16.** $5\frac{1}{2}$	**17.** $4\frac{1}{2}$	**18.** $4\frac{2}{3}$
$4\frac{1}{5}$	$1\frac{1}{3}$	$\frac{3}{4}$	$7\frac{3}{4}$	$\frac{5}{6}$	$9\frac{1}{8}$
$\frac{1}{4}$	$9\frac{7}{8}$	$1\frac{5}{6}$	$\frac{2}{3}$	$6\frac{1}{8}$	$8\frac{7}{12}$
$8\frac{4}{5}$	$5\frac{1}{2}$	$2\frac{1}{2}$	$8\frac{5}{6}$	$3\frac{2}{3}$	$8\frac{3}{4}$

REDUCING MIXED NUMBERS TO IMPROPER FRACTIONS

Anna has $\$6\frac{3}{4}$ and she wishes to change it all to *quarters*. How many *quarters* will she get for $\$6\frac{3}{4}$?

Since $1 = \frac{4}{4}$, then $6 = \frac{24}{4}$. Hence $6\frac{3}{4} = \frac{24}{4} + \frac{3}{4}$, or $\frac{27}{4}$.

A quick way to change $6\frac{3}{4}$ to $\frac{27}{4}$ is to multiply 6 by 4 and add 3, which gives 27. Then write 27 over 4, which gives $\frac{27}{4}$.

$$6\tfrac{3}{4} = \tfrac{27}{4}$$

To reduce a mixed number to an improper fraction, multiply the whole number by the denominator, add the numerator, and place the result over the denominator.

Reduce the following to improper fractions:

1. $6\frac{1}{2}$	**4.** $5\frac{1}{6}$	**7.** $2\frac{7}{8}$	**10.** $8\frac{3}{8}$	**13.** $5\frac{2}{3}$	**16.** $8\frac{5}{16}$
2. $4\frac{2}{3}$	**5.** $9\frac{5}{6}$	**8.** $8\frac{3}{5}$	**11.** $6\frac{2}{3}$	**14.** $9\frac{5}{12}$	**17.** $2\frac{7}{10}$
3. $5\frac{1}{5}$	**6.** $7\frac{3}{7}$	**9.** $4\frac{5}{6}$	**12.** $3\frac{4}{5}$	**15.** $7\frac{3}{10}$	**18.** $2\frac{25}{100}$

19. Edith is making silk roses. If each rose takes $\frac{1}{4}$ yd. of ribbon, how many roses can she make from $2\frac{3}{4}$ yd.?

Getting Ready to Subtract Fractions

Tell what numbers should be put in place of the stars:

1. \$6 equals \$5 and 4 quarters. Then $6 = 5\frac{*}{4}$.

2. $\$6\frac{1}{4}$ equals \$5 and * quarters. Then $6\frac{1}{4} = 5\frac{*}{4}$.

3. $\$5\frac{1}{2}$ equals \$4 and * quarters. Then $5\frac{1}{2} = 4\frac{*}{4}$.

4. $\$3\frac{1}{2}$ equals \$2 and * dimes. Then $3\frac{1}{2} = 2\frac{*}{10}$.

5. $5\frac{3}{4}$ in. equals 4 in. and * eighths. Then $5\frac{3}{4} = 4\frac{*}{8}$.

6. $7 = 6\frac{*}{2}$	**10.** $7\frac{3}{4} = 6\frac{*}{4}$	**14.** $7\frac{1}{2} = 6\frac{*}{8}$
7. $8 = 7\frac{*}{4}$	**11.** $9\frac{1}{5} = 8\frac{*}{5}$	**15.** $9\frac{3}{4} = 8\frac{*}{8}$
8. $5 = 4\frac{*}{8}$	**12.** $6\frac{1}{2} = 5\frac{*}{4}$	**16.** $3\frac{1}{4} = 2\frac{*}{16}$
9. $6 = 5\frac{*}{2}$	**13.** $3\frac{1}{2} = 2\frac{*}{8}$	**17.** $2\frac{7}{8} = 1\frac{*}{16}$

TIMING THE 50-YARD DASH

1. A boy 11 yr. old should run 50 yd. in $8\frac{3}{5}$ sec. It takes Arthur $9\frac{1}{10}$ sec. What part of a second longer does Arthur take than he should?

Since $\frac{1}{10}$ is smaller than $\frac{3}{5}$, you have to change $9\frac{1}{10}$ to $8\frac{11}{10}$ before you can subtract.

The difference is $\frac{1}{2}$, hence it takes Arthur $\frac{1}{2}$ sec. longer than the time allowed.

Check the work by going over it again.

$$9\tfrac{1}{10} = 8\tfrac{11}{10}$$
$$8\tfrac{3}{5} = 8\tfrac{6}{10}$$
$$\tfrac{5}{10} = \tfrac{1}{2}$$

2. A girl 12 yr. old should run 50 yd. in 9 sec. If a boy 12 yr. old is allowed $8\frac{1}{5}$ sec., how much more time is allowed a girl than a boy to run 50 yd.?

3. An 11-year old girl is allowed $9\frac{1}{5}$ sec. for the 50-yard dash. Ruth can do it in $8\frac{7}{10}$ sec. How much less than the time allowed does it take Ruth?

Practice in Subtracting Fractions

Subtract. Check the work by going over it again:

1. $4\frac{7}{8}$	6. 8	11. $8\frac{1}{6}$	16. $7\frac{2}{3}$	21. $6\frac{1}{2}$	26. $9\frac{3}{4}$
$2\frac{3}{8}$	$4\frac{5}{16}$	$5\frac{5}{6}$	$5\frac{1}{6}$	$3\frac{5}{6}$	$5\frac{1}{3}$
2. $5\frac{9}{10}$	7. $5\frac{1}{4}$	12. $4\frac{3}{16}$	17. $8\frac{1}{2}$	22. $8\frac{5}{12}$	27. $7\frac{2}{3}$
$4\frac{3}{10}$	$2\frac{3}{4}$	$1\frac{7}{16}$	$3\frac{5}{12}$	$6\frac{5}{6}$	$6\frac{3}{5}$
3. $5\frac{15}{16}$	8. $9\frac{1}{8}$	13. $4\frac{7}{8}$	18. $7\frac{3}{4}$	23. $7\frac{1}{3}$	28. $8\frac{2}{3}$
$2\frac{3}{16}$	$3\frac{5}{8}$	$4\frac{1}{2}$	$2\frac{7}{16}$	$1\frac{5}{6}$	$1\frac{3}{4}$
4. 7	9. $4\frac{1}{3}$	14. $8\frac{9}{10}$	19. $7\frac{3}{4}$	24. $6\frac{1}{2}$	29. $8\frac{3}{8}$
$5\frac{3}{4}$	$2\frac{2}{3}$	$4\frac{2}{5}$	$2\frac{7}{8}$	$5\frac{7}{10}$	$6\frac{1}{3}$
5. 9	10. $9\frac{1}{10}$	15. $5\frac{3}{8}$	20. $9\frac{1}{2}$	25. $5\frac{1}{2}$	30. $9\frac{3}{4}$
$4\frac{7}{8}$	$7\frac{7}{10}$	$3\frac{3}{16}$	$5\frac{15}{16}$	$4\frac{1}{3}$	$6\frac{2}{3}$

IMPROVEMENT TEST No. 2

Test 2 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
5484	7074	1537	7421	5612
629	9737	6106	598	341
8432	4461	9722	779	21
265	970	70	6444	8815
9075	994	8799	8870	2309
6115	7046	553	7208	6821
7236	8568	5838	917	6743
660	5139	6373	4454	97

Test 2 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
94213	49101	99752	32510
61904	41941	70864	13778
5.	**6.**	**7.**	**8.**
97059	88730	43730	84041
41298	59760	20475	63488
9.	**10.**	**11.**	**12.**
94831	98360	48364	57000
19254	58456	29075	20631

Test 2 C — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.	5.
4952	8673	7683	6972	2549
76	507	48	93	28

Test 2 D — Division (Time 4 min. after copying)

1. $29\overline{)15586}$ 2. $62\overline{)32064}$ 3. $47\overline{)42065}$

4. $88\overline{)54208}$ 5. $31\overline{)64418}$

USING FRACTIONS IN PROBLEMS

1. A party of girl scouts are hiking to a village 14 mi. away. When they have walked $7\frac{1}{4}$ mi., how many miles have they still to walk?

2. Amy's scout troop is hiking from Hightown to Cobham, a distance of $12\frac{1}{4}$ mi. At Glenville they see a sign, "To Cobham, $4\frac{1}{2}$ mi." How much of the distance have they covered by that time?

3. Olive and Cora hike $4\frac{3}{4}$ mi. to Blue River. They walk $1\frac{5}{8}$ mi. along the river and then reach home by walking $5\frac{1}{2}$ mi. along the road. How many miles in all do they hike?

4. May weighs 85 lb. Her baby sister weighs $16\frac{3}{16}$ lb. How much more does May weigh than the baby?

5. On a 5-hour automobile trip with his father, Dan kept count of the number of miles they drove each hour. His record was as follows: $23\frac{1}{2}$ mi., $21\frac{9}{10}$ mi., 25 mi., $26\frac{4}{5}$ mi., and $23\frac{7}{10}$ mi. How many miles did they drive?

6. John's father is $6\frac{1}{4}$ ft. tall, while John is $4\frac{2}{3}$ ft. tall. How much shorter is John than his father?

7. Hugh keeps chickens. On Monday he collected $2\frac{5}{12}$ doz. eggs; on Tuesday, $3\frac{1}{4}$ doz.; and on Wednesday, $3\frac{1}{2}$ doz. How many dozen eggs did he collect in all?

USING FRACTIONS IN THE STORE

1. Bert works in a store. When he sells anything he makes out a *sales slip* like the one shown below. On this slip "$\frac{3}{4}$ lb. butter @ .48" means that the butter costs 48¢ a pound. What does the 36 stand for?

In finding $\frac{3}{4}$ of 48¢, Bert does the work in his head, like this: $\frac{1}{4}$ of 48 = 12; hence $\frac{3}{4}$ of 48 = 3 × 12, or 36.

In finding $\frac{3}{4}$ of 51¢, however, Bert cannot do the work mentally because 51¢ is not exactly divisible by 4. Hence he first multiplies 51 by 3 and then divides the result by 4, thus:

$$\frac{3}{4}\times 51=\frac{3\times 51}{4}=\frac{153}{4}=38\tfrac{1}{4}$$

The answer is $38\frac{1}{4}$¢, which Bert calls 39¢.

THE ECONOMY MARKET

Date Jan. 3 19~

Sold by #15 Am't. Rec'd $5.00

¾ lb. butter	@ .48		36
¾ lb. coffee	@ .51		39
2½ lb. prunes	@ .17		43
2½ doz. eggs	@ .65	1	63
		2	81

In retail stores a fraction of a cent is usually counted as another cent.

2. When you find $\frac{3}{4}$ of 24¢, you are multiplying 24 by $\frac{3}{4}$ and you may write it like this: $\frac{3}{4}\times 24$. The sign × means " of " when written after a fraction.

3. Multiply a whole number by a fraction in this way:

If the whole number is exactly divisible by the denominator, first divide by the denominator and then multiply by the numerator. Otherwise first multiply by the numerator and then divide by the denominator.

By this rule you always multiply by the numerator and divide by the denominator. In certain cases you *divide first* and then multiply, while in other cases you *multiply first* and then divide.

MAKING OUT SALES SLIPS

Make out sales slips for the following and add each:

1. $\frac{3}{4}$ lb. Swiss cheese @ 60¢
$\frac{3}{8}$ lb. tea @ 64¢
$2\frac{1}{2}$ lb. butter @ 56¢

2. $1\frac{1}{2}$ doz. rolls @ 15¢
$\frac{2}{3}$ doz. sweet buns @ 25¢
$2\frac{3}{4}$ doz. cookies @ 18¢
1 loaf of bread @ 9¢

3. $2\frac{3}{4}$ lb. lamb chops @ 48¢
$2\frac{7}{8}$ lb. bacon @ 40¢
$6\frac{1}{4}$ lb. chicken @ 50¢

4. $\frac{5}{8}$ lb. cheese @ 30¢
$\frac{1}{4}$ lb. tea @ 70¢
$\frac{3}{4}$ lb. butter @ 50¢
$\frac{3}{4}$ doz. candles @ 65¢

5. Make out the sales slip that Rose receives when she buys: $3\frac{1}{2}$ yd. of gingham @ 59¢, $4\frac{3}{4}$ yd. of ribbon @ 22¢, $\frac{7}{8}$ yd. of velvet @ \$2.50, and 3 cards of buttons @ 15¢.

6. Mary bought $2\frac{1}{2}$ yd. of cotton crêpe @ 75¢, $\frac{3}{4}$ yd. of organdie @ 89¢, $\frac{1}{2}$ doz. buttons @ 28¢, and 2 spools of thread @ 10¢. Make out the sales slip.

Practice in Multiplying by a Fraction

Multiply the following mentally:

1. $\frac{3}{4}$ of 28
2. $\frac{2}{3}$ of 18
3. $\frac{7}{8}$ of 16
4. $\frac{2}{3} \times 45$
5. $\frac{3}{5} \times 35$
6. $\frac{5}{6} \times 30$
7. $\frac{5}{8} \times 40$
8. $\frac{3}{4} \times 160$
9. $\frac{3}{10} \times 150$

Multiply. Call a fraction of a cent another cent:

10. $\frac{3}{5}$ of \$.12
11. $\frac{2}{3}$ of \$.80
12. $\frac{3}{4}$ of \$.35
13. $\frac{5}{8}$ of \$.50
14. $\frac{7}{8}$ of \$.20
15. $\frac{4}{5}$ of \$.28
16. $1\frac{5}{6} \times$ \$.40
17. $5\frac{3}{4} \times$ \$.25
18. $2\frac{3}{8} \times$ \$.30
19. $2\frac{2}{3} \times$ \$.96
20. $6\frac{7}{8} \times$ \$.45
21. $6\frac{5}{8} \times$ \$.65
22. $3\frac{3}{8} \times$ \$.60
23. $5\frac{3}{4} \times$ \$.44
24. $2\frac{2}{3} \times$ \$.30
25. $8\frac{5}{12} \times$ \$.36
26. $7\frac{5}{12} \times$ \$.60
27. $9\frac{3}{16} \times$ \$.20

MULTIPLYING FRACTIONS BY WHOLE NUMBERS

1. How high is a pile of 9 boards if each is $\frac{5}{8}$ in. thick?

You know that 9×5 *books* $= 45$ *books*. In the same way, 9×5 *eighths* $= 45$ *eighths*, or $5\frac{5}{8}$, as shown at the right.

$$9 \times \frac{5}{8} = \frac{9 \times 5}{8} = \frac{45}{8} = 5\frac{5}{8}$$

To multiply a fraction by a whole number, multiply the numerator by the whole number and divide the result by the denominator.

Explain each step in the following problems:

2. Multiply $\frac{7}{8}$ by 12.

$$12 \times \frac{7}{8} = \frac{\overset{3}{\cancel{12}} \times 7}{\underset{2}{\cancel{8}}} = \frac{21}{2} = 10\frac{1}{2}$$

3. Multiply $18\frac{3}{4}$ by 7.

$$\begin{array}{r} 18\frac{3}{4} \\ 7 \\ \hline 5\frac{1}{4} \\ 126 \\ \hline 131\frac{1}{4} \end{array}$$

Exercises

1. It is $\frac{3}{4}$ mi. around a running track. How far does Bert run if he goes around the track 5 times?

Multiply mentally. Reduce the result, if possible:

2. $5 \times \frac{3}{8}$	**5.** $6 \times \frac{2}{5}$	**8.** $7 \times \frac{4}{5}$	**11.** $9 \times \frac{2}{3}$
3. $8 \times \frac{1}{2}$	**6.** $4 \times \frac{2}{3}$	**9.** $2 \times \frac{5}{8}$	**12.** $3 \times \frac{3}{5}$
4. $3 \times \frac{3}{8}$	**7.** $6 \times \frac{2}{3}$	**10.** $3 \times \frac{5}{8}$	**13.** $6 \times \frac{1}{6}$

Multiply the following. Cancel when possible:

14. $8 \times \frac{5}{16}$	**16.** $48 \times \frac{7}{12}$	**18.** $5 \times \frac{7}{10}$	**20.** $24 \times \frac{5}{6}$
15. $8 \times \frac{3}{4}$	**17.** $25 \times \frac{9}{10}$	**19.** $6 \times \frac{5}{12}$	**21.** $36 \times \frac{7}{8}$

Multiply the following mixed numbers:

22. $12 \times 6\frac{1}{2}$	**24.** $15 \times 16\frac{2}{3}$	**26.** $9 \times 12\frac{5}{6}$
23. $6 \times 33\frac{1}{3}$	**25.** $15 \times 37\frac{1}{2}$	**27.** $16 \times 35\frac{7}{12}$

MULTIPLYING A FRACTION BY A FRACTION

Review this rule, which you learned last year:

To multiply a fraction by a fraction, multiply the numerators together for a new numerator and the denominators together for a new denominator.

This rule may also be used to multiply mixed numbers if each mixed number is first reduced to an improper fraction.

Study the steps in working these problems:

1. Multiply $\frac{4}{9}$ by $\frac{1}{8}$.

$$\frac{1}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{9} = \frac{1 \times 1}{2 \times 9} = \frac{1}{18}$$

First cancel 4 from 8 and from 4, thinking "$8 \div 4 = 2$ and $4 \div 4 = 1$." Notice that the numerator of the result is 1×1, which equals 1.

2. Multiply $5\frac{5}{8}$ by $3\frac{1}{5}$.

$$\frac{\cancel{16}^{2}}{\cancel{5}_{1}} \times \frac{\cancel{45}^{9}}{\cancel{8}_{1}} = \frac{2 \times 9}{1 \times 1} = 18$$

Change $3\frac{1}{5}$ to $\frac{16}{5}$ and $5\frac{5}{8}$ to $\frac{45}{8}$. Cancel 5, thinking "$5 \div 5 = 1$ and $45 \div 5 = 9$." Then cancel 8, thinking "$8 \div 8 = 1$ and $16 \div 8 = 2$."

3. In ex. 1 above you could have done the work without canceling, thus: $\frac{1}{8} \times \frac{4}{9} = \frac{4}{72}$. Then you would have had to reduce $\frac{4}{72}$ to $\frac{1}{18}$, which is the same result you got above. By canceling you were saved the trouble of reducing the result to lowest terms.

When you cancel in multiplying fractions, you divide a numerator and a denominator by the same number.

Exercises

1. In copying a scout map John wishes to make each line $\frac{7}{10}$ as long as it is on the original map. How long should he make a line that is $\frac{15}{16}$ in. on the original map?

2. A cake recipe calls for $2\frac{2}{3}$ cups of sugar. How much sugar is needed for $1\frac{1}{4}$ times the recipe?

GRADED EXERCISES

Multiply. Check the work by going over it again:

Group			
A.	1. $\frac{3}{4}\times\frac{1}{4}$	2. $4\frac{2}{3}\times\frac{2}{3}$	3. $2\frac{1}{2}\times3\frac{1}{2}$
	4. $\frac{3}{8}\times\frac{3}{4}$	5. $4\frac{1}{3}\times2\frac{1}{2}$	6. $2\frac{1}{5}\times1\frac{1}{3}$
B.	7. $\frac{4}{5}\times1\frac{1}{2}$	8. $12\frac{1}{2}\times\frac{3}{5}$	9. $3\frac{1}{8}\times3\frac{2}{5}$
	10. $\frac{7}{12}\times4\frac{4}{5}$	11. $5\frac{3}{4}\times5\frac{3}{5}$	12. $2\frac{1}{2}\times2\frac{2}{3}$
C.	13. $\frac{5}{9}\times\frac{3}{4}$	14. $4\frac{1}{10}\times\frac{5}{6}$	15. $1\frac{1}{5}\times1\frac{7}{12}$
	16. $\frac{3}{10}\times\frac{5}{8}$	17. $\frac{4}{5}\times\frac{7}{8}$	18. $2\frac{1}{2}\times1\frac{3}{10}$
D.	19. $\frac{5}{9}\times7\frac{1}{2}$	20. $7\frac{1}{2}\times\frac{3}{10}$	21. $\frac{7}{8}\times2\frac{2}{5}$
	22. $\frac{9}{10}\times\frac{15}{16}$	23. $7\frac{1}{2}\times4\frac{3}{10}$	24. $5\frac{3}{5}\times1\frac{5}{12}$
E.	25. $\frac{1}{2}\times\frac{8}{9}$	26. $\frac{1}{8}\times13\frac{1}{3}$	27. $6\frac{2}{5}\times\frac{1}{8}$
	28. $\frac{1}{7}\times\frac{14}{15}$	29. $\frac{1}{6}\times2\frac{2}{5}$	30. $1\frac{1}{5}\times\frac{1}{2}$
F.	31. $\frac{1}{5}\times\frac{5}{6}$	32. $\frac{1}{8}\times1\frac{1}{3}$	33. $1\frac{3}{5}\times\frac{1}{16}$
	34. $\frac{1}{10}\times1\frac{1}{9}$	35. $2\frac{1}{2}\times\frac{1}{15}$	36. $3\frac{1}{5}\times\frac{1}{16}$
G.	37. $\frac{4}{9}\times18$	38. $\frac{5}{6}\times24$	39. $15\times\frac{3}{5}$
	40. $\frac{3}{5}\times25$	41. $\frac{5}{6}\times72$	42. $88\times\frac{7}{8}$
H.	43. $\frac{2}{5}\times\frac{15}{16}$	44. $3\frac{3}{4}\times\frac{2}{3}$	45. $2\frac{5}{8}\times1\frac{1}{3}$
J.	46. $\frac{8}{9}\times\frac{3}{8}$	47. $2\frac{2}{3}\times\frac{3}{16}$	48. $1\frac{1}{4}\times\frac{4}{15}$
K.	49. $5\frac{5}{8}\times3\frac{1}{5}$	50. $2\frac{2}{5}\times\frac{5}{6}$	51. $\frac{3}{5}\times3\frac{1}{3}$
L.	52. $\frac{9}{10}\times9\frac{1}{6}$	53. $3\frac{3}{10}\times4\frac{1}{6}$	54. $14\frac{2}{5}\times\frac{5}{16}$

NOTE. Each of the above groups represents a different difficulty.

DIVIDING FRACTIONS

1. In the division of fractions you must always *invert* the divisor. To invert means *to turn upside down.* Thus, $\frac{2}{3}$ inverted is $\frac{3}{2}$, while $\frac{5}{9}$ inverted is $\frac{9}{5}$. You may also *invert* a whole number. For example, to invert 3 (which equals $\frac{3}{1}$) you write it as $\frac{1}{3}$. If you invert $\frac{1}{9}$, you get $\frac{9}{1}$, or 9.

You have already learned the following rule:

To divide a fraction by a fraction, invert the divisor and multiply.

This rule may also be used for dividing mixed numbers or for dividing by a whole number. Be sure always to invert the divisor before multiplying.

2. A fast airplane flying at the rate of $2\frac{1}{2}$ mi. a minute will take how many minutes to travel 75 mi.?

You must divide 75 by $2\frac{1}{2}$. First change $2\frac{1}{2}$ to $\frac{5}{2}$. Next invert $\frac{5}{2}$ to $\frac{2}{5}$. Then multiply as shown here.

$$75 \div \frac{5}{2} = 75 \times \frac{2}{5} = \frac{\overset{15}{\cancel{75}} \times 2}{\underset{1}{\cancel{5}}} = 30$$

To check, go over the work or multiply 30 by $2\frac{1}{2}$, which gives 75.

3. In 3 yr. Lena gained $28\frac{1}{2}$ lb. in weight. Find her average gain each year.

You must divide $28\frac{1}{2}$ by 3. First change $28\frac{1}{2}$ to $\frac{57}{2}$. Since 3 is the divisor, invert it to $\frac{1}{3}$ and multiply as shown here.

$$\frac{57}{2} \div 3 = \frac{\overset{19}{\cancel{57}}}{2} \times \frac{1}{\underset{1}{\cancel{3}}} = \frac{19}{2} = 9\frac{1}{2}$$

You can prove that $9\frac{1}{2}$ is the correct result because $3 \times 9\frac{1}{2} = 28\frac{1}{2}$.

4. Mr. King's car travels 15 mi. on 1 gal. of gasoline. How many gallons will he use on a trip of $67\frac{1}{2}$ mi.?

5. Mrs. Lee bought $3\frac{1}{2}$ yd. of curtain goods. How many curtains, each $\frac{7}{8}$ yd. long, can she make from this?

GRADED EXERCISES

Divide. Check the work by going over it again:

Group			
A.	1. $\frac{1}{2} \div \frac{1}{3}$	2. $\frac{2}{5} \div \frac{1}{3}$	3. $\frac{2}{3} \div 3$
	4. $\frac{3}{8} \div 2$	5. $\frac{4}{5} \div 3$	6. $\frac{15}{16} \div 2$
	7. $\frac{3}{4} \div 2$	8. $\frac{5}{6} \div 2$	9. $3\frac{1}{2} \div 1\frac{2}{3}$
B.	10. $\frac{7}{8} \div \frac{3}{16}$	11. $\frac{3}{8} \div \frac{5}{16}$	12. $2\frac{4}{5} \div \frac{2}{3}$
C.	13. $\frac{2}{3} \div \frac{4}{5}$	14. $\frac{5}{6} \div \frac{2}{3}$	15. $\frac{11}{12} \div \frac{5}{6}$
D.	16. $\frac{6}{5} \div 9$	17. $\frac{9}{16} \div 1\frac{1}{5}$	18. $7\frac{1}{5} \div 24$
	19. $\frac{4}{5} \div 6$	20. $7\frac{1}{2} \div 3\frac{1}{3}$	21. $3\frac{3}{5} \div 27$
	22. $\frac{13}{16} \div \frac{5}{24}$	23. $2\frac{1}{4} \div 12$	24. $12\frac{3}{5} \div 18$
	25. $\frac{15}{16} \div 12$	26. $2\frac{2}{5} \div 16$	27. $11\frac{1}{4} \div 20$
E.	28. $\frac{3}{8} \div \frac{1}{4}$	29. $8 \div 3\frac{1}{5}$	30. $\frac{15}{16} \div 5$
	31. $\frac{8}{9} \div 2$	32. $8\frac{1}{3} \div 5$	33. $5 \div 7\frac{1}{2}$
	34. $\frac{9}{16} \div 3$	35. $5\frac{2}{5} \div 9$	36. $14\frac{2}{5} \div 12$
F.	37. $\frac{1}{10} \div \frac{4}{5}$	38. $\frac{1}{8} \div 1\frac{7}{8}$	39. $\frac{5}{8} \div 10$
	40. $\frac{2}{3} \div 8$	41. $1\frac{1}{3} \div 16$	42. $\frac{1}{3} \div 3\frac{1}{3}$
	43. $\frac{5}{6} \div 10$	44. $1\frac{3}{4} \div 14$	45. $2\frac{1}{2} \div 25$
G.	46. $\frac{7}{8} \div \frac{1}{16}$	47. $16 \div 2\frac{2}{3}$	48. $27 \div \frac{3}{8}$
H.	49. $\frac{3}{8} \div \frac{9}{16}$	50. $12\frac{1}{2} \div 9\frac{3}{8}$	51. $2\frac{1}{4} \div 3\frac{3}{8}$
J.	52. $\frac{5}{16} \div \frac{5}{8}$	53. $\frac{4}{5} \div 1\frac{3}{5}$	54. $2\frac{2}{3} \div 13\frac{1}{3}$
K.	55. $\frac{3}{4} \div \frac{3}{16}$	56. $4\frac{2}{3} \div 1\frac{5}{9}$	57. $12\frac{1}{2} \div \frac{5}{8}$
L.	58. $\frac{5}{16} \div \frac{5}{6}$	59. $3\frac{3}{5} \div 2\frac{1}{10}$	60. $1\frac{15}{16} \div 1\frac{1}{24}$

DECIMALS

Reading and Writing Decimals

A **decimal** is a short way of writing a fraction like $\frac{7}{10}$ or $\frac{15}{100}$ without writing the denominator. Thus, .7 means $\frac{7}{10}$ and is read *7 tenths*. .15 means $\frac{15}{100}$ and is read *15 hundredths*. .165 means $\frac{165}{1000}$ and is read *165 thousandths*. .6 is called a 1-place decimal; .24 is a 2-place decimal; and .236 is a 3-place decimal.

There are also decimals of four, five, and six places. Thus, .1029 = $\frac{1029}{10000}$ and is read *1029 ten-thousandths*. .178407 = $\frac{178407}{1000000}$ and is read *178,407 millionths*. To read a decimal, read it as a whole number and then give it the name of its last decimal place.

Memorize the names of the decimal places in the following chart, beginning at the decimal point.

Hundreds	Tens	Units	Decimal point (read "and")	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
4	6	2	.	3	0	6	8	4	9
WHOLE NUMBERS				DECIMALS					

The number 462.306849 is a **mixed decimal**; it is made up of a whole number and a decimal. Read it *462 and 306,849 millionths*, reading *and* at the decimal point.

In reading dollars and cents, however, the word *and* is often omitted. Thus, $2.46 may be read *2 dollars, 46 cents*.

In business, a number like 5.628 may be read *five-point-six-two-eight*. How is the decimal point read here?

READING DECIMALS

A zero may be placed before a decimal like .3 to call attention to the decimal point. Thus, 0.3 means .3.

Read these decimals:

1.	.7	0.9	.235	0.9160	.01032
2.	.08	0.08	.050	0.0003	.00008

Read each of these in two different ways:

3.	9.704	6.0085	29.23730	7.008975
4.	5.007	3.9816	30.05068	8.000001

How the Position of a Figure Affects Its Value

1. In the numbers at the right, the value of the 3 depends on its position in the number. Each time the 3 is moved 1 place to the left, its value becomes 10 times as great as before.

In 263 the 3 represents 3
In 237 the 3 represents 30
In 325 the 3 represents 300

2. Now study these decimals. Does the value of the 3 here also depend on its position? Since $10 \times .003 = .03$ and $10 \times .03 = .3$, you see that each time the 3 is moved 1 decimal place to the left, its value is multiplied by 10.

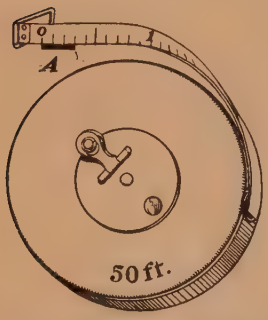
In .513 the 3 represents .003
In .431 the 3 represents .03
In .348 the 3 represents .3

If any figure of a number is moved 1 *place to the left, its value becomes* 10 *times as great as before.*

3. Write these numbers under one another in the order of their value. Keep the decimal points in line.

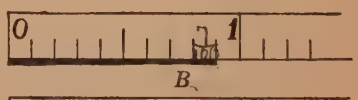
700 .7 .07 70 .007 7 .0007 7000

HOW FRANK'S FATHER MEASURES LAND

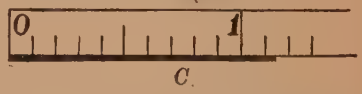


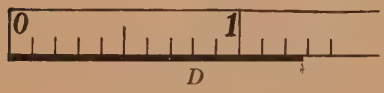
Frank's father is a surveyor who measures land. He has a steel tape on which each foot is divided into 100 equal parts. Hence each part is $\frac{1}{100}$ of a foot, or .01 ft., long. The shaded line A in the picture, which covers 3 small spaces, represents $\frac{3}{100}$ ft., or .03 ft. The length from 0 to 1, which covers 10 small spaces, represents .10 ft., or .1 ft. Hence the number 1 on the tape stands for .1 ft. and the number 2 stands for .2 ft., or .20 ft.

1. How many hundredths of a foot long is shaded line B?



Line C is a little longer than .11 ft. Measured to the *nearest hundredth* of a foot, we say that C is .11 ft. long because its length is nearer to .11 than to .12.





2. Is the shaded line D closer to .13 or to .12? Then how long is D, to the *nearest hundredth* of a foot?

3. If you measure only to the *nearest tenth* of a foot, line B is .1 ft. long. How long is line C, to the nearest tenth of a foot? How long is line D?

4. Express the following lengths to the nearest tenth of a foot: 5.29 ft., 7.21 ft., 4.38 ft., 6.42 ft.

MEASURING TO HUNDREDTHS OF A FOOT

It is very hard to measure accurately. If you measure land with a long tape, stretching the tape tightly, you may get a length of 86.23 ft. But if you let the tape sag a little, the length may be 86.29 ft. or even 86.32 ft. Land is measured to the *nearest hundredth* of a foot, if it is very valuable; otherwise it is measured to the *nearest tenth* of a foot or to the *nearest foot.*

It is very convenient to measure lengths to tenths and hundredths of a foot. Mary's height, measured in feet and inches, might be 5 ft. $3\frac{1}{2}$ in.; but with the foot divided into hundredths, it would be 5.29 ft. Which is shorter to write, 5.29 ft. or 5 ft. $3\frac{1}{2}$ in.?

Exercises

1. To express 8.48 ft. to the nearest tenth of a foot, Ada thinks "8.48 is nearer to 8.50 than to 8.40; hence 8.48 ft., to the nearest tenth, is 8.5 ft." Is Ada right?

2. Give each of these lengths to the nearest tenth of a foot: 7.32 ft., 8.29 ft., 3.17 ft., 4.12 ft., 5.09 ft., 3.01 ft.

3. Make a ruler of cardboard with the foot marked into tenths and hundredths. To do this, place the edge of the cardboard along the line at *C* on page 30 to get the right length for each space.

4. Find the length of your classroom, to the nearest tenth of a foot; to the nearest whole foot.

Measure the following to the nearest tenth of a foot; also measure each to the nearest hundredth of a foot:

5. The height of each pupil in your class.

6. The length and the width of your desk; of the door.

WHAT IS WRONG WITH THESE STATEMENTS?

Some of the numbers below need decimal points to make the statements sensible. Put the decimal points where they belong in the numbers that need them.

1. Kenneth is 11 yr. old and weighs 875 pounds.
2. Alice sold her ice skates to Jenny for $200.
3. A fast airplane traveled 135 mi. an hour.
4. John can run 50 yd. in 83 sec.
5. Mary is 10 yr. old and is 49 ft. tall.
6. Dick paid $25 for a bag of 10 marbles.
7. The flagpole in the school yard is 30 ft. high.
8. Mrs. Jones paid $48 a pound for a turkey.
9. Mr. Smith drives his car 216 mi. an hour.
10. Henry's mother bought him a suit for $1875 and a pair of tennis shoes for $139.
11. Tom is 12 yr. old and is 585 in. tall.

The decimal points below are in the wrong places. Put them where they belong to make the statements sensible.

12. Sam can walk a mile in 2.3 min.
13. Will rides his bicycle .93 mi. an hour.
14. Mr. James bought a new automobile for $12.35.
15. Nat is 11 yr. old and weighs 9.2 lb.
16. Arthur was paid $2.5 an hour for running errands
17. A fast train traveled 4.8 mi. per hour.
18. Edward paid $.01 for a circus ticket.
19. Janet paid $.065 a quart for ice cream.
20. Grace received $8.50 change from a dollar bill.

CHANGING THE NAME OF A DECIMAL

Every decimal has the same name as its *understood* denominator. Thus, every 2-place decimal, like .45, has the name *hundredths*, while every 3-place decimal, like .162, has the name *thousandths*. A decimal has the same name as its last decimal place. It is often convenient to change a decimal to one with a different name.

You can change the name of a decimal without changing its value by annexing or removing zeros at the right of it.

Thus, $.4 = .40$, or $.400$, because $\frac{4}{10} = \frac{40}{100} = \frac{400}{1000}$.

Also, $.530 = .53$ because $\frac{530}{1000} = \frac{53}{100}$.

If two decimals have different names, you can change them to decimals with the same name by annexing zeros at the right of one of them. Thus, .23 and .465 have different names; but if you change .23 to .230, you see that .230 and .465 now have the same name.

Exercises

1. *Change to hundredths:* .10 .70 .30 .80 .20 .50
2. *Change to thousandths:* .900 .6 .43 .500 .050 .800
3. *Change to hundredths:* .810 .700 .980 .300 .050
4. *Change to tenths:* .90 .80 .600 .500 .40

In each of the following, pick out all the tenths, then all the hundredths, then all the thousandths:

5. .14 .030 .7 .008 .050 .05
6. .9 .17 .60 .5 .130 .3

In each example, give both decimals the same name:

7. .21 and .209
8. .85 and .868
9. .81 and .9
10. .64 and .037
11. .02 and .018
12. .05 and .3185

COMPARING DECIMALS

In a race, an American airplane averaged 149.6 mi. an hour, while a French airplane averaged 149.57 mi. an hour. Which airplane went faster?

You have learned that you cannot compare 3 ft. and 37 in. unless you first give them the same name. If you change 3 ft. to 36 in., can you tell easily whether 36 in. or 37 in. is larger?

Likewise, in the above problem, you cannot compare the decimals .6 and .57 because they have different names. But if you change .6 to .60, so that both decimals have the name *hundredths*, you can easily compare .60 and .57. Since .60 is greater than .57, you see at once that .6 is greater than .57.

Hence the American airplane flew faster.

To compare two decimals, give them the same name by annexing zeros at the right of one of the decimals.

Exercises

In each problem, tell which decimal is larger:

1. .4 or .36	**5.** .89 or .900	**9.** .28 or .2652
2. .6 or .60	**6.** .45 or .448	**10.** .030 or .0281
3. .8 or .82	**7.** .27 or .273	**11.** .003 or .0028
4. .1 or .09	**8.** .02 or .009	**12.** .201 or .2009

13. James lives 1.3 mi. from school. Arthur lives 1.25 mi. from school. Who lives farther away?

14. Fred found that his plot in the school garden was 14.8 ft. long. Is this more or less than $14\frac{3}{4}$ ft.?

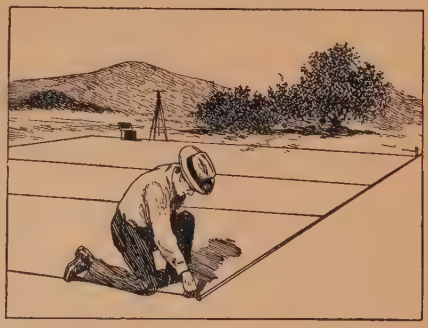
15. Express these decimals as thousandths and arrange them in the order of their size, with the largest on top:

.61 .8 .295 .791 .78 .82 .7

221

MEASURING VALUABLE LAND

A surveyor near our house measured three valuable lots that were side by side. The first lot measured 35.78 ft. in width; the second measured exactly 25 ft.; and the third measured exactly 25.3 ft. How wide were the three lots together?



Since the land was valuable, each lot was measured to the *nearest hundredth* of a foot. Thus, the first lot was 35.78 ft. wide. In saying that the second lot was exactly 25 ft. wide, the surveyor meant 25.00 ft.; likewise, by "exactly 25.3 ft.," he meant 25.30 ft.

You could add the decimals as in *A*, but it is easier to keep them in the correct columns if you annex zeros, as in *B*. *B* shows also, without further explanation, that all the measurements were made to the nearest hundredth of a foot, which it is important for us to know.

A	*B*
35.78	35.78
25.	25.00
25.3	25.30
86.08	86.08

The width of the three lots was 86.08 ft.

If the land were less valuable, it would be measured to the *nearest tenth* of a foot and the numbers would be written as in *C*. Still less valuable land is measured to the *nearest whole foot* and the results are written as in *D*.

C	*D*
35.8	36
25.0	25
25.3	25
86.1	86

When several similar things are measured, each measurement is made to the same number of decimal places; that is, each is made to tenths or each to hundredths or each to thousandths.

GRADED EXERCISES IN ADDING DECIMALS

Add the following and check the work:

1.
6.67
5.74
.45
8.36

2.
8.698
.845
8.050
5.333

3.
7.0625
7.9995
.8478
4.1607

4.
5.05625
4.37075
.56699
2.68527

5.
6.02
7.89
8.52
1.10
.12
4.73
2.89

6.
5.666
.711
9.375
7.166
1.426
.150
8.719

7.
3.9668
1.4400
2.3842
.0434
.7313
.8835
.2575

8.
2.00975
.57775
1.00001
7.43920
3.43456
.30299
2.11223

9.
6.28
3.2
.88
9.4

10.
3.02
9.875
5.8
7.333

11.
9.166
6.2
1.75
4.

12.
5.05
6.004
.56751
9.88272

Copy in columns and add. Then annex zeros to give each decimal the same number of places and add again. Do you get the same answer both ways?

13. 41.3, 56.98, 5.16, 2.333, 82.05

14. 23.006, 19, 4.28, 1.09, 5.4

15. 6.4, 2.07, 65, 9.555, 4.375

16. 18.05, .033, .75, 9.685, 21.5

17. 2.075, 13.95, 4.375, 5.0005

18. 9.5, 14, 24.65, .0525, 9.533

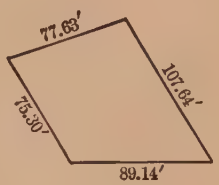
19. 3.0005, 98.125, 4.333, .127

20. 9.8333, 4.17, 9.005, 98, 56.2505

PROBLEMS

1. Last week Dick rode his new bicycle 16.4 mi. on Monday, 9.8 mi. on Tuesday, 11.5 mi. on Wednesday, 13 mi. on Thursday, and 17.2 mi. on Saturday. How many miles did he ride last week?

2. The lengths of the four sides of a lot are 77.63 ft., 107.64 ft., 89.14 ft., and 75.30 ft. Find the distance around the lot. This distance is called the *perimeter*.



3. On modern dairy farms milk is often weighed to tenths of a pound. In one week a prize cow gave these amounts of milk: 49.3 lb., 47.6 lb., 51.2 lb., 52.3 lb., 46.5 lb., 45.2 lb., and 48.2 lb. Find the total number of pounds for the week.

4. Three iron rods have these lengths: 25.753 ft., 18.654 ft., and 17.740 ft. If the rods are laid end to end in a straight line, what distance will they cover?

5. Sam ran 100 yd. each of 6 days. His records were 15.1 sec., 15.6 sec., 15.5 sec., 14.9 sec., 15.3 sec., and 14.8 sec. Find his average time for 100 yd.

6. Mr. Fox found that five very thin sheets of metal measured in thickness .0052 in., .0183 in., .0091 in., .0112 in., and .0060 in. How thick were the five sheets together?

7. The teacher asked Lucy to tell some uses of decimals in measuring. Lucy said that surveyors measure to tenths and hundredths of a foot, rain is measured to hundredths of an inch, stop watches measure time to tenths of a second, and speedometers measure distances to tenths of a mile. What other uses can you think of?

USING DECIMALS

1. Mr. Smith drives his car 22.8 mi. an hour. If the speed limit is 25 mi. an hour, how much faster could he drive and still keep within the limit?

$$\begin{array}{r} 25.0 \\ 22.8 \\ \hline 2.2 \end{array}$$

First write 25 as 25.0 and then subtract.
Check by seeing whether $2.2 + 22.8 = 25$.

2. Last year Alice could swim the length of the 60-foot pool in 41.3 sec. Now she does it in 39.9 sec. By how much has she lowered her time?

3. The speedometer on Mr. Hart's car read 987.6 mi. on Sunday and 1177.5 mi. at the end of the week. How many miles did he drive that week?

4. When the English unit of money, the pound, is worth \$4.8665, how much less than \$5 is it worth?

5. A surveyor found a bridge 27.325 ft. high at one end and 29.002 ft. high in the middle. How much higher was the middle of the bridge than the end?

6. Raymond found that the water in a pan was 2.7 in. deep before a storm and 5.25 in. deep after it. How many inches of rain fell during the storm?

7. This railroad time-table shows, in the left-hand column, how many miles each station is from Denver. How far is it from Denver to Georgetown? from Arvada to Golden? from Golden to Forks Creek? from Forks Creek to Lawson? from Lawson to Georgetown?

MILES		P.M.
0.0	Denver	3.15
7.6	Arvada	3.41
15.9	Golden	4.02
28.7	Forks Creek	5.00
44.2	Lawson	6.06
49.9	Georgetown	6.26

8. Bring to class a time-table of a railroad near your home. Make up and solve five problems about it like those in ex. 7.

GRADED EXERCISES IN SUBTRACTING DECIMALS

Subtract the following and check the work:

1.	2.006 1.423	**8.**	8.232 4.987	**15.**	.9141 .6597	**22.**	1.7507 .0605
2.	8.587 6.802	**9.**	.8345 .3360	**16.**	.1556 .0796	**23.**	6.6370 5.3599
3.	9.130 8.894	**10.**	.5404 .3564	**17.**	2.717 .709	**24.**	9.9158 1.4789
4.	.8809 .1815	**11.**	5.005 4.000	**18.**	8.232 .446	**25.**	9.0625 .3287
5.	925.0 159.5	**12.**	2.714 .021	**19.**	150.3 3.4	**26.**	.00066 .00018
6.	9.939 8.9	**13.**	4.57 1.711	**20.**	.3292 .187	**27.**	11.166 3.
7.	6. 5.875	**14.**	5. .166	**21.**	7. 1.505	**28.**	.808 .23064

Copy the following. Then subtract and check:

29. 2 − .27 **32.** 9 − 5.9876 **35.** .766 − .0065

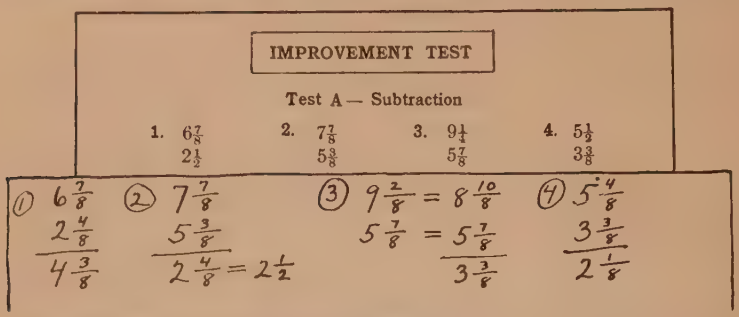
30. 6 − 1.42 **33.** 9.3 − 7.951 **36.** .532 − .1463

31. 9 − 1.3975 **34.** 1.72 − .887 **37.** 74 − 36.428

38. 84 − 49.062 **40.** 92.86 − 3.1875

39. 87.812 − 38.7974 **41.** 75.636 − 12.898

TAKING AN IMPROVEMENT TEST IN FRACTIONS


IMPROVEMENT TEST
Test A — Subtraction
1. $6\frac{7}{8}$ $2\frac{1}{2}$
2. $7\frac{7}{8}$ $5\frac{3}{8}$
3. $9\frac{1}{4}$ $5\frac{7}{8}$
4. $5\frac{1}{2}$ $3\frac{3}{8}$
① $6\frac{7}{8}$ $2\frac{4}{8}$ $4\frac{3}{8}$
② $7\frac{7}{8}$ $5\frac{3}{8}$ $2\frac{4}{8} = 2\frac{1}{2}$
③ $9\frac{2}{8} = 8\frac{10}{8}$ $5\frac{7}{8} = 5\frac{7}{8}$ $3\frac{3}{8}$
④ $5\frac{4}{8}$ $3\frac{3}{8}$ $2\frac{1}{8}$

While you are studying decimals and other topics, you can improve your skill with common fractions by taking Improvement Tests in fractions, such as those on page 41.

In taking an Improvement Test in the addition or subtraction of fractions, place the edge of a wide sheet of paper just under the top row of examples. Copy ex. 1 on your paper under ex. 1 of the test; but, instead of copying the example exactly as it stands in the test, change the fractions, as you copy, to fractions having a common denominator. When you have worked the first example, slide your paper slightly to the left and copy ex. 2 in the same way. Write the numbers of each example in a circle, as shown in the illustration above.

When you finish the top row of examples in a test, fold it under on your paper and do the next row along the folded edge. Then fold this row under, and so on.

Study the illustration at the top of this page and tell how each example there was changed as it was copied.

For a test in the multiplication or division of fractions, first copy all the examples on paper, allowing sufficient space between them for working them.

IMPROVEMENT TEST No. 3

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Test 3 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$8\frac{3}{4}$	$9\frac{7}{8}$	$7\frac{1}{2}$	$4\frac{1}{2}$	$7\frac{1}{5}$
$2\frac{1}{8}$	4	$8\frac{2}{3}$	6	4
5	$2\frac{1}{4}$	$5\frac{5}{6}$	$3\frac{2}{3}$	$2\frac{1}{2}$
$8\frac{9}{16}$	$3\frac{1}{2}$	$1\frac{1}{6}$	$9\frac{3}{4}$	$6\frac{7}{10}$

Test 3 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$7\frac{5}{8}$	$3\frac{7}{8}$	6	$7\frac{2}{5}$
$4\frac{1}{8}$	$1\frac{1}{2}$	$3\frac{3}{8}$	$2\frac{1}{2}$
5.	**6.**	**7.**	**8.**
$9\frac{1}{4}$	$8\frac{1}{2}$	7	$2\frac{2}{3}$
$1\frac{5}{8}$	$6\frac{1}{3}$	$6\frac{3}{16}$	$1\frac{1}{6}$
9.	**10.**	**11.**	**12.**
$1\frac{1}{2}$	$9\frac{1}{3}$	$8\frac{3}{4}$	$6\frac{4}{5}$
$\frac{3}{10}$	$5\frac{3}{4}$	$2\frac{3}{5}$	$4\frac{7}{10}$

Test 3 C — Multiplication (Time 3 min. after copying)

1. $\frac{3}{4}$ of 36	**2.** $\frac{5}{8} \times \frac{1}{10}$	**3.** $\frac{3}{5} \times 3\frac{1}{6}$
4. $\frac{2}{3}$ of 10	**5.** $1\frac{1}{2} \times \frac{14}{15}$	**6.** $2\frac{5}{8} \times 16$
7. $\frac{5}{6}$ of 14	**8.** $6\frac{3}{10} \times \frac{2}{3}$	**9.** $3\frac{1}{5} \times 3\frac{3}{4}$

Test 3 D — Division (Time 3 min. after copying)

1. $\frac{3}{4} \div 3$	**2.** $9 \div \frac{3}{16}$	**3.** $2\frac{1}{4} \div 12$
4. $\frac{8}{9} \div 4$	**5.** $\frac{3}{8} \div \frac{5}{12}$	**6.** $18 \div 2\frac{2}{3}$
7. $\frac{1}{4} \div \frac{1}{6}$	**8.** $1\frac{1}{2} \div \frac{9}{16}$	**9.** $4\frac{1}{5} \div 8\frac{2}{5}$

NOTE. In tests in the addition or subtraction of fractions, the time allowed includes the time required for copying each example in the common denominator form. In tests in the multiplication or division of fractions, the given time does not include time for copying.

CHANGING DECIMALS TO COMMON FRACTIONS

The Weather Bureau reported that .75 in. of rain fell yesterday in Bedford. Write .75 in. another way.

Since $.75 = \frac{75}{100} = \frac{3}{4}$, you see that .75 in. is equal to $\frac{3}{4}$ in.
In the same way, 6.75 may be written as $6\frac{3}{4}$.

To change a decimal to a common fraction, omit the decimal point and write the decimal over its understood denominator. Then reduce the fraction to lowest terms.

Thus, $.375 = \frac{375}{1000} = \frac{3}{8}$ and $.33\frac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{33\frac{1}{3} \times 3}{100 \times 3} = \frac{1}{3}$.

The number of zeros in the understood denominator is the same as the number of places in the decimal. Since .625 has 3 decimal places, its understood denominator has 3 zeros and $.625 = \frac{625}{1000}$.

Change to common fractions or to mixed numbers:

1.	.25	.50	$.16\frac{2}{3}$	6.50	$1.37\frac{1}{2}$
2.	.65	$.62\frac{1}{2}$	$.87\frac{1}{2}$	.800	$8.62\frac{1}{2}$
3.	.80	$.83\frac{1}{3}$	$.66\frac{2}{3}$	.025	$3.12\frac{1}{2}$

Frequently Used Decimals

Memorize the following because they are often used:

$.25 = \frac{1}{4}$	$.12\frac{1}{2} = \frac{1}{8}$	$.33\frac{1}{3} = \frac{1}{3}$
$.50 = \frac{1}{2}$	$.37\frac{1}{2} = \frac{3}{8}$	$.66\frac{2}{3} = \frac{2}{3}$
$.75 = \frac{3}{4}$	$.62\frac{1}{2} = \frac{5}{8}$	$.16\frac{2}{3} = \frac{1}{6}$
$1.00 = 1$	$.87\frac{1}{2} = \frac{7}{8}$	$.83\frac{1}{3} = \frac{5}{6}$

Change mentally to fractions or to mixed numbers:

1.	.25	1.75	$.16\frac{2}{3}$	$6.87\frac{1}{2}$	$2.66\frac{2}{3}$
2.	$.62\frac{1}{2}$	3.50	$.37\frac{1}{2}$	$1.33\frac{1}{3}$	$5.12\frac{1}{2}$

MULTIPLYING DECIMALS

1. Edna is going to ride 9.4 mi. on a train. If the fare is \$.036 a mile, how much will her ticket cost?

To multiply .036 by 9.4, use this rule:

To multiply decimals, first multiply as with whole numbers. Then, beginning at the right, point off as many decimal places in the product as there are in the multiplier and the multiplicand together.

Since 9.4 has 1 decimal place and .036 has 3 places, point off 4 places in the product, which gives .3384.

\$.3384 may be written \$.34, which is correct to the nearest cent. Edna's ticket will cost \$.34.

To prove that .3384 is the right product, change .036 to $\frac{36}{1000}$ and 9.4 to $9\frac{4}{10}$, or $\frac{94}{10}$. Then multiply as shown below:

$$\frac{94}{10} \times \frac{36}{1000} = \frac{3384}{10000} = .3384$$

.036
9.4
144
324
.3384

Study the following problems:

A.	*B.*	*C.*	*D.*
1.95	.23	.015	.36
46	.35	.05	500
11 70	115	.00075	180.00
78 0	69		
89.70	.0805		

2. In *A*, how many decimal places are there in 1.95 and 46 together? how many in the answer?

3. In *B*, .23 has 2 places and .35 has 2 places; hence you must point off 4 decimal places in the answer. To do this, write a 0 before 805, making .0805.

4. In *C*, why do you write three 0's in the answer?

5. In *D*, you get 180.00 as answer. Why can you cross out the last two 0's in this answer?

ARE AMERICANS FOND OF SWEETS?

1. During 1913 the people of the United States used an average of 85.32 lb. of sugar per person. How many pounds of sugar did an average family of 7 use that year? If the sugar cost $.055 a pound, how much did the family of 7 pay for sugar for the year?

Find the result correct to the nearest cent.

2. During 1923 the people of this country used an average of 104.24 lb. of sugar per person. How many pounds did a family of 7 use? At an average of $.101 a pound, how much did the family spend for sugar in 1923?

3. In 1925 the average price of sugar dropped to $.072 a pound. Work ex. 2 again, using $.072 instead of $.101 as the price per pound.

4. How much more did the family of 7 spend for sugar in 1923 than in 1913? in 1925 than in 1913?

Some Other Uses of Decimals

1. Last year it cost Mr. Ball $.094 a mile to drive to and from his office each day, the round trip being 14.2 mi. How much, to the nearest cent, did each day's trip cost?

2. This year Mr. Ball has a larger car and it costs an average of $.112 a mile to run it. How much does each day's trip of 14.2 mi. cost now?

3. Last winter a town in Colorado had the following snowfalls: October, 9.9 in.; November, 14.0 in.; December, 79.3 in.; January, 23.9 in.; February, 20.2 in.; March, 63.2 in.; April, 16.8 in. How much snow fell last winter in that town?

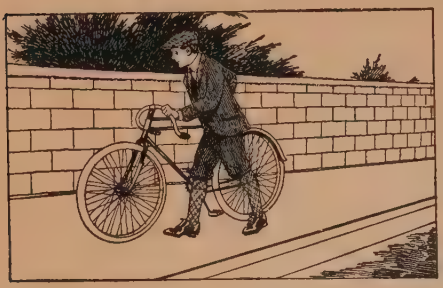
GRADED EXERCISES IN MULTIPLYING DECIMALS

Multiply. Check the work by going over it again. Practice the hard groups until you make no mistakes.

Group				
A.	**1.** 1.38 × 76	**2.** 653 × .38	**3.** 8.72 × .8	**4.** 7.6 × .21
	5. 9.82 × 7.2	**6.** 5.61 × .7	**7.** .903 × 54	**8.** .29 × 1.6
B.	**9.** 54.6 × .09	**10.** 116 × .006	**11.** 350 × .002	**12.** 5.74 × .06
C.	**13.** .017 × 7	**14.** .079 × 3	**15.** .046 × 5	**16.** .038 × 9
D.	**17.** .64 × .12	**18.** .006 × .4	**19.** .048 × .3	**20.** .142 × .05
	21. .02 × .03	**22.** .052 × .14	**23.** .607 × .09	**24.** .141 × .13
	25. .006 × .007	**26.** .076 × .56	**27.** .0009 × .05	**28.** 3.29 × .018
E.	**29.** 300 × .03	**30.** .54 × 800	**31.** .58 × 600	**32.** 5.8 × 5000
F.	**33.** .900 × .29	**34.** .706 × .300	**35.** .9000 × .063	**36.** .305 × .700

Note. The examples are arranged in groups according to difficulty.

MEASURING DISTANCES WITH A BICYCLE



1. Will finds that the distance around his bicycle wheel, which is called its **circumference,** is 7.13 ft. This bicycle travels 7.13 ft. each time the wheel goes around once. To find the length of a new sidewalk, Will pushes his bicycle from one end of the walk to the other and finds that the wheel goes around 9 times. How long is the walk?

2. Will also measures the length of the school playground with his bicycle. The wheel goes around $28\frac{1}{2}$ times. Change $28\frac{1}{2}$ to 28.5 and find, to the nearest hundredth of a foot, how long the playground is. Use the circumference of the wheel as given in ex. 1.

3. Find, to the nearest hundredth of a foot, the width of the playground if the wheel goes around $16\frac{1}{4}$ times in crossing it. Change $16\frac{1}{4}$ to 16.25 before multiplying.

4. How far does Will's bicycle travel when the wheel goes around 83 times? $102\frac{1}{2}$ times? $99\frac{1}{4}$ times?

5. Edna's bicycle wheel has a circumference of 6.67 ft. How far, to the nearest hundredth of a foot, does it travel when the wheel goes around 93 times? $42\frac{1}{2}$ times? $62\frac{1}{4}$ times? $18\frac{1}{2}$ times?

6. Harold has a big hoop with a circumference of 20.4 ft. If the hoop goes around 47 times while Harold rolls it down a hill, how long is the hill?

PUT THE DECIMAL POINT IN THE RIGHT PLACE

Place the decimal point mentally in each product. Read the results like this: "eight-point-four."

1. 12 × .7 = 84	**2.** .7 × .9 = 63	**3.** .41 × .6 = 246	**4.** 6.7 × 5.8 = 3886
5. .42 × 2.3 = 966	**6.** .37 × .57 = 2109	**7.** .127 × 1.4 = 1778	**8.** .25 × .24 = 600
9. 49 × .06 = 294	**10.** 219 × .007 = 1533	**11.** 467 × .09 = 4203	**12.** 54.2 × .08 = 4336
13. .08 × 4 = 32	**14.** .09 × 7 = 63	**15.** .023 × 5 = 115	**16.** .066 × 8 = 528
17. .04 × 2 = 8	**18.** .06 × .09 = 54	**19.** 4.31 × .02 = 862	**20.** .049 × .03 = 147
21. .06 × .2 = 12	**22.** .25 × .26 = 650	**23.** .176 × .4 = 704	**24.** .027 × 3.7 = 999
25. 300 × .18 = 5400	**26.** 7000 × .006 = 42000	**27.** .456 × 50 = 22800	**28.** .49 × 400 = 19600
29. .270 × 89 = 24030	**30.** .026 × .40 = 1040	**31.** .705 × .80 = 56400	**32.** 5.63 × .900 = 506700

MULTIPLYING BY 10, 100, AND 1000

Our school is having a picnic at Echo Lake. Each fare is $1.25. How much will it cost if 10 children go? if 100 children go? if 1000 children go?

A	*B*	*C*
$1.25	$1.25	$1.25
10	100	1000
$12.50	$125.00	$1250.00

In *A*, you can get the answer $12.50 quickly by moving the decimal point in $1.25 one place to the right.

In *B*, you can get $125.00 quickly by moving the decimal point in $1.25 two places to the right.

In *C*, you can get $1250.00 by moving the decimal point in $1.25 three places to the right; but this time you must also insert a 0 before the decimal point because $1.25 has only two decimal places.

To multiply a number by 10, 100, *or* 1000, *move the decimal point as many places to the right as there are zeros in the multiplier.*

Exercises

Multiply mentally by 10 *and also by* 100:

1.	$6.43	$.58	$15.00	$19.	.8
2.	$2.85	$.27	$12.84	$275.	.09
3.	$1.54	$.42	$12.625	$158.	5.9

Multiply mentally by 1000 *and also by* 10:

4.	9.114	$6.125	$8.35	6.3	7
5.	8.206	$8.375	$4.26	2.4	30

6. How much do 10 baseballs cost at $.65 each?

7. How much do 10 yd. of silk cost at $1.25 a yard?

8. How much do 100 tickets cost at $.15 each?

DIVIDING BY 10, 100, AND 1000

1. Ruth saved $14.60 in 10 wk. How much, on the average, did she save each week?

Since you multiply by 10 by moving the decimal point one place to the right, you divide by 10 by moving the decimal point one place to the left.

$14.60 ÷ 10 = $1.46

2. If 10 sheets of tinfoil are .14 in. thick, how thick is each sheet?

To divide by 10, place a zero before .14 and move the decimal point one place to the left.

.14 ÷ 10 = .014

3. Explain how each number below is divided by 100.

$135 \div 100 = 1.35$ $\$15.00 \div 100 = \$.15$ $1.3 \div 100 = .013$

In a whole number, like 135, the decimal point is understood to be at the right of the units' place.

To divide by 10, 100, *or* 1000, *move the decimal point as many places to the left as there are zeros in the divisor.*

Exercises

Divide the following mentally by 10:

1.	$15.00	$5.50	29.4	50	.8
2.	$16.20	$4.00	18.6	83	.26

Divide the following mentally by 100:

3.	$198	$23.00	61.2	$5	6.8
4.	$210	$96.00	40.3	$8	7.1

Divide mentally by 1000 *and also divide by* 10:

5.	$5000	$780	143.1	$90	6

6. If 10 hats cost $27.00, how much does 1 hat cost?

7. Jack sold 100 tickets for the school play for $125.00. How much did he get for each ticket?

BUYING BY THE HUNDRED

Meat, cotton, and many other things are sold by the *hundredweight*, which equals 100 lb. and is written "cwt." Express and freight rates are also given per 100 pounds. Certain kinds of roofing are sold by the 100 square feet, while water and gas are often sold by the 100 cubic feet.

Exercises

1. If chicken feed costs \$3.60 per 100 lb., how much must George pay for 35 lb. of it? for 165 lb.?

If 100 lb. cost \$3.60, then 35 lb. cost $\frac{35}{100}$ of \$3.60.
Since $.35 \times \$3.60 = \$1.26$, George pays \$1.26 for 35 lb.
Similarly, for 165 lb., George pays $1.65 \times \$3.60$, or \$5.94.

2. If the express rate is \$2.28 per hundredweight, how much does it cost to ship a trunk weighing 225 lb.?

3. How much must John pay for the freight on a desk weighing 185 lb., if the rate is \$1.26 per hundredweight?

4. If meat scraps cost \$3.86 per hundredweight, how much does Henry pay for 45 lb. for his chickens?

5. How many hundredweight are there in 216 lb.? in 869 lb.? in 4260 lb.? in 1096 lb.? in 86 lb.?

6. In buying hardwood flooring, we add $\frac{1}{3}$ of the actual floor surface to allow for matching. How much does the maple flooring cost for a room $22\frac{1}{2}$ ft. long and 18 ft. wide at \$10.90 per 100 sq. ft.?

7. How much does a metal ceiling 24 ft. wide and 36 ft. long cost at \$6.98 per 100 sq. ft.?

8. At 32¢ per 100 cu. ft., what is the cost of the water needed to fill a swimming tank 60 ft. long and 20 ft. wide to a depth of $4\frac{1}{4}$ ft.?

IMPROVEMENT TEST No. 4

Test 4 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
6926	7983	5778	4859	8646
4645	6909	8663	4609	6716
9098	5782	6875	8597	3557
6833	7597	7248	5754	7455
6.	**7.**	**8.**	**9.**	**10.**
3145	1986	9171	6061	9375
3384	8097	7878	9478	4996
2934	2223	2919	6511	7474
8466	2346	2844	7985	6539

Test 4 B — Subtraction (Time 3 min.)

1.	2.	3.	4.	5.
6921	7032	6403	8711	9206
5474	2523	4520	2066	7913
6.	**7.**	**8.**	**9.**	**10.**
8679	4343	9644	8573	5502
3699	2569	8985	1768	2974
11.	**12.**	**13.**	**14.**	**15.**
5301	6297	8225	7688	8914
1333	1618	5468	4979	1218

Test 4 C — Addition (Time 4 min.)

1.	2.	3.	4.	5.
5938	7649	9523	1895	8658
6575	4639	1449	367	6261
696	380	3685	6668	7759
8234	2029	8750	851	6808
7743	873	1372	7119	443
888	9956	354	507	7891
6976	5769	885	9292	4471
4125	7643	9528	6748	529

DIVIDING A DECIMAL BY A WHOLE NUMBER

1. Mr. Dodd drove a truck 209 mi. on 25 gal. of gasoline. How many miles did he average per gallon?

Dividing as in *A*, you get $8\frac{9}{25}$. To avoid the remainder, write 209 as 209.00 and divide as in *B*. The decimal point in 8.36 comes directly above the decimal point in 209.00. 8.36 is equal to $8\frac{9}{25}$ because $.36 = \frac{36}{100} = \frac{9}{25}$.

Mr. Dodd averaged 8.36 mi. per gallon.

A. $8\frac{9}{25}$

```
25)209
   200
     9
```

B. 8.36

```
25)209.00
   200
    90
    75
   150
   150
```

2. Mr. King drove a truck 198.4 mi. on 23 gal. of gasoline. How many miles per gallon was that?

You could write the quotient as $8.6\frac{6}{23}$; but it is better to annex two 0's after 198.4, making 198.400, as in *D*. This gives 8.626 as the answer, to three decimal places.

To write the answer correct to the nearest hundredth, change 8.626 to 8.63. To find it to the nearest tenth, you would carry the quotient only to two decimal places. This would give 8.62, which is 8.6 correct to the nearest tenth.

Mr. King averaged 8.63 mi. per gallon.

C. $8.6\frac{6}{23}$

```
23)198.4
   184
   144
   138
     6
```

D. 8.626

```
23)198.400
   184
   144
   138
    60
    46
   140
   138
     2
```

To divide a decimal by a whole number, place the decimal point in the quotient above the decimal point in the dividend. If there is a remainder, annex zeros to the dividend and continue to divide.

Carry the quotient to three decimal places for a result correct to the nearest hundredth, or to two decimal places for a result correct to the nearest tenth.

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GRADED EXERCISES

Divide. If there is a remainder, annex 0's after the dividend and continue to divide until the quotient is exact.

GROUP

A. **1.** $51\overline{)166.26}$ **2.** $92\overline{)3256.8}$ **3.** $45\overline{)80.55}$

4. $56\overline{)255.92}$ **5.** $33\overline{)1735.8}$ **6.** $79\overline{)291.51}$

7. $67\overline{)240.53}$ **8.** $81\overline{)4187.7}$ **9.** $62\overline{)452.6}$

B. **10.** $28\overline{)217}$ **11.** $52\overline{)429}$ **12.** $64\overline{)400}$

13. $24\overline{)114}$ **14.** $25\overline{)93}$ **15.** $96\overline{)60}$

16. $88\overline{)77}$ **17.** $64\overline{)56}$ **18.** $72\overline{)27}$

19. $48\overline{)42}$ **20.** $40\overline{)25}$ **21.** $54\overline{)5157}$

22. $22\overline{)1595}$ **23.** $74\overline{)6845}$ **24.** $42\overline{)3129}$

C. **25.** $45\overline{)431.1}$ **26.** $36\overline{)343.8}$ **27.** $56\overline{)142.8}$

Divide. Carry the quotient to two decimal places; then write the result correct to the nearest tenth:

D. **28.** $56\overline{)544}$ **29.** $32\overline{)265}$ **30.** $85\overline{)798}$

31. $73\overline{)444}$ **32.** $91\overline{)835}$ **33.** $24\overline{)175}$

34. $57\overline{)275}$ **35.** $73\overline{)305}$ **36.** $48\overline{)236}$

37. $92\overline{)769}$ **38.** $55\overline{)546.1}$ **39.** $61\overline{)239.1}$

40. $73\overline{)359.2}$ **41.** $92\overline{)269.6}$ **42.** $29\overline{)265.1}$

43. $37\overline{)127.4}$ **44.** $56\overline{)358.9}$ **45.** $78\overline{)465.6}$

Divide. Carry the quotient to three decimal places; then write the result correct to the nearest hundredth:

E. **46.** $22\overline{)119}$ **47.** $74\overline{)141}$ **48.** $84\overline{)196}$

49. $69\overline{)248}$ **50.** $85\overline{)668}$ **51.** $43\overline{)208}$

52. $36\overline{)255}$ **53.** $23\overline{)124}$ **54.** $66\overline{)158.9}$

55. $71\overline{)262.6}$ **56.** $53\overline{)222.1}$ **57.** $79\overline{)377.8}$

PROBLEMS IN DIVISION OF DECIMALS

1. Charlotte gained 34.5 lb. in 3 yr. What was the average number of pounds she gained per year?

2. On Mr. Owen's farm a cow gave 132.3 lb. of milk in 1 wk. How many pounds per day was that?

3. Fred wants to buy a scout uniform costing $9.45. If he works 35 days, how much money will he have to save each day in order to save $9.45?

4. During August Mr. Sage used 68 gal. of gasoline, which cost him $15.64 in all. Find the average price per gallon of the gasoline.

5. Mr. Jackson earns $55 a week, working 44 hr. How much does he earn per hour?

6. Mr. Brant earns $60 a week, working 44 hr. Find, to the nearest cent, how much he earns per hour.

7. It takes a train 45 min. to go from Barton to Scoville. The distance is 31 mi. Find, to the nearest tenth of a mile, how far the train goes per minute.

8. A truck used 14 gal. of gasoline for a trip of 85.6 mi. Find, to the nearest tenth of a mile, how many miles it traveled on 1 gal. of gasoline.

9. On another trip the same truck went 148 mi. on 23 gal. of gasoline. Did it make as many miles per gallon as it did in ex. 8?

10. For $150 the Manor School hires a steamboat to take 602 pupils on an excursion. Find, to the nearest cent, each child's share of the expense.

11. If Irene's father goes 22 mi. an hour in his automobile, how long will a trip of 118.8 mi. take him?

CHANGING FRACTIONS TO DECIMALS

Guy runs an adding machine in a store. On it he can add whole numbers and decimals; but there are no keys for adding fractions. To add $15\frac{3}{8}$ and $53\frac{1}{2}$, he changes the fractions to decimals and adds on the machine as in *A*. The sum is 68.875. To see if Guy is right, Ted adds the fractions in pencil as in *B*, getting $68\frac{7}{8}$. Does $68.875 = 68\frac{7}{8}$?

A.

$$\begin{array}{r} 15.375 \\ 53.5 \\ \hline 68.875 \end{array}$$

B.

$$\begin{array}{r} 15\frac{3}{8} = 15\frac{3}{8} \\ 53\frac{1}{2} = 53\frac{4}{8} \\ \hline 68\frac{7}{8} \end{array}$$

To change $\frac{3}{8}$ to a decimal Guy writes 3 as 3.000 and divides by 8, as shown at the right.

$$\begin{array}{r|l} 8 & 3.000 \\ \hline & .375 \end{array}$$

To change a common fraction to a decimal, write a decimal point after the numerator, annex zeros, and divide by the denominator.

Exercises

Change to exact decimals:

1. $\frac{1}{4}$ $\frac{1}{5}$ $\frac{7}{20}$ $\frac{2}{5}$ $\frac{17}{50}$ $\frac{3}{20}$ $\frac{8}{25}$

2. $\frac{5}{8}$ $\frac{3}{4}$ $\frac{23}{25}$ $\frac{3}{5}$ $\frac{3}{25}$ $\frac{19}{20}$ $\frac{49}{50}$

3. $\frac{9}{25}$ $\frac{1}{8}$ $\frac{43}{125}$ $\frac{7}{8}$ $\frac{113}{250}$ $\frac{3}{16}$ $\frac{5}{16}$

Add by changing the fractions to decimals as Guy did. Then add the fractions as Ted did. Are the answers equal?

4. $2\frac{1}{4}$ / $5\frac{1}{2}$

5. $6\frac{1}{8}$ / $2\frac{3}{4}$

6. $3\frac{1}{5}$ / $2\frac{7}{10}$

7. $16\frac{7}{8}$ / $22\frac{3}{4}$

8. $12\frac{5}{8}$ / $3\frac{3}{4}$

CHANGING FRACTIONS TO INCOMPLETE DECIMALS

1. Change $\frac{5}{6}$ to a decimal.

In changing $\frac{5}{6}$ to a decimal, you find that the quotient never comes out exact, no matter how many 0's you annex. In such a case the resulting decimal is called an **incomplete decimal.** It is usually sufficient to carry the division to three or four decimal places. Thus, in the case of $\frac{5}{6}$, call the answer .833 or .8333. To show that the exact value is a little more than .83 you may write it as .83+.

$$\begin{array}{r} .8333\frac{1}{3} \\ 6\overline{)5.0000} \\ \end{array}$$

2. Change $\frac{19}{43}$ to a decimal, to the nearest thousandth.

Carry the division to four decimal places, which gives .4418. Since the fourth figure is more than 5, make the third figure 1 larger. Thus, .4418 is .442 to the *nearest thousandth.* When the fourth figure is less than 5, drop it. Thus, .4413 would be .441 to the nearest thousandth.

If you had need for an answer correct only to the *nearest hundredth,* then you would carry the division to three decimal places, which would give .441. Writing .441 correct to the nearest hundredth, you get .44.

$$\begin{array}{r} .4418 \\ 43\overline{)19.0000} \\ 172 \\ \hline 180 \\ 172 \\ \hline 80 \\ 43 \\ \hline 370 \\ 344 \\ \hline 26 \end{array}$$

Exercises

Change to decimals, correct to the nearest hundredth:

1.	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{4}{9}$	$\frac{19}{21}$	$\frac{5}{11}$	$\frac{7}{9}$	$\frac{2}{13}$
2.	$\frac{4}{7}$	$\frac{7}{11}$	$\frac{1}{7}$	$\frac{9}{11}$	$\frac{2}{19}$	$\frac{6}{7}$	$\frac{2}{17}$
3.	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{3}{7}$	$\frac{5}{63}$	$\frac{13}{15}$	$\frac{5}{9}$	$\frac{1}{19}$

Change to decimals, correct to the nearest thousandth:

4.	$\frac{5}{7}$	$\frac{8}{15}$	$\frac{29}{45}$	$\frac{5}{36}$	$\frac{25}{64}$	$\frac{17}{18}$	$\frac{283}{665}$
5.	$\frac{53}{71}$	$\frac{28}{34}$	$\frac{22}{43}$	$\frac{177}{431}$	$\frac{61}{81}$	$\frac{22}{59}$	$\frac{18}{111}$
6.	$\frac{39}{76}$	$\frac{69}{75}$	$\frac{27}{52}$	$\frac{181}{544}$	$\frac{5}{32}$	$\frac{17}{37}$	$\frac{12}{999}$

243.

WHEN DECIMALS ARE MORE CONVENIENT THAN FRACTIONS

Mt. Mansfield in Vermont is 4364 ft. high. What part of a mile high is it?

Since 1 mi. = 5280 ft., Mt. Mansfield is $\frac{4364}{5280}$ of a mile high. Reducing $\frac{4364}{5280}$ to lowest terms, you get $\frac{1091}{1320}$, which is still an awkward fraction. It is much simpler to change $\frac{4364}{5280}$ directly to a decimal. Doing this, you get $4364 \div 5280 = .826$. Hence Mt. Mansfield is .83 mi. high, to the nearest hundredth of a mile.

Exercises

Express each decimal to the nearest hundredth.

1. The steamship *Leviathan* is 907 ft. long. What decimal part of a mile long is it?

2. The Woolworth Building in New York is 792 ft. high. What decimal part of a mile high is it?

3. Ed went from Boston to India via the Panama Canal in 61 da. What decimal part of a year (365 da.) did the trip take?

4. James walks $5\frac{1}{2}$ blocks to get to school. What decimal part of a mile is that if 1 block = 264 ft.?

5. Mt. Vesuvius in Italy has an altitude of 4300 ft. What decimal part of a mile high is it?

6. In a weight-lifting contest a man lifts 217 lb. What decimal part of a ton does he lift?

7. Ella picks 27 lb. of cherries. What part of a bushel is that if 1 bu. of cherries = 50 lb.?

8. It is 3180 mi. from New York to San Francisco and 909 mi. from New York to Chicago. In traveling from New York to San Francisco, what part of the trip have you finished when you reach Chicago?

A FRACTION AT THE END OF A DECIMAL

1. You know that you can reduce $\frac{1}{8}$ to a decimal by dividing the numerator by the denominator.

In *A* the division is carried to hundredths and the remainder is expressed as a fraction, giving $.12\frac{1}{2}$.

In *B* the division is carried to thousandths, giving .125. $.12\frac{1}{2} = .125$.

A	*B*
8)1.00	8)1.000
$.12\frac{1}{2}$	.125

When $\frac{1}{2}$ occurs at the end of a decimal, you may always write 5 in place of it, as shown below.

$.37\frac{1}{2} = .375$ $.62\frac{1}{2} = .625$ $.4\frac{1}{2} = .45$ $.112\frac{1}{2} = .1125$

2. If you reduce $\frac{5}{16}$ to a decimal, you get $.31\frac{1}{4}$ by carrying the result to two places and .3125 by carrying it to four places. Then $.31\frac{1}{4} = .3125$. When $\frac{1}{4}$ occurs at the end of a decimal, you may write 25 in place of it; $\frac{3}{4}$ at the end of a decimal may be replaced by 75.

$.9\frac{1}{4} = .925$ $.06\frac{1}{4} = .0625$ $.17\frac{3}{4} = .1775$ $.05\frac{3}{4} = .0575$

3. In like manner, $\frac{1}{3}$ at the end of a decimal becomes $33\frac{1}{3}$ or 33, while $\frac{2}{3}$ may be replaced by $66\frac{2}{3}$ or 67.

$.8\frac{1}{3} = .833\frac{1}{3}$ or .833 $.16\frac{2}{3} = .1666\frac{2}{3}$ or .1667

Exercises

Write the following entirely as decimals:

1. $.07\frac{1}{2}$	6. $9.06\frac{3}{4}$	11. $4.00\frac{1}{4}$	16. $.116\frac{1}{4}$
2. $.09\frac{1}{4}$	7. $1.83\frac{1}{3}$	12. $19.4\frac{1}{4}$	17. $.105\frac{1}{2}$
3. $.28\frac{3}{4}$	8. $2.50\frac{1}{4}$	13. $2.58\frac{1}{3}$	18. $1.00\frac{1}{2}$
4. $2.2\frac{1}{2}$	9. $7.36\frac{1}{2}$	14. $1.26\frac{1}{3}$	19. $4.66\frac{2}{3}$
5. $3.7\frac{3}{4}$	10. $9.21\frac{2}{3}$	15. $9.01\frac{3}{4}$	20. $.106\frac{2}{3}$

21. Find the cost of driving 65 mi. at $\$.08\frac{3}{4}$ a mile.

PREPARING TO DIVIDE BY A DECIMAL

1. Study carefully these problems in division:

A. $3\overline{)15}$ $\quad$ *B.* $30\overline{)150}$ $\quad$ *C.* $300\overline{)1500}$

5 $\quad$ 5 $\quad$ 5

In *B* both the dividend and the divisor are 10 times as great as in *A*; but both *A* and *B* have 5 as quotient.

In *C* both the dividend and the divisor are 100 times as great as in *A*, but the quotient is still 5.

Multiplying both the dividend and the divisor by 10, *by* 100, *or by* 1000 *does not change the quotient.*

2. If the divisor is a decimal, you can change it to a whole number by the above rule. For example, if you multiply both numbers in *D* by 10, you get *E*.

D $\quad$ *E*

$3.1\overline{)15.5}$ $\quad$ $31\overline{)155}$

DIVIDING BY A DECIMAL

1. John wants to buy a bicycle for \$29.25. If he saves \$2.25 a week, in how many weeks will he have \$29.25?

In dividing 29.25 by 2.25, first change the divisor to a whole number. To do this, multiply both 2.25 and 29.25 by 100 by moving each decimal point 2 places to the right. Then divide 2925 by 225.

John can save \$29.25 in 13 wk.

$2.25\overline{)29.25}$ $\quad$ $225\overline{)2925}$

13

225

675

675

Divide, first changing each divisor to a whole number:

2. \$13.75 ÷ \$1.25

3. \$31.95 ÷ \$0.71

4. \$27.95 ÷ \$2.15

5. \$26.25 ÷ \$1.75

6. \$70.55 ÷ \$0.85

7. \$20.25 ÷ \$1.35

8. \$25.50 ÷ \$0.75

9. \$85.33 ÷ \$0.53

DICK SAVES TIME IN DIVIDING DECIMALS

How long will it take an airplane to fly $138\frac{3}{4}$ mi. if it goes $1\frac{1}{2}$ mi. a minute?

Dick changes $1\frac{1}{2}$ to 1.5 and $138\frac{3}{4}$ to 138.75. He must divide 138.75 by 1.5. To make the divisor, 1.5, a whole number, he multiplies both 1.5 and 138.75 by 10 by moving each decimal point 1 place to the right. He uses a caret (ʌ) to indicate the new position of each decimal point. He places the decimal point in the quotient directly above the caret in 138.7ʌ5.

```
        92.5          Check
1.5ʌ)138.7ʌ5          92.5
      135              1.5
       37             4625
       30              925
       75            138.75
       75
```

The airplane flies $138\frac{3}{4}$ mi. in 92.5 min., or $92\frac{1}{2}$ min.

By using the carets Dick is saved the trouble of erasing the decimal points or of writing the problem over again as you had to do at the bottom of page 59. Dick can also easily tell what the original dividend and divisor are when he wishes to check his work.

Dick checks by multiplying 92.5 by the original divisor, 1.5. If the product equals the original dividend, 138.75, the work is right.

In dividing by a decimal, place a caret (ʌ) as many places to the right of the decimal point in both divisor and dividend as there are decimal places in the divisor. Place the decimal point in the quotient directly above the caret in the dividend.

If the work is right, the quotient and the divisor together have as many decimal places as the dividend has.

Exercises

Divide the following and check the work:

1. 112.32 ÷ 1.3
2. 105.14 ÷ 1.4
3. 163.28 ÷ 2.6
4. 194.15 ÷ 5.5
5. \$24.05 ÷ \$.13
6. \$84.06 ÷ \$.18
7. \$200.34 ÷ \$.53
8. \$303.55 ÷ \$.65

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PROBLEMS IN DIVIDING BY DECIMALS

In these problems first change all fractions to decimals.

1. The first day that Ellen had her new bicycle she rode $13\frac{3}{4}$ mi. in $2\frac{1}{2}$ hr. How fast did she go per hour?

2. May Smith earns \$13.75 a week. As she works only half a day on Saturday, she works $5\frac{1}{2}$ da. a week in all. How much does she earn per day?

3. During a rainstorm that lasted 9.5 hr., 1.71 in. of rain fell. Find, to the nearest hundredth of an inch, the average rainfall per hour.

4. Charles and his sister want to buy a radio for \$21.25. Charles can save \$.75 a week and his sister can save \$.50. How long will it take them to save \$21.25?

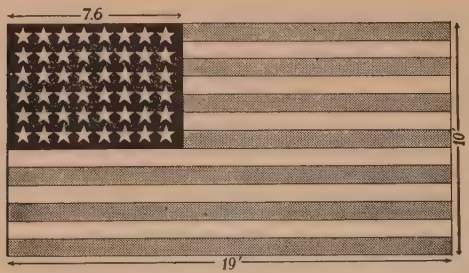
5. A bus driver can go 6.4 mi. on 1 gal. of gasoline. How many gallons will he use in driving 102.4 mi.?

6. Jim rides a mile in $7\frac{1}{2}$ min. on his bicycle. He finds that it takes him $26\frac{1}{4}$ min. to ride to the village. How many miles is it to the village?

Divide the following and check the work:

7. $5.1\overline{)173.91}$	**15.** $6.2\overline{)271.56}$	**23.** $6.43\overline{)280.348}$
8. $8.8\overline{)28.776}$	**16.** $.96\overline{)39.264}$	**24.** $2.16\overline{)122.904}$
9. $.55\overline{)44.495}$	**17.** $7.8\overline{)652.86}$	**25.** $3.12\overline{)285.168}$
10. $.73\overline{)51.246}$	**18.** $4.5\overline{)336.15}$	**26.** $21.4\overline{)1074.28}$
11. $.79\overline{)4.2739}$	**19.** $3.7\overline{)17.242}$	**27.** $2.97\overline{)244.431}$
12. $6.1\overline{)347.09}$	**20.** $8.2\overline{)597.78}$	**28.** $.481\overline{)29.6777}$
13. $.31\overline{)1.7577}$	**21.** $7.2\overline{)349.92}$	**29.** $73.4\overline{)663.536}$
14. $.025\overline{)1.3175}$	**22.** $.87\overline{)49.851}$	**30.** $4.28\overline{)13.7388}$

HOW TO MAKE AN AMERICAN FLAG



The proportions of the United States flag as fixed by official regulations are as follows:

Its length is 1.9 times its width. Each stripe is $\frac{1}{13}$ of the width. The length of the blue field is .76 of the width of the flag, and the width of the blue field equals 7 stripes, or $\frac{7}{13}$ of the flag's width.

The flag shown above has a width of 10 ft. Its other dimensions are found as follows:

The length of the flag $= 1.9 \times 10$ ft., or 19 ft.
The length of the blue field $= .76 \times 10$ ft., or 7.6 ft.
The width of the blue field $= \frac{7}{13}$ of 10 ft., or 5.4 ft.

Exercises

1. Pauline is making a United States flag 4.75 ft. wide. How long should she make the flag?

2. How wide, to the nearest tenth of an inch, should Pauline make each stripe? How long and how wide should the blue field be?

3. Irene's class is making a flag 12 ft. long. How wide, to the nearest tenth of a foot, should the flag be? Give the dimensions of its blue field.

4. How wide are the stripes in the regular flag, 10 ft.×19 ft., shown in the picture on page 62?

5. The flag at Albert's school is 8 ft.×14 ft. Is it an official United States flag? How long should a flag 8 ft. wide be to have the right shape?

Arthur sees these flags advertised in a catalogue. Which of them are of the official proportions?

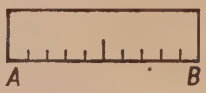
6. $4\frac{1}{2}$ ft.×8 ft. 6 ft.×11.4 ft. 10 ft.×19 ft.

7. 5 ft.×9.5 ft. 8 ft.×12 ft. 12 ft.×21 ft.

8. Caroline found that each stripe of a very large regulation flag used in a street parade was 18 in. wide. She could not get the other measures. How long and how wide was the flag?

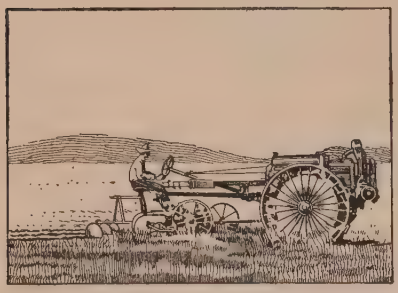
9. A gardener wishes to lay out a flower bed in the shape of a regulation United States flag. If the bed is 60 ft. wide, find, to the nearest tenth of a foot: (*a*) the length of the bed; (*b*) the dimensions of the blue field; (*c*) the width of each stripe.

10. Line AB is 1 in. long. It is divided into tenths. Hence each small space equals $\frac{1}{10}$ in., or .1 in. Make a ruler of cardboard 12 in. long and divide each inch into tenths. Place the edge of the cardboard along line AB to get the right length for each small space.



11. On the blackboard draw an American flag 22 in. wide. What will be the length of the flag, to the nearest tenth of an inch? What will be the width of each stripe? the length and the width of the blue field? Use the ruler you made in ex. 10 to measure the tenths of an inch.

IS IT CHEAPER TO BUY A TRACTOR?



1. Dan's father bought a 3-plow tractor for his 340-acre farm. This tractor can plow 8.6 A. a day. How many days will it take it to plow 111.8 A.?

2. Dan used to plow an average of 2.6 A. a day with a 3-horse team. How long did it take him to plow 111.8 A.?

3. How many days' time was saved by plowing the 111.8 A. with the tractor instead of with horses?

4. The tractor used an average of 2.9 gal. of gasoline per acre. How many gallons, to the nearest whole gallon, were needed for the 111.8 A.?

5. After getting the tractor, Dan's father sold 3 horses that were costing $196 each yearly, and did without a hired man who worked 72 da. for $2.75 a day. Find the total yearly savings on these items.

6. Dan read that the average costs per acre for plowing with the tractor were: cost of tractor, 62¢; repairs, 16.5¢; gasoline, 98¢; oil, 15.5¢; wages of operator, 47.5¢. Find the total cost per acre.

7. A day's plowing with horses cost $5.20 for the horses and plow and $3.25 for wages. Find the total cost per acre if 2.6 A. were plowed in a day.

8. How much cheaper per acre is the plowing with a tractor than with horses? Have you found also that a tractor is quicker than horses?

HARDER WORK IN DECIMALS

1. Charles pays $2.34 for a railroad ticket to Long Lake. At $.036 a mile, for how many miles is the ticket?

Since there are 3 decimal places in the divisor, .036, and only 2 places in the dividend, 2.34, write a 0 after 2.34 so that you can put the caret 3 places to the right of the decimal point, thus: 2.340∧. Place the decimal point in the quotient above the caret.

The quotient is 65.

Hence the ticket is for 65 mi.

Check by multiplying .036 by 5.

```
      65.
.036∧)2.340∧
     2 16
     ----
      180
      180
```

2. Divide .15254 by 5.8.

Place the decimal point in the quotient directly above the caret. Place 2 in the quotient directly above the 2 of 152. Write 0 before 2 in the quotient to fill the space at the right of the decimal point. The quotient, .0263, has 4 decimal places. This is correct because .0263 and 5.8 together have 5 decimal places and the dividend also has 5 places.

To check, multiply .0263 by 5.8.

```
       .0263
5.8∧)1.∧5254
     1 16
     ----
      365
      348
      ---
      174
      174
```

The quotient and the divisor together should have as many decimal places as the dividend has.

Some Airplane Records

Find answers to the nearest tenth of a mile.

1. A monoplane flew 2520.5 mi. from Mineola, N. Y., to San Diego, Calif., in 26.75 hr. Find its speed per hour.

2. In 1919 Alcock and Brown flew 1962 mi. across the Atlantic Ocean in 16.2 hr. Find the average speed per hour.

3. In 1925 Fernand Lasne broke a world record by flying 621 mi. in 4.02 hr. Find his speed per hour.

GRADED EXERCISES IN DIVIDING BY DECIMALS

Divide and check. In short division, place the decimal point in the quotient below the caret:

1. .3)$\underline{.36}$
2. .8)$\underline{8.8}$
3. .9)$\underline{.54}$
4. .03)$\underline{.177}$
5. .5)$\underline{.0285}$
6. .8)$\underline{.0664}$
7. .02)$\underline{1.814}$
8. .004)$\underline{.336}$
9. .9)$\underline{.0333}$
10. .08)$\underline{55.2}$
11. .04)$\underline{26.8}$
12. .005)$\underline{2.45}$

Divide the following and check the work:

13. 4.1)147.19
14. .94)11.656
15. .63)224.91
16. 7.7)329.56
17. 5.3)43.778
18. 6.5)2996.5
19. .022)11.616
20. .028)10.304
21. .092)23.736
22. 7.2).42552
23. 6.4).51968
24. 7.6).19304
25. 2.9).24389
26. 7.7).41811
27. 3.9).20007
28. .83)65238
29. .51)16269
30. .29)1270.2
31. .26)1747.2
32. .022)135.08
33. .074)286.38

Divide. Carry the quotient to the nearest hundredth:

34. 8.6)753.96
35. 4.6)24.476
36. .57)1.3393
37. .53)3.1119
38. 9.2)45.341
39. 5.8)38.161
40. .96)7.3058
41. .39)34.7254
42. 7.7)399
43. .52)438
44. 4.6)1493
45. 7.2)49.171

PUT THE DECIMAL POINT IN THE RIGHT PLACE

In the following the correct figures of the quotients are given, but the decimal points are omitted. Put the decimal point in the right position in each quotient and insert any missing 0's. Give the results orally, thus: "one-point-two."

1. .6)$\overline{.72}$ — 12

2. .8)$\overline{.24}$ — 3

3. .5)$\overline{1.65}$ — 33

4. 2.7)$\overline{7.02}$ — 26

5. .39)$\overline{.0819}$ — 21

6. 8.7)$\overline{.2001}$ — 23

7. .73)$\overline{6394.8}$ — 87 6

8. 11.1)$\overline{410.7}$ — 3 7

9. .035)$\overline{3.255}$ — 93

10. .496)$\overline{49.6}$ — 1

11. .019)$\overline{1.026}$ — 54

12. .124)$\overline{793.6}$ — 6 4

13. .079)$\overline{1.817}$ — 23

14. 38.4)$\overline{1.5744}$ — 41

15. .053)$\overline{28.461}$ — 537

16. 26.4)$\overline{.16632}$ — 63

17. 4.88)$\overline{.28304}$ — 58

18. .051)$\overline{647.7}$ — 12 7

19. 62.5)$\overline{153.125}$ — 245

20. 6.25)$\overline{1531.25}$ — 2 45

21. 62.5)$\overline{15.3125}$ — 245

22. 6.25)$\overline{.153125}$ — 245

23. 6.25)$\overline{15312.5}$ — 24 5

24. 62.5)$\overline{.153125}$ — 245

25. .625)$\overline{15312.5}$ — 24 5

26. .0625)$\overline{15.3125}$ — 245

27. .0625)$\overline{153.125}$ — 245

CAN YOU FIND WALTER'S MISTAKES?

Check each of Walter's problems by multiplying the quotient by the divisor. If the work is wrong, find the mistakes and do the problems correctly. Tell how Walter can avoid these mistakes another time.

1. $7\overline{).7}$
$\underline{1}$

2. $.9_\wedge\overline{)9.0_\wedge}$
$\underline{1\ 0}$

3. $.4_\wedge\overline{).8_\wedge0}$
$\underline{2\ 0}$

4.
$$\begin{array}{r} 3\ 7. \\ 3.2_\wedge\overline{)11.8_\wedge4} \\ \underline{9\ 6} \\ 2\ 24 \\ \underline{2\ 24} \end{array}$$

5.
$$\begin{array}{r} .253 \\ 7.5_\wedge\overline{).1_\wedge8975} \\ \underline{1\ 50} \\ 397 \\ \underline{375} \\ 225 \\ \underline{225} \end{array}$$

6.
$$\begin{array}{r} .736 \\ 5.4_\wedge\overline{).3_\wedge9744} \\ \underline{3\ 78} \\ 194 \\ \underline{162} \\ 324 \\ \underline{324} \end{array}$$

7.
$$\begin{array}{r} 2.83 \\ .69\overline{)195.27} \\ \underline{138} \\ 572 \\ \underline{552} \\ 207 \\ \underline{207} \end{array}$$

8.
$$\begin{array}{r} 4.53 \\ .44\overline{)19.9_\wedge32} \\ \underline{17\ 6} \\ 2\ 33 \\ \underline{2\ 20} \\ 1\ 32 \\ \underline{1\ 32} \end{array}$$

9.
$$\begin{array}{r} 93\ 1. \\ .87\overline{)8099.7_\wedge} \\ \underline{783} \\ 269 \\ \underline{261} \\ 87 \\ \underline{87} \end{array}$$

10.
$$\begin{array}{r} 76\ 10. \\ .84_\wedge\overline{)6392.40_\wedge} \\ \underline{588} \\ 512 \\ \underline{504} \\ 84 \\ \underline{84} \end{array}$$

11.
$$\begin{array}{r} 52.3 \\ .037_\wedge\overline{)19.35_\wedge1} \\ \underline{18\ 5} \\ 85 \\ \underline{74} \\ 111 \\ \underline{111} \end{array}$$

12.
$$\begin{array}{r} 2.4\ 3 \\ 5.3\overline{)128_\wedge7.9} \\ \underline{106} \\ 227 \\ \underline{212} \\ 159 \\ \underline{159} \end{array}$$

IMPROVEMENT TEST No. 5

Test 5 A — Addition (Time 4 min.)

1. $6\frac{5}{8}$ + 4	2. $8\frac{2}{3}$ + $5\frac{1}{3}$	3. $6\frac{3}{4}$ + $7\frac{3}{8}$	4. $9\frac{1}{2}$ + $2\frac{3}{5}$
5. $8\frac{7}{8}$ + $6\frac{7}{8}$	6. $1\frac{1}{2}$ + $8\frac{9}{10}$	7. $6\frac{2}{3}$ + $9\frac{1}{8}$	8. $7\frac{2}{3}$ + $\frac{1}{2}$
9. $2\frac{2}{5}$ + $3\frac{1}{3}$	10. $2\frac{5}{8}$ + $5\frac{7}{16}$	11. $7\frac{1}{6}$ + $3\frac{1}{4}$	12. $8\frac{1}{3}$ + $1\frac{3}{4}$

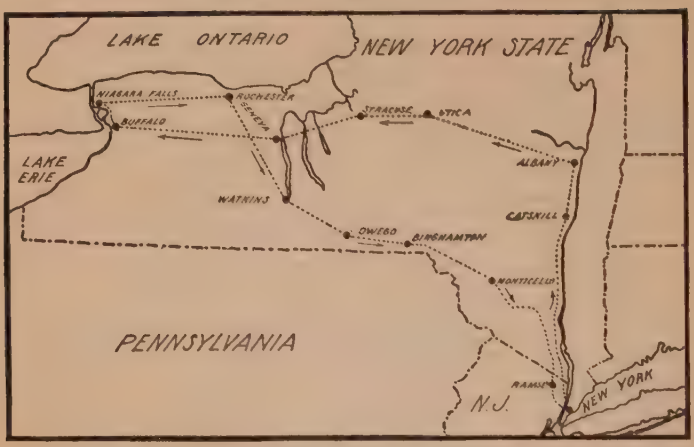
Test 5 B — Multiplication (Time 4 min. after copying)

1. \$1.24 × $3\frac{1}{2}$	2. \$2.16 × $1\frac{3}{4}$	3. $17\frac{1}{4}$ × 9
4. $\frac{1}{4} \times \frac{1}{2}$	5. $\frac{2}{3} \times 15$	6. $6\frac{3}{4} \times \frac{8}{9}$
7. $\frac{2}{3} \times \frac{5}{8}$	8. $1\frac{1}{4} \times 12$	9. $4\frac{2}{3} \times 4\frac{1}{2}$
10. $\frac{9}{10} \times \frac{5}{6}$	11. $10 \times 2\frac{1}{4}$	12. $1\frac{1}{5} \times 1\frac{7}{8}$

Test 5 C — Division (Time 4 min. after copying)

1. $\frac{5}{6} \div \frac{1}{12}$	2. $2\frac{1}{3} \div 2$	3. $\frac{1}{16} \div \frac{1}{16}$
4. $\frac{3}{4} \div \frac{3}{16}$	5. $6\frac{1}{4} \div 5$	6. $\frac{1}{8} \div 1\frac{1}{4}$
7. $3 \div \frac{9}{10}$	8. $27 \div 4\frac{1}{2}$	9. $5\frac{1}{3} \div 2\frac{2}{3}$
10. $\frac{9}{16} \div \frac{15}{16}$	11. $7\frac{1}{2} \div 10$	12. $3\frac{3}{5} \div 1\frac{1}{8}$

AN AUTOMOBILE TRIP



Mr. Wright and five scouts are driving from New York to Niagara Falls. They find these routes in a book:

ROUTE 26: *New York to Niagara Falls*

MILES FROM NEW YORK	
0.0	New York
151.7	Albany
244.9	Utica
296.0	Syracuse
340.4	Geneva
452.0	Buffalo
478.8	Niagara Falls

ROUTE 27: *Niagara Falls to Rochester*

MILES FROM NIAGARA FALLS	
0.0	Niagara Falls
23.0	Lockport
37.2	Middleport
42.5	Medina
53.4	Albion
68.5	Brockport
88.2	Rochester

ROUTE 28: *Rochester to New York*

MILES FROM ROCHESTER	
0.0	Rochester
80.0	Watkins
138.1	Owego
158.9	Binghamton
258.8	Monticello
328.1	Ramsey
349.0	New York

1. The scouts go by Route 26. Make a table showing the distance from each town to the next on this route.

2. They return by way of Rochester, using Routes 27 and 28. Make a table combining Routes 27 and 28 so that all the distances read from Niagara Falls. To do this, add 88.2 mi. to every distance now given in Route 28.

3. Make a table like that in ex. 1 to show the distances between the towns in Routes 27 and 28.

4. What is the total distance from New York to Niagara Falls and return along the routes selected?

5. How much longer is the trip to Niagara Falls than the trip back along the routes selected?

6. Mr. Wright's car averages 16.8 mi. on 1 gal. of gasoline. How many gallons, to the nearest whole gallon, are needed for the trip from New York to Albany?

7. When the scouts leave Syracuse, on the way to Niagara, they have to make a detour because the road is being repaired. When they reach Buffalo, they find that they have gone 176.8 mi. since leaving Syracuse. How many extra miles have they gone because of the detour?

8. If the speedometer reads 11,265 mi. when they leave New York and 12,294.1 mi. when they return, how many miles do they really travel? How much more is this than the total distance on Routes 26, 27, and 28?

9. It costs Mr. Wright 8.4¢ a mile to operate his car. The 5 scouts and Mr. Wright are each to pay $\frac{1}{6}$ of this cost. How much will each scout pay per mile?

10. Using the total distance found in ex. 8 and the cost per mile found in ex. 9, find, to the nearest cent, how much the transportation for the entire trip costs each scout. How much would it cost each scout to travel the same distance on the railroad at 3.6¢ a mile? How much does each scout save by going with Mr. Wright?

11. If the total driving time for the entire trip is 50.2 hr., what is the average rate of driving per hour?

DECIMALS

1. Which is longer, 14.9 ft. or 14.875 ft.?

Reduce to common fractions or to mixed numbers:

2. .35 **3.** .12½ **4.** 2.25 **5.** 4.625 **6.** .225

Reduce the following to decimals:

7. $\frac{1}{4}$ **8.** $\frac{5}{6}$ **9.** $\frac{3}{8}$ **10.** $\frac{7}{8}$ **11.** $\frac{2}{3}$

Multiply. Check the work by going over it again:

12. .057 × 7 **15.** 6.92 × 6.7 **18.** .049 × 5 **21.** 40.5 × 100

13. 4000 × .002 **16.** 8900 × 7.2 **19.** 1.9 × 800 **22.** 5.64 × .300

14. 195 × 10 **17.** 2.5 × 100 **20.** 326.7 × .08 **23.** .09 × 1000

Divide each of the following by 10, 100, *and* 1000:

24. 600 **25.** $50 **26.** 38 **27.** 9.4 **28.** .7

Divide. Check the work by multiplying:

29. .4)$\overline{.48}$ **32.** 7.9)$\overline{301.78}$ **35.** 5.6)$\overline{.18144}$

30. .8)$\overline{5.6}$ **33.** .73)$\overline{371.57}$ **36.** .83)$\overline{4913.6}$

31. .6)$\overline{.0282}$ **34.** .094)$\overline{89.582}$ **37.** .063)$\overline{242.55}$

Divide, annexing 0's *to the dividend. If the quotient does not come out exact, carry it to three decimal places and find the answer to the nearest hundredth:*

38. 24)$\overline{188}$ **41.** 56)$\overline{402}$ **44.** 5.6)$\overline{332}$

39. 75)$\overline{186}$ **42.** 8.7)$\overline{94.5}$ **45.** 46)$\overline{4439}$

40. 28)$\overline{194.6}$ **43.** .28)$\overline{14.92}$ **46.** 93)$\overline{439.9}$

REVIEW OF FRACTIONS

Add. Check the work by going over it:

1.	2.	3.	4.	5.	6.
$3\frac{1}{8}$	$6\frac{3}{4}$	$9\frac{1}{2}$	$2\frac{2}{3}$	$2\frac{5}{10}$	$8\frac{1}{4}$
$4\frac{5}{8}$	$2\frac{1}{2}$	$3\frac{5}{8}$	$3\frac{1}{6}$	$9\frac{1}{10}$	$9\frac{1}{2}$
$7\frac{3}{8}$	$5\frac{1}{4}$	$4\frac{3}{4}$	$1\frac{1}{2}$	$6\frac{1}{2}$	$2\frac{5}{12}$

7.	8.	9.	10.	11.	12.
$5\frac{3}{8}$	$3\frac{2}{3}$	$1\frac{1}{4}$	$9\frac{7}{10}$	$2\frac{5}{8}$	$5\frac{5}{6}$
$7\frac{1}{2}$	$\frac{1}{2}$	$6\frac{3}{5}$	$8\frac{3}{5}$	$6\frac{1}{12}$	$4\frac{3}{4}$
$9\frac{7}{16}$	$2\frac{1}{3}$	$4\frac{3}{4}$	$9\frac{2}{5}$	$2\frac{1}{4}$	$\frac{2}{3}$
$4\frac{3}{8}$	$\frac{3}{4}$	$\frac{4}{5}$	$7\frac{1}{2}$	$1\frac{2}{3}$	$5\frac{1}{3}$

Subtract. Check the work by going over it:

13.	14.	15.	16.	17.	18.
$8\frac{7}{16}$	$5\frac{1}{8}$	6	9	$7\frac{3}{4}$	$4\frac{1}{2}$
$3\frac{1}{16}$	$4\frac{7}{8}$	$2\frac{3}{4}$	$8\frac{5}{8}$	$3\frac{1}{2}$	$2\frac{3}{8}$

19.	20.	21.	22.	23.	24.
$6\frac{1}{4}$	$7\frac{1}{5}$	$3\frac{1}{6}$	$7\frac{3}{4}$	$9\frac{3}{10}$	$8\frac{1}{2}$
$3\frac{5}{8}$	$5\frac{3}{10}$	$\frac{1}{2}$	$4\frac{1}{3}$	$5\frac{1}{2}$	$1\frac{9}{16}$

Multiply. In the answers for ex. 33 to 36, *call a fraction of a cent another cent:*

25. $\frac{5}{8}\times\frac{1}{2}$	**29.** $12\times\frac{7}{16}$	**33.** $9\times\$.37\frac{1}{2}$
26. $\frac{7}{16}\times\frac{4}{5}$	**30.** $3\frac{1}{4}\times\frac{1}{2}$	**34.** $3\frac{5}{8}\times\$2.40$
27. $\frac{3}{4}\times 18$	**31.** $6\frac{1}{4}\times 3\frac{1}{5}$	**35.** $1\frac{3}{4}\times\$1.50$
28. $3\times 11\frac{3}{4}$	**32.** $12\frac{1}{2}\times 16\frac{2}{3}$	**36.** $5\frac{1}{2}\times\$1.73$

Divide. Check the work by going over it:

37. $\frac{1}{2}\div 2$	**41.** $\frac{9}{8}\div\frac{6}{5}$	**45.** $4\frac{1}{2}\div 6$
38. $\frac{3}{4}\div 6$	**42.** $28\div\frac{1}{4}$	**46.** $5\frac{1}{4}\div 7$
39. $\frac{7}{8}\div\frac{1}{2}$	**43.** $1\frac{7}{8}\div\frac{3}{4}$	**47.** $7\frac{1}{2}\div 2\frac{1}{2}$
40. $\frac{5}{12}\div\frac{3}{8}$	**44.** $15\div 3\frac{1}{7}$	**48.** $4\frac{3}{8}\div 3\frac{1}{2}$

HOW CAN THESE PROBLEMS BE SOLVED?

1. Dick knows how long he takes to walk a mile and how many miles it is to the village. How can he tell how long he will take to walk to the village?

2. Mabel knows how many people will be at her birthday party and how much ice cream costs per quart. If she plans to serve 6 people with each quart, how can she find how much all the ice cream will cost?

3. Hilda knows the amount of money she had in the savings bank on January 1 and the amounts she deposited and withdrew during January. Tell her how to find what amount she should have in the bank on January 31.

4. Hazel is planning to stay at a camp 6 wk. next summer. She knows the railroad fare and the weekly rate at camp. How can she find the total expense?

5. A number of boys are planning to rent a canoe for 5 wk. next summer. If they know the rate per week, how can they find each boy's share in the rent of the canoe?

6. Al works afternoons at 35¢ an hour. He keeps a record of the time he arrives at work and the time he leaves. How can he find his week's earnings?

7. At Easter time Wilda colored $3\frac{1}{2}$ doz. Easter eggs. She knows the cost of the dye she used and the cost of the eggs per dozen. How can she find the total expense for the Easter eggs?

8. Mrs. Adams knows how many cups of sugar she needs for jelly and how much sugar costs per pound. Counting 2 cups to a pound, how can she find the cost of the sugar for the jelly?

CHAPTER II

STUDYING A BILL

A storekeeper often lets a reliable customer open an *account* with him and *charge* the things bought at his store. At the end of the month he sends a *bill* showing how much the customer owes him. When the bill is paid, the storekeeper *receipts* it. Here is a typical bill:

JONES, KING, AND COMPANY
Clothiers

Chicago, Nov. 1, 19–

Mr. Charles Reed
893 Beech St.
Chicago

Oct.	2	4 Neckties	@ .85	3 40	
	10	6 Collars	@ .25	1 50	
	15	2 Pairs Gloves	@2.10	4 20	
	24	1 Boys' Suit	@16.50	16 50	
					25 60

Received Payment
Nov. 4, 19—
JONES, KING, AND COMPANY
per M. K.

The sign @ means "at." The total charge for each item, such as the $3.40 for neckties, is an *extension*. $25.60 is the *total amount*. When was this bill receipted? by whom?

A bill should give these facts: (1) the place and the date; (2) the merchant's name; (3) the customer's name; (4) the date, the quantity, the price, and the extension for each item; (5) the total amount of the bill.

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MAKING OUT BILLS

Make out and receipt bills for the following. Make up the names, the addresses, and the dates:

1. $2\frac{3}{4}$ doz. eggs @ 56¢; $3\frac{1}{2}$ lb. sweet butter @ 64¢; $1\frac{3}{4}$ lb. Swiss cheese @ 60¢; 2 pt. cream @ 35¢.

2. 1 boys' mackinaw overcoat @ $7.45; 2 ties @ $.95; 3 pr. boys' sport hose @ $1.25; 1 cap @ $2.15.

3. $2\frac{1}{2}$ doz. rolls @ 15¢; 4 loaves bread @ 9¢; $1\frac{3}{4}$ doz. buns @ 18¢; 3 doz. cookies @ 20¢.

4. $6\frac{1}{2}$ lb. chicken @ 52¢; $1\frac{1}{4}$ lb. sausage @ 42¢; $2\frac{3}{4}$ lb. steak @ 60¢; $3\frac{1}{2}$ lb. lamb chops @ 50¢.

5. 1 wheelbarrow @ $4.25; 2 axes @ $1.18; 1 garden hose @ $14.60; 16 lb. nails @ $.12; 5 bolts @ $.35.

6. 5 tennis balls @ $.85; 2 pr. sport hose @ $1.15; 2 catchers' mitts @ $2.90; 1 football @ $6.75; 2 tennis rackets @ $3.75.

7. $\frac{1}{2}$ doz. linen handkerchiefs @ $3.50; 2 pr. kid gloves @ $2.55; 2 pr. woolen blankets @ $6.98; 7 Turkish towels @ $.49.

8. 1 tool chest @ $9.65; 2 pr. pliers @ $.42; 1 steel hammer @ $.65; 3 chisels @ $.68.

9. 2 Axminster rugs @ $20.95; 15 sq. yd. printed linoleum @ $1.56; 11 yd. curtain goods @ $.75.

10. 3 jars strawberry jam @ 35¢; 5 packages buckwheat flour @ 25¢; 6 packages crackers @ 15¢.

11. 2 bottles tomato catsup @ 24¢; 3 jars olives @ 26¢; $2\frac{1}{2}$ lb. walnuts @ 43¢; 4 cans peaches @ 39¢.

12. $5\frac{1}{2}$ yd. cotton crêpe @ 48¢; 3 spools sewing silk @ 12¢; $11\frac{1}{2}$ yd. braid @ 35¢; $2\frac{3}{4}$ yd. ribbon @ 39¢.

SHOWING CREDITS IN BILLS

Sometimes the *charges* on monthly bills are placed in one column and the *credits* in the next column. If a payment is made during the month or if some article is returned, the merchant gives *credit* for these items.

WARREN BROTHERS
Athletic Goods

Dick Rankin, Baseball Manager
Roosevelt School,
Flint, Mich.

Detroit, Apr. 30, 19~

Date		Items		Charges		Credits	
Apr	2	9 gray baseball suits	@ 5.50	49	50		
	6	6 pr. baseball shoes	@ 3.25	19	50		
	6	By cash on account				25	00
	8	3 catchers' mitts	@ 1.50	4	50		
	11	1 catchers' mitt returned	@ 1.50			1	50
				73	50	26	50
			Credit	26	50		
			Balance	47	00		

The total charges above are $73.50; the total credits are $26.50. Hence the *balance* due is $47.00.

Exercises

Make out bills for these items. Supply names and dates:

1. 3 scout hats @ $1.60; 6 scout handkerchiefs @ 25¢; credit, 1 scout hat @ $1.60 returned; 2 sweaters @ $7.00; 2 khaki middy blouses @ $1.75; 3 pr. stockings @ $.55; credit, 1 sweater @ $7.00 returned.

2. 1 small Chinese rug @ $49.50; cash on account, $25.00; 3 chairs @ $6.76; 2 lamps @ $18.75; 1 table @ $24.80; 2 tablecloths @ $2.85.

MAKING OUT RECEIPTS

You have seen how a bill is receipted. Sometimes, if no bill is presented, a separate receipt is written to show that one person has received money from another.

Buffalo, N. Y. April 10, 19~

$16.25

Received from George Miller

Sixteen and 25/100 Dollars

for repairing automobile.

John Harris

Supply the dates and the names of cities:

1. Bob Dugan sold George Price his bicycle for $13.50. Write the receipt which Bob gave George.

2. Esther paid her dentist, Dr. J. M. Green, $3.50 for filling her tooth. Write a receipt for this.

3. Mr. Rex bought a car for $2500 from the Monroe Company. He paid $700 cash. Write a receipt for this payment.

4. Mrs. Fuller pays her landlord, Mr. Greeley, $50 rent each month. Make out the rent receipt which Mrs. Fuller receives from Mr. Greeley for rent for March.

Write a receipt for each. Supply names and dates:

5. $50 for building a garage.

6. $10 for the rent of a tent for one month.

7. $3.60 for milk for August.

8. $15 for the first payment on a radio set.

9. $4.75 for a telephone bill for January.

IMPROVEMENT TEST No. 6

Test 6 A — Reduction (Time 4 min. after copying)

Change to common fractions in lowest terms:

1. .24 2. .35 3. .37½ 4. .050 5. .036

6. .60 7. .06 8. .12½ 9. .075 10. .875

Change to exact decimals:

11. $\frac{9}{20}$ 12. $\frac{3}{8}$ 13. $\frac{4}{5}$ 14. $\frac{21}{25}$ 15. $\frac{13}{50}$

Test 6 B — Reduction (Time 4 min. after copying)

Change to decimals, correct to the nearest hundredth:

1. $\frac{4}{7}$ 2. $\frac{5}{12}$ 3. $\frac{7}{12}$ 4. $\frac{9}{16}$

5. $\frac{8}{9}$ 6. $\frac{5}{32}$ 7. $\frac{4}{15}$ 8. $\frac{13}{36}$

Test 6 C — Multiplication (Time 4 min. after copying)

1. 64 × .12	2. 28.7 × 1.2	3. 3000 × .003	4. 2940 × .41
5. 273 × .28	6. 39.6 × .07	7. .0653 × 9	8. 37.00 × .52
9. .036 × .36	10. 419 × .006	11. .0009 × .05	12. .108 × 800

Test 6 D — Division (Time 4 min. after copying)

1. 5.4)$\overline{.18414}$ 2. .81)$\overline{33858}$ 3. 4.6)$\overline{24.472}$

BUYING BY PARCEL POST

Packages may be sent through the mail by **parcel post.** The amount of postage depends on the weight of the package and on the distance it is sent. All places within 50 mi. of a post office are in Zone 1; all those from 51 mi. to 150 mi. away are in Zone 2, etc. The table below shows the rate for each zone. The map on page 81 shows how the zones are marked from Philadelphia. Any fraction of a pound is counted as an extra pound. Thus, $3\frac{1}{4}$ lb. is counted as 4 lb.

WEIGHT	ZONES						
	1 AND 2	3	4	5	6	7	8
	1 MI. TO 150 MI.	151 MI. TO 300 MI.	301 MI. TO 600 MI.	601 MI. TO 1000 MI.	1001 MI. TO 1400 MI.	1401 MI. TO 1800 MI.	ALL OVER 1800 MI.
First pound	7¢	8¢	8¢	9¢	10¢	12¢	13¢
Each additional pound	1¢	2¢	4¢	6¢	8¢	10¢	12¢

Exercises

In the following the distances are figured from Philadelphia. Use the map on page 81 *to find the zone in which each city is located.*

1. Find the cost of sending a 14-pound package from Philadelphia to Richmond, Va., in Zone 3.

For Zone 3, the first pound costs 8¢; the 13 additional pounds cost 2¢ each, or 26¢. Find the total cost.

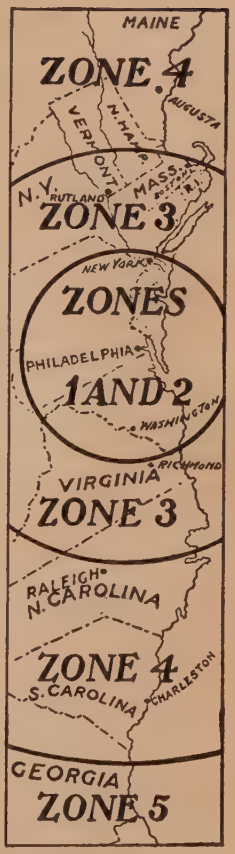
2. A farmer near Washington, D. C., sends a package of eggs weighing 8 lb. to a man in Philadelphia. How much does he pay for postage?

3. A girl in Raleigh, N. C., orders a lamp costing $17.65 from Philadelphia. The shipping weight is 18 lb. and the postage is extra. How much money should she send for the lamp and the postage?

4. How much will it cost to send from Philadelphia to Augusta, Maine, a tennis racket weighing 2 lb.? a package of books weighing 18 lb.?

5. George can buy pure Vermont maple sugar in Rutland, Vt., for 24¢ a pound. It costs him 38¢ a pound in Philadelphia, where he lives. How much will he save after paying the postage if he sends to Rutland for 10 lb. of maple sugar?

6. Paul sends from Charleston, S. C., to Philadelphia for the following articles: baseball bat, $2\frac{1}{4}$ lb.; glove, 8 oz.; baseball, 8 oz.; mask, $1\frac{1}{2}$ lb.; uniform, 3 lb.; shoes, $2\frac{1}{2}$ lb. How much postage must he send for the entire package?



Find the cost of mailing the following packages:

7. 17 lb. to Zone 8

8. $5\frac{3}{4}$ lb. to Zone 6

9. $11\frac{5}{8}$ lb. to Zone 7

10. 20 lb., 900 miles

11. $7\frac{1}{4}$ lb., 280 miles

12. $3\frac{1}{2}$ lb., 1900 miles

13. Using the map above, make up five parcel-post problems. Use Philadelphia as one of the cities in each problem.

SOLVING HARDER PROBLEMS

Guy Jones works in a factory which employs 156 men. He gets 45¢ an hour for regular work and $1\frac{1}{2}$ times that rate for overtime. Last week he worked 53 hr., of which 8 hr. were overtime. What were his earnings last week?

In solving harder problems, ask yourself these questions:

1. *Do I understand the problem?* What do "overtime" and "$1\frac{1}{2}$ times that rate" mean?

2. *What am I asked to find?* You are here asked to find Guy's earnings for the week.

3. *What facts are given?* Four of the facts given in the problem must be used to solve it. What are they? Are any facts given that are not needed?

4. *What steps shall I take to obtain the answer?* To find Guy's earnings you must answer these questions:

(1) How many hours of regular work did Guy do?
53 hr. − 8 hr. = 45 hr., the number of hours of regular work.

(2) How much did he earn for regular work?
45×45¢ = \$20.25, his earnings for regular work.

(3) How much did he earn for 1 hour of overtime?
$1\frac{1}{2}$×45¢ = $67\frac{1}{2}$¢, his earnings for 1 hr. of overtime.

(4) How much did he earn in all for overtime?
8×$67\frac{1}{2}$¢ = \$5.40, his earnings for 8 hr. of overtime.

(5) What were Guy's total wages?
\$20.25 + \$5.40 = \$25.65, his total wages.

5. *Have I found what was asked for?* Read the problem again to see if you have answered the question asked.

6. *Is the work right?* Go over all your steps to see if your thinking is right. Then check your computations.

CAN YOU DO THESE HARDER PROBLEMS?

1. Mr. Coss and 3 of his friends rent a fishing boat. They have the boat 5 hr. at $.65 an hour, and they pay $1.55 for bait. Find how much each man must pay if they share the total expenses equally.

2. Mrs. Peel is buying a vacuum cleaner on the installment plan. The clerk says that there are two ways of paying for it. The first plan is to pay $15 cash and then pay $4.50 a month for 11 mo. The second plan is to pay $25 cash and $5.75 a month for 6 mo. Which plan is cheaper and how much cheaper?

3. Henry and Jim wanted to sell their canoe. They put a 3-line advertisement in the *Daily News* for 4 days at 45¢ a line each day. Tom saw the advertisement and bought the canoe for $37.50. If Henry and Jim divided the money equally, how much did each boy get from the sale after the advertising expense was deducted?

4. Mr. Bates and his family are making an automobile trip of 3312 mi. across the United States. If they drive 8 hr. a day at an average speed of 18 mi. an hour and allow 5 days for stopovers and repairs, how many days must they allow for the trip?

5. Each of the 32 children in the primary grades of our school gave 15¢ to a Thanksgiving fund for poor families. Each of the 27 children in the grammar grades gave twice as much as that. How much did our school contribute?

WHAT MUST YOU KNOW TO WORK THESE PROBLEMS?

1. Jack wants to find out at what time he must leave his house in the morning in order to reach school on time. Kitty says that he must first know how long it takes him to reach school and at what time he is due there. Then he must subtract the time it takes him from the time when he is due. Is Kitty right?

2. Carl needs some rope to make a swing on the apple tree. What must he know and then what must he do to find how much the rope will cost?

3. Mr. Johnson is buying an automobile on the installment plan. He is paying for it in equal monthly installments. What must he know and then what must he do to find how many payments he will have to make?

4. What must you know and then what must you do to find how much more money you have in the savings bank this June than you had last September?

5. How can a grocer find out how many $3\frac{1}{2}$-pound bags of flour he can fill from a barrel of flour?

6. Suppose that an airplane makes a trip from New York to Denver. What must you know and what must you do to find how fast per hour the airplane travels?

7. If you know the number of miles your automobile can travel on 1 gal. of gasoline, what else must you know and what must you do to find how much gasoline you need to make a certain trip?

8. During the summer Arthur sells books at $2.00 each. Mr. Dick asks Arthur how much his profits average per week. How can Arthur find out?

GIVE THE MISSING FACT

In each problem a fact is missing which you must know before you can find the answer. Supply the missing fact and work the problem.

1. There are 24 sandwiches for the children at the picnic. How many will each child get?

The missing fact is the number of children at the picnic. If there are 6 children, each will get 24 sandwiches ÷ 6, or 4 sandwiches.

2. Arthur waited at the station for a train to arrive. It was due at 3.15 P.M., but was delayed. How late was the train?

3. Mr. Smith buys a paper every Sunday. How much does he spend in 1 year for his Sunday papers?

4. How much will $4\frac{1}{2}$ doz. new marbles cost James?

5. Harry is saving $\frac{2}{5}$ of the money he earns this summer. How much will he save this summer?

6. Janet wants a new couch cover. She sees some material costing $1.14 a yard. If she buys this material, how much will the cover cost?

7. In Jennie's family they use $10\frac{1}{2}$ doz. rolls a week. How much is spent each week for rolls?

8. The 27 children in our class are going by boat to Indian Point. How much will all the tickets cost?

9. Ruth has $5 to spend for goods for her dress. How much per yard can she pay for the goods?

10. Mr. Owen is driving 1053 mi. from New York to Chicago. How long will the trip take?

11. Tom is buying a typewriter for $60 on the installment plan. How long will it take him to pay for it?

WHAT QUESTION SHOULD BE ASKED?

The following problems are not complete because no question has been asked. Finish each problem by asking a question which can be answered by using the facts given. Then find the answer to your question.

1. Some boys rented a rowboat on Bronx Lake for 6 hours at 50¢ an hour.

The missing question is " How much did they pay in all? " Answer this question.

2. Each of the 47 children in our class is giving a nickel to buy some plants for the schoolroom.

3. Nell was queen of the June walk. She needed $2\frac{1}{8}$ yd. of white satin at \$1.12 a yard for her dress and $1\frac{3}{4}$ yd. of tinsel at \$.28 a yard.

4. Pat was born in 1918 and his sister Kathleen was born in 1925.

5. Janet has read 117 pages of a 205-page book.

6. Jack and Tom bought a small tent for \$11.95. At the end of the summer they sold it for \$5.50.

7. Last month Jim worked after school for Mr. Dean for 49 hr. at 18¢ an hour. When Mr. Dean paid him he handed him a 10-dollar bill.

8. This morning Ida rode her bicycle 4 mi. in 20 min.

9. It is 710 mi. by train from New Orleans to St. Louis and you pass through Memphis, Tenn., on the way. It is 394 mi. from New Orleans to Memphis.

10. It took Ned $3\frac{1}{2}$ hr. to clean Mrs. Fisk's cellar. He received 28¢ an hour. He gave half of what he earned to his mother and spent 15¢ for the movies.

ARE YOU A GOOD THINKER?

In each problem, some numbers are given that are not needed for finding the answer. Tell which numbers are not needed and then solve the problem.

1. Henry is going to camp for 14 da. He needs \$45 for board and fare. He has \$38.79 and his father is giving him the rest. How much is his father giving him?

In finding the answer, you do not use the fact that Henry is going to camp for 14 da.

2. A 36-car freight train traveled 475 miles in 25 hr. Find its average speed per hour.

3. Mr. and Mrs. Felton rent a 5-room apartment for \$57 a month. How much rent will they pay in $\frac{1}{2}$ yr.?

4. Seven persons are going on an automobile trip of 1210 mi. If they ride 220 mi. a day, how many days will it take them to make the trip?

5. Jack wants to buy a radio set for \$38.50. He has worked 17 da. and has earned \$29.75 in all. How much more must he earn to buy the set?

6. At a sale Milton bought for \$14.95 a suit originally marked \$18.75. How much money did he have left from the \$20.00 which he had saved?

7. Frank attends school from 8.30 A.M. to 12.30 P.M. and from 1.30 P.M. to 3 P.M. How much time has he for lunch?

8. John gets \$1.50 a week. Last week he paid 35¢ for $3\frac{1}{2}$ doz. marbles. How much money had he left?

9. There are 1396 children at the Lawrence School. If 964 of them contribute 10¢ each to a charity, how much money is collected at the school?

LIVING MORE THAN 1000 YEARS

Think very carefully in working these problems.

1. It is said that Methuselah lived 969 yr. How long would it have taken him to haul $\frac{1}{2}$ ton of stone on an ox cart to a city 20 mi. away and to return, if his oxen went 2 mi. an hour and he traveled 10 hr. a day?

2. If Methuselah worked 300 da. a year at the rate given in ex. 1, how many tons could he have hauled to the city in 900 yr.?

3. How many years would it take a modern man with an auto truck carrying 3 tons at a time to haul the same number of tons as in ex. 2, if he made 2 round trips daily to the city 20 mi. distant and worked 300 da. a year?

4. How many years would it have taken Methuselah, working 300 days a year, to cart as much stone to the city (see ex. 1 and 2) as a modern man, working at the rate given in ex. 3, could haul in 50 yr.?

5. How many men pulling an average of 60 lb. each would it have taken, when the pyramids of Egypt were built, to raise as much as an engineer does to-day with a modern derrick, if the derrick lifts 8 tons at a time and works 3 times as fast as the ancient human power hoist?

You see that to-day a man who lives to be 80 yr. old can accomplish a great deal more than a man in olden times could if he lived 1000 yr.

PERCENTAGE

A New Way to Write Hundredths

You have learned in your work in decimals that .37 is a short way of writing $\frac{37}{100}$. Another way to write $\frac{37}{100}$ is 37%, which is read *37 per cent.* *Per cent* means *hundredths.* The sign % stands for *per cent.*

You can now write hundredths in these four ways:

$$\frac{37}{100} = .37 = 37\% = 37 \text{ per cent}$$

Similarly, $\frac{6}{100} = .06 = 6\% = 6$ per cent

This new method of writing hundredths is called **percentage.** It is used every day in business and is often found in newspapers and magazines.

How Per Cents Are Used

1. If 96% of the 450 pupils in our school were present yesterday, how many pupils were present?

This means that $\frac{96}{100}$ of 450 pupils were present yesterday. Another way to write $\frac{96}{100}$ of 450 is $.96 \times 450$. Hence 96% of 450 means $.96 \times 450$.

Multiplying 450 by .96, you find that 432 pupils were present yesterday.

```
  450
  .96
 27 00
405 0
------
432.00
```

2. Sarah sees this sign in a store. How much cheaper will skates which regularly cost $2.40 be on Saturday than on other days?

The amount "taken off" the price on Saturday is 15% of $2.40, which means $.15 \times \$2.40$, or $.36. Hence on Saturday the skates will be $.36 cheaper than on other days.

USING DECIMALS IN PERCENTAGE PROBLEMS

In percentage there are no new rules for adding, subtracting, multiplying, or dividing, because percentage problems are worked by changing the per cents to decimals.

Exercises

Write each of the following as a decimal:

1. 17%	**5.** 1%	**9.** $\frac{55}{100}$	**13.** $\frac{70}{100}$
2. 23%	**6.** 90%	**10.** $\frac{98}{100}$	**14.** $\frac{6}{100}$
3. 36%	**7.** 7%	**11.** $\frac{76}{100}$	**15.** $\frac{5}{100}$
4. 45%	**8.** 9%	**12.** $\frac{1}{100}$	**16.** $\frac{9}{100}$

17. 2 per cent **19.** 95 per cent

18. 83 per cent **20.** 60 per cent

Write each of these as a per cent, using the sign %:

21. .63	**24.** .01	**27.** $\frac{45}{100}$	**30.** $\frac{9}{100}$
22. .24	**25.** .05	**28.** $\frac{76}{100}$	**31.** $\frac{1}{100}$
23. .40	**26.** .07	**29.** $\frac{80}{100}$	**32.** $\frac{6}{100}$

33. You learned on page 89 that 96% of 450 means .96×450. What does 96% of 868 mean?

Tell what numbers should be put in place of the dots:

34. 54% of 250 means .54×250, which equals 135.

35. 15% of 620 means . . .×620, which equals . . .

36. 8% of 150 means .08×150, which equals . . .

37. 4% of 425 means . . .×425, which equals . . .

38. 1% of 900 means . . .×900, which equals . . .

39. 5% of 60 means . . .×. . ., which equals . . .

40. 32% of 75 means . . .×. . ., which equals . . .

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LEARNING THE LANGUAGE OF PERCENTAGE

In studying percentage, you are learning a new kind of language, which business men use every day for talking about *hundredths*. Read each problem carefully to be sure that you understand this new language.

1. Lola received a mark of 85% on a spelling test of 40 words. What decimal part of the test had she right? How many words did she have right?

2. Will's arithmetic test was marked 65%. What does this mean? If there were 20 examples on the test, how many did Will get right?

3. In a spelling test of 50 words, the marks of some of the pupils were as follows: Alice, 98%; George, 86%; Fred, 74%; Tony, 68%; Marion, 92%; Julia, 94%. How many words did each of these pupils get right?

4. The Stars played 25 baseball games and won 76% of them. How many games did they win?

5. The principal said that 18% of the 350 children in the Walker School were underweight. What did he mean by that? How many children were underweight?

6. Tom's older brother saves 15% of all he earns. How much does he save in a year if he earns $1000 a year? if he earns $1200 a year?

7. Mrs. Cox gets 5% off on a gas bill of $2.40 for paying it promptly. How much reduction does she get?

8. Of the 5225 children in the Waterville schools, 68% have money in the School Savings Bank. How many of the children have money in the bank?

THE LANGUAGE OF PERCENTAGE (*continued*)

9. The wages in a machine factory were increased 5%. What does this mean? If a man earned $5.00 a day before the increase, how much more will he get now?

10. On a spelling test of 100 words, Roy spelled 89 words correctly. Sarah said that he got $\frac{89}{100}$ of the test right. Say this in percentage language.

11. Joe is trying to save 35¢ out of every dollar he earns. What part of each dollar will he save? Say this in percentage language.

12. Albert sorted apples before packing them in a barrel. He found, on an average, 7 bad apples in every 100 apples. What part of each 100 apples was bad? Say this in percentage language.

13. The school superintendent found that .87 of the pupils in his city had not been absent or tardy for six months. Say this in percentage language.

14. Mr. Law raises chickens. He says that 73% of the eggs which he put in the incubator hatched. What does this mean? What other fact do you need to know if you wish to find out how many eggs hatched?

Tell what each of these statements means:

15. Milk is about 87% water.

16. Good milk contains as much as 5% butter fat.

17. Kansas produces 13% of our wheat supply.

18. About 45% of the world's gold comes from Africa.

19. The population of the United States is about 7% of the population of the entire world.

20. Dick bought an overcoat marked "85% wool."

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EXERCISES IN FINDING PER CENTS

Find each of the following per cents:

1. 56% of 25	**12.** 5% of $300	**23.** 6% of 950
2. 35% of 80	**13.** 85% of $40	**24.** 35% of 680
3. 65% of 60	**14.** 24% of $75	**25.** 45% of 480
4. 68% of 50	**15.** 7% of $350	**26.** 7% of 8500
5. 56% of 75	**16.** 6% of $240	**27.** 85% of 1320
6. 2% of 850	**17.** 3% of $754	**28.** 82% of 7750
7. 5% of 360	**18.** 9% of $198	**29.** 5% of 12,460
8. 8% of 250	**19.** 76% of $355	**30.** 1% of 93,500
9. 5% of 240	**20.** 93% of $298	**31.** 9% of 18,500
10. 6% of 150	**21.** 16% of $258	**32.** 3% of 65,900
11. 8% of 825	**22.** 85% of $298	**33.** 76% of 19,350

34. In Reed City, 92% of the 25,850 schoolchildren were promoted. How many children were promoted?

35. Mrs. Day plans to buy on the installment plan furniture which costs $159.70. The clerk offers her a reduction of 8% if she will pay the total amount in cash. How much will she save by paying cash?

Since 8% of 159.70 means .08×159.70, multiply as shown here. The result is $12.776, which, to the nearest cent, is $12.78.

$$\begin{array}{r} \$159.70 \\ .08 \\ \hline \$12.7760 \end{array}$$

Mrs. Day will save $12.78 by paying cash.

Find each answer correct to the nearest cent:

36. 1% of $258.85	**40.** 31% of $998.60
37. 2% of $217.60	**41.** 65% of $250.90
38. 5% of $440.50	**42.** 11% of $500.75
39. 12% of $985.55	**43.** 93% of $233.60

A SHORT WAY TO FIND 25% OF A NUMBER

1. Since 25% of 160 means .25 of 160, you may find 25% of \$160 by multiplying \$160 by .25. This gives \$40 as the result.

2. You know also that $25\% = \frac{25}{100}$, which can be reduced to $\frac{1}{4}$. Hence $25\% = \frac{1}{4}$. Instead of finding 25% of \$160 by multiplying \$160 by .25, you can get the same answer in a shorter way by taking $\frac{1}{4}$ of \$160.

	\$160
	.25
	8 00
	32 0
	\$40.00
4)	\$160
	\$ 40

3. In the same way, $50\% = \frac{50}{100}$, which can be reduced to $\frac{1}{2}$; hence $50\% = \frac{1}{2}$. Similarly, $75\% = \frac{75}{100} = \frac{3}{4}$.

4. *Remember these short cuts:*

25% of anything is the same as $\frac{1}{4}$ of it.
50% of anything is the same as $\frac{1}{2}$ of it.
75% of anything is the same as $\frac{3}{4}$ of it.

Exercises

1. On an arithmetic test Benjamin received a mark of 50%. If there were 16 problems in the test, how many did Benjamin get right?

2. Eleanor got $\frac{3}{4}$ of her problems right to-day. What per cent was her paper marked?

3. Mr. Benson buys an automobile for \$1200. He pays 50% cash when he gets the car and the balance in monthly installments. How much cash does he pay?

4. Two stores were selling basket balls at the same price. One store now advertises them at "25% less than regular price." The other store offers them at "$\frac{1}{4}$ off." Which store offers the better bargain?

USING THE SHORT CUTS

1. The motor on a new Kent car gives 25% more power than last year's model. If the old motor was 32 horse power, what is the horse power of the new motor?

2. India produces about $\frac{1}{4}$ of the world's crop of cane sugar. Say this in percentage language.

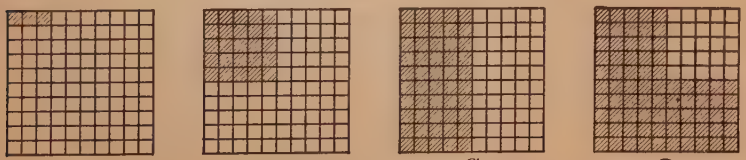
3. Jane was marked 75% on a spelling test of 60 words. How many words did she have right?

4. Jerry and Frank decide to share the expenses of a camping trip "fifty-fifty." What does this mean? Say it in percentage language.

Use the short cuts to find the following:

5. 25% of 64	**10.** 25% of \$1.64	**15.** 75% of \$3.60
6. 50% of 80	**11.** 25% of \$5.20	**16.** 50% of \$2.87
7. 75% of 76	**12.** 75% of \$8.48	**17.** 75% of \$4.56
8. 25% of 48	**13.** 50% of \$5.00	**18.** 25% of \$1.25
9. 75% of 40	**14.** 50% of \$1.35	**19.** 25% of \$2.70

20. Into how many small squares is A divided? Each small square is what part of A? what per cent of A?



A *B* *C* *D*

21. $\frac{1}{4}$ of B is shaded. By counting the small squares in the shaded part, show that $\frac{1}{4} = \frac{25}{100} = 25\%$.

22. In C, show that $\frac{1}{2} = \frac{50}{100} = 50\%$.

23. In D, show that $\frac{3}{4} = \frac{75}{100} = 75\%$.

A SHORT WAY TO FIND 10% OR 20% OF A NUMBER

1. Louis, the treasurer of the athletic association, says that 10% of the 130 members have not paid their dues. How many have not yet paid?

$10\% = \frac{10}{100}$, which can be reduced to $\frac{1}{10}$. Therefore $10\% = \frac{1}{10}$. A short way to find 10% of 130 is to take $\frac{1}{10}$ of 130, which gives 13.

2. Prove the following by changing each per cent to a fraction and reducing the fraction to lowest terms. Then memorize them:

$10\% = \frac{1}{10}$	$30\% = \frac{3}{10}$	$60\% = \frac{3}{5}$
$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$80\% = \frac{4}{5}$

3. About 10% of the inhabitants of Missouri live in Kansas City. Say this in another way.

4. Jack earns \$15 a week. If his salary is increased 20%, how much more will he earn each week?

5. About $\frac{4}{5}$ of the moving pictures made in America are also used abroad. Say this in another way.

Use the short cuts to find the following per cents:

6. 10% of 70	**13.** 60% of 280	**20.** 10% of \$1.30
7. 20% of 45	**14.** 40% of 260	**21.** 20% of \$9.85
8. 60% of 85	**15.** 20% of 800	**22.** 10% of \$0.60
9. 40% of 45	**16.** 30% of 580	**23.** 40% of \$8.50
10. 20% of 15	**17.** 10% of 240	**24.** 20% of \$6.70
11. 10% of 20	**18.** 10% of 550	**25.** 10% of \$0.40
12. 20% of 95	**19.** 60% of 780	**26.** 80% of \$0.75

MORE SHORT CUTS IN FINDING PER CENTS

1. Nat saw this advertisement in a magazine. What does it mean?

BOYS' SUITS
BUY DIRECTLY FROM THE FACTORY
AND
SAVE $33\frac{1}{3}\%$
BOSTON SUIT COMPANY

$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100}$, which may be changed to a smaller fraction in this way:

$$\frac{33\frac{1}{3}}{100} = \frac{33\frac{1}{3}\times 3}{100\times 3} = \frac{100}{300} = \frac{1}{3}$$

Hence to save $33\frac{1}{3}\%$ of the price means to save $\frac{1}{3}$ of it.

2. You often see $66\frac{2}{3}\%$ and $12\frac{1}{2}\%$ in advertisements. These per cents are changed to fractions in this way:

$$66\tfrac{2}{3}\% = \frac{66\frac{2}{3}}{100} = \frac{66\frac{2}{3}\times 3}{100\times 3} = \frac{200}{300} = \frac{2}{3} \qquad 12\tfrac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$$

3. *Memorize the following:*

$33\frac{1}{3}\% = \frac{1}{3}$	$16\frac{2}{3}\% = \frac{1}{6}$	$12\frac{1}{2}\% = \frac{1}{8}$	$62\frac{1}{2}\% = \frac{5}{8}$
$66\frac{2}{3}\% = \frac{2}{3}$	$83\frac{1}{3}\% = \frac{5}{6}$	$37\frac{1}{2}\% = \frac{3}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$

Exercises

1. Ida got $\frac{2}{3}$ of the examples on a test right. What per cent should her paper be marked?

2. Hugh bought a bicycle for $27.75. After several months he sold it to Bill for $66\frac{2}{3}\%$ of what he had paid for it. How much did Bill pay?

3. Show how to change $16\frac{2}{3}\%$ to $\frac{1}{6}$. Show also how to change $37\frac{1}{2}\%$ to $\frac{3}{8}$ and $62\frac{1}{2}\%$ to $\frac{5}{8}$.

4. Show how to change $83\frac{1}{3}\%$ to $\frac{5}{6}$ and $87\frac{1}{2}\%$ to $\frac{7}{8}$.

IMPORTANT PRODUCTS OF THE WORLD

1. Jack reads that $33\frac{1}{3}\%$ of the 10,770,000 bu. of flaxseed produced annually in this country comes from Minnesota. Say this in another way. How many bushels of flaxseed does Minnesota produce annually?

2. Mexico produced $37\frac{1}{2}\%$ of the $146,000,000 worth of silver produced in the world one year. Find the value of Mexico's silver output that year.

3. Brazil produced $12\frac{1}{2}\%$ of the world's rubber supply one year. If the whole world produced 600,000,000 lb. of rubber that year, how much did Brazil produce?

4. The United States produced 11,000,000 bales of cotton one year. If $16\frac{2}{3}\%$ of this amount came from North Carolina and South Carolina together, how many bales did these two states produce that year?

Finding Per Cents by Short Cuts

Find $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, *and* $16\frac{2}{3}\%$ *of each number:*

1. 72	**4.** 234	**7.** 444	**10.** $3.42	**13.** $2.88
2. 96	**5.** 282	**8.** 270	**11.** $2.28	**14.** $4.35
3. 84	**6.** 156	**9.** 516	**12.** $4.98	**15.** $6.27

Find $12\frac{1}{2}\%$, $37\frac{1}{2}\%$, $62\frac{1}{2}\%$, *and* $87\frac{1}{2}\%$ *of each:*

16. 64	**19.** 192	**22.** 344	**25.** $4.64	**28.** $6.24
17. 96	**20.** 312	**23.** 840	**26.** $5.36	**29.** $10.00
18. 56	**21.** 168	**24.** 608	**27.** $2.32	**30.** $15.92

Find the following per cents:

31. $33\frac{1}{3}\%$ of $15.87
32. $16\frac{2}{3}\%$ of $17.04
33. $66\frac{2}{3}\%$ of $15.51
34. $12\frac{1}{2}\%$ of $1540.80
35. $37\frac{1}{2}\%$ of $1800.00
36. $87\frac{1}{2}\%$ of $1555.50

IMPROVEMENT TEST No. 7

Test 7 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$6\frac{3}{8}$	$5\frac{3}{8}$	$9\frac{1}{2}$	$3\frac{3}{4}$	$6\frac{1}{6}$
$1\frac{1}{2}$	$2\frac{3}{4}$	$8\frac{3}{4}$	$2\frac{3}{5}$	$5\frac{1}{2}$
$\frac{1}{4}$	$1\frac{1}{2}$	$\frac{2}{3}$	$4\frac{1}{5}$	$4\frac{3}{4}$
$2\frac{3}{16}$	$8\frac{1}{2}$	$6\frac{1}{2}$	$1\frac{1}{10}$	$2\frac{5}{12}$

Test 7 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$1\frac{1}{2}$	9	$7\frac{1}{4}$	$6\frac{5}{16}$
$\frac{1}{3}$	$7\frac{5}{8}$	$2\frac{1}{5}$	$3\frac{3}{4}$
5.	**6.**	**7.**	**8.**
$8\frac{3}{4}$	7	$8\frac{1}{8}$	$9\frac{1}{12}$
$\frac{1}{3}$	$6\frac{1}{12}$	$5\frac{3}{4}$	$8\frac{1}{2}$
9.	**10.**	**11.**	**12.**
$2\frac{9}{10}$	8	$3\frac{1}{6}$	$9\frac{3}{8}$
$\frac{3}{5}$	$5\frac{7}{10}$	$1\frac{3}{4}$	$4\frac{1}{16}$

Test 7 C — Multiplication (Time 3 min. after copying)

1. $\frac{3}{8} \times \frac{1}{2}$	2. $5\frac{1}{6} \times 9$	3. $7 \times 62\frac{1}{2}$
4. $\frac{5}{6} \times \frac{3}{8}$	5. $9\frac{3}{4} \times 20$	6. $8\frac{5}{8} \times 48$
7. $\frac{3}{10} \times 55$	8. $2\frac{3}{4} \times 42$	9. $13\frac{1}{2} \times 1\frac{1}{3}$

Test 7 D — Division (Time 3 min. after copying)

1. $2\frac{1}{2} \div 5$	2. $5 \div \frac{3}{4}$	3. $2\frac{1}{4} \div \frac{3}{4}$
4. $1\frac{2}{3} \div 3$	5. $6 \div \frac{4}{5}$	6. $6\frac{1}{2} \div \frac{1}{4}$
7. $3\frac{3}{4} \div 10$	8. $9 \div 3\frac{3}{5}$	9. $1\frac{1}{4} \div 3\frac{1}{8}$

ESTIMATING THE ANSWER IN PERCENTAGE

In percentage, you sometimes *estimate* an answer mentally to find out approximately what it will be.

Arthur wanted to know about how large 49% of \$240 is. Since 49% is almost 50%, or $\frac{1}{2}$, Arthur found $\frac{1}{2}$ of \$240 mentally and got \$120. Is \$120 more or less than the exact answer? Find the exact answer.

To estimate 34% of 90, find $\frac{1}{3}$ of 90, because 34% is close to $33\frac{1}{3}\%$, which equals $\frac{1}{3}$.

Exercises

1. How can you estimate 26% of \$400? Will the estimate be more or less than the exact answer?

2. Louise got a mark of 52% on a spelling test of 50 words. About how many words did she spell correctly?

3. Harvey heard that 19% of the 450 boys in his school failed in spelling. About how many boys failed?

Think of 19% as about 20%, which equals $\frac{1}{5}$.

First estimate the approximate answer for each problem. Then find the correct answer:

4. 51% of 200	**9.** 9% of 850	**14.** 76% of \$1.28
5. 19% of 150	**10.** 30% of 660	**15.** 11% of \$9.40
6. 17% of 360	**11.** 13% of 800	**16.** 18% of \$6.00
7. 74% of 880	**12.** 41% of 250	**17.** 52% of \$2.25
8. 89% of 500	**13.** 48% of 150	**18.** 12% of \$2.40

19. If 39% of the 1600 children in a school are underweight, how many are underweight?

Blanche thinks of 39% as nearly $37\frac{1}{2}\%$, or $\frac{3}{8}$, while Howard thinks of it as almost 40%, or $\frac{2}{5}$. How can you tell whose estimate will be closer to the exact answer?

A NEW WAY TO SPEAK ABOUT THE WHOLE OF ANYTHING

1. Ben was marked 100% on his arithmetic test. If there were 8 examples in the test, how many examples did he get right? What is 100% of 8 examples?

Since per cent means hundredths, $100\% = \frac{100}{100}$, or 1. Hence 100% of 8 examples means 1×8 examples, or 8 examples.

Ben got all 8 examples right.

$$\mathbf{100\% = \frac{100}{100} = 1 = \text{one unit} = \text{all} = \text{the whole}}$$

2. You see that 100% of anything means all of it.

100% of \$250 equals $1 \times \$250$, or *all of* \$250.

100% of a number means the entire number.

100% of a class means the whole class.

Exercises

1. The sign on this new building reads "100% fireproof." What does this mean? Are the walls of this building fireproof? the ceilings? the floors?

Tell what each of these means:

2. At a factory 100% of the workmen have savings bank accounts.

3. Ethel's dress is 100% wool.

4. 100% of the pupils in our class belong to the Red Cross.

Say each of these in percentage language:

5. All of our school attended the baseball game.

6. Laura had all the words in a spelling test right.

7. Beatrice attended school every day last term.

SUBTRACTING FROM 100%

1. Claude saved 25% of his salary last summer. What per cent of his salary did he spend?

All of Claude's salary is 100% of it. If he saved 25% of his salary, he spent 75% of it. You find 75% by subtracting 25% from 100%.

2. Sidney has 75% of his problems right. What per cent of them are wrong?

3. In Harriet's class, 20% of the children were absent during a storm. What per cent of them were present?

4. The Lang School won 65% of the baseball games they played. What per cent of the games did they lose?

5. If 90% of Miss Murray's class passed in arithmetic, what per cent of the class did not pass?

6. A blanket is guaranteed to be 75% camel's hair and the rest pure wool. What per cent of the blanket is wool?

7. In buying furniture Mrs. Blake agreed to pay 20% of the price in cash and the rest later. What per cent of the price did she pay later?

8. Edwin sold his sled for 10% less than it cost him. What per cent of the cost did he get for it?

9. Maud spends 50% of her allowance for lunches and 20% for car fare. If she saves the rest, what per cent of her allowance does she save?

10. In Ruth's class 24% of the pupils are underweight and 12% are overweight. What per cent of the class are the right weight?

11. India supplies 35% of the world's tea, China supplies 26%, and Ceylon supplies 25%. What per cent of the total supply of tea comes from the rest of the world?

SELLING MAGAZINES

1. When Lewis Brooks read this advertisement, he wrote the following letter:

Radio Monthly
Chicago, Ill.
Dear Sirs:

I should like to get subscriptions for the Radio Monthly. Your advertisement says that you pay 30% commission. Does this mean that I may keep 30% of each $2.00 that I collect for subscriptions?

Yours truly,
Lewis Brooks

AGENTS WANTED

to get subscriptions for the

RADIO MONTHLY

Price $2.00 a year

30% Commission to Agents

2. What reply do you think the Radio Monthly sent Lewis in answer to his question?

3. The first week Lewis got 8 subscriptions and collected $16. He kept 30% of this amount. How much did he keep?

Lewis kept 30% of $16, or $4.80. This $4.80 is called his **commission.** The remaining $11.20, which he sent to the company, is called the **net proceeds.**

$16	$16.00
.30	4.80
$4.80	$11.20

4. Lewis got 17 subscriptions at $1.50 each for another magazine whose agents get 20% commission. Find his commission and the net proceeds.

5. In the summer Polly sells books on a commission of 35%. What is her commission on sales amounting to $50? on sales of $210? on sales of $340?

SELLING ON COMMISSION

1. An auctioneer agrees to sell a piano for Mrs. Harris at a commission of 4%. This means that his pay will be 4% of the price he gets for the piano. What is his commission if he sells the piano for $350? How much will Mrs. Harris receive?

2. Mr. Blake sold his house for $8000 through an agent who charged 2% commission. How much did the agent receive? Find Mr. Blake's net proceeds.

3. Dick sells Mr. Miller's fruit and vegetables during the summer and gets 10% commission. One week he sold $145.60 worth. Find his commission that week.

4. Lillian works in a dress store. She gets $15 a week salary and besides that she receives a commission of 2% on all her sales. If her sales are $585 in one week, find her total earnings that week.

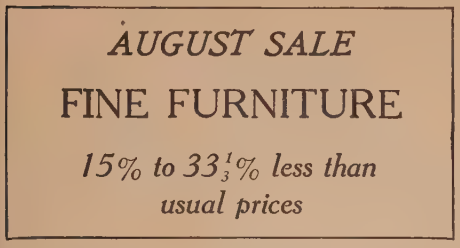
5. Jack sells a radio set for $75 and gets $15 commission. Show that he really receives 20% commission.

Find the commission and the net proceeds on each sale. The first number is the amount of the sales, while the second number is the rate of commission:

6. $950, 40%	**13.** $178, $16\frac{2}{3}\%$	**20.** $650, $16\frac{2}{3}\%$
7. $365, 35%	**14.** $250, $33\frac{1}{3}\%$	**21.** $480, $12\frac{1}{2}\%$
8. $380, 10%	**15.** $475, 20%	**22.** $725, 28%
9. $177, 20%	**16.** $350, 15%	**23.** $1195, 3%
10. $485, 15%	**17.** $160, $12\frac{1}{2}\%$	**24.** $3650, 4%
11. $375, 30%	**18.** $340, 10%	**25.** $4980, 5%
12. $640, 25%	**19.** $243, 45%	**26.** $4230, 2%

BUYING AT A SALE

When you buy anything at less than the usual price, you are receiving a **discount.** If you buy a table marked \$30 during a sale at $33\frac{1}{3}\%$ less than its marked price, the discount is $33\frac{1}{3}\%$ of \$30, or \$10. Hence you get the table for \$30 − \$10, or \$20.



Exercises

Find all answers correct to the nearest cent.

1. Mrs. Thomas buys furniture for Jack's room. Here are the regular prices, on which she receives a discount of 25%. How much does she pay for each article?

Enameled bed . .	\$15.60	Chiffonier . .	\$18.00
Writing desk . .	22.85	Radio table . .	4.75
Bookcase	14.00	Stool	1.69

2. Mrs. Thomas also buys these things for Anna's room and gets a discount of 15% from the regular prices. Find the sale price of each article.

Wooden bed . . .	\$22.50	Bureau . . .	\$18.40
Rocking chair . .	5.00	Lamp	8.00
Sewing table . . .	3.25	Pillow	2.30

3. Mrs. Thomas buys these articles at $33\frac{1}{3}\%$ less than the regular prices. How much does she pay for each?

China closet . . .	\$35.00	Chair	\$12.60
Dining-room table .	36.75	Couch	29.40

4. Work ex. 3 again using 40% as the rate of discount.

BUYING GOODS AT A DISCOUNT

1. If footballs are selling at $33\frac{1}{3}\%$ off, how much does a football marked \$10.85 cost?

2. If fur coats are sold at 35% less than the marked price, find the cost of a coat marked \$175.

3. Mr. Davis will sell Frank a bicycle for \$28.75 on the installment plan. If Frank pays cash, he gets 8% discount. How much does he save by paying cash?

4. A flashlight, formerly marked \$3.80, now costs \$3.23. Jack says that it is being sold at a discount of 15%. Show whether or not Jack is right.

5. Mrs. Stone sells at a discount of 25% a girl's coat which was formerly marked \$17. She also offers a coat whose price was \$22.50 at a discount of $33\frac{1}{3}\%$. Find the sale price of each coat.

6. The Ajax Store is selling tennis rackets marked \$5.00 at $33\frac{1}{3}\%$ less than the marked price. The Sports Store is selling similar rackets at \$3.50. Which is the better bargain?

7. If Mrs. Loft pays her electric light bill before the fifth of the month, she gets 5% discount. How much does she save on a bill for \$3.87 by paying promptly?

Find, to the nearest cent, the discount on each amount. The amounts given are the regular prices:

8. \$4.50, $16\frac{2}{3}\%$	**12.** \$6.45, 25%	**16.** \$3.95, $12\frac{1}{2}\%$
9. \$8.50, 40%	**13.** \$9.60, 15%	**17.** \$8.67, $33\frac{1}{3}\%$
10. \$7.74, $33\frac{1}{3}\%$	**14.** \$10.98, 30%	**18.** \$5.98, $37\frac{1}{2}\%$
11. \$3.75, 50%	**15.** \$15.46, 20%	**19.** \$6.72, 35%

INTEREST

Mr. Thompson rents a house at the seashore. He pays the owner $75 a month rent for the use of the house.

Mr. Fields borrows $500 from Mr. Clark. He must pay Mr. Clark 6% of the $500, or $30, for the use of this money for 1 year. The $500 which he borrows is the **principal;** the $30 which he pays for the use of it is the **interest.** Interest is paid for the use of money, just as rent is paid for the use of a house.

The $30 interest is 6% of $500; 6% is called the **rate of interest.** Unless otherwise agreed, the rate of interest is understood to be the charge for 1 year's use of the money. If Mr. Fields borrows the money for 2 years, he has to pay twice as much interest, or $60.

When money is put in the savings bank, the bank pays interest on it because the bank has the use of the money and often lends it to other people.

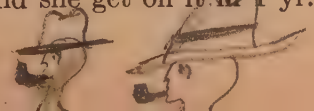
Exercises

Find the interest on each amount for 1 *yr.:*

1. $200 at 6%	**7.** $35 at 4%	**13.** $1450 at 6%
2. $350 at 5%	**8.** $80 at 5%	**14.** $2600 at 5%
3. $800 at 4%	**9.** $75 at 6%	**15.** $1000 at 3%
4. $150 at 7%	**10.** $90 at 7%	**16.** $1250 at 4%
5. $750 at 6%	**11.** $110 at 3%	**17.** $16.25 at 8%
6. $250 at 7%	**12.** $175 at 6%	**18.** $27.50 at 7%

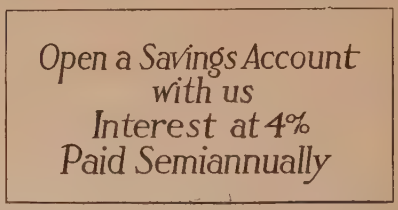
19. Fred borrowed $50 from his brother for 1 yr. at 5% interest. How much interest did he pay?

20. Amy put $25 in a savings bank which pays 4% interest. How much interest did she get on it in 1 yr.?



PAYING INTEREST SEMIANNUALLY

Roger saw this sign on the window of a savings bank.



A bank that pays interest semiannually pays interest every 6 mo. instead of every year. If Roger has \$25 in this bank, the interest on it for 1 yr. at 4% is \$1; but instead of making Roger wait a year for his interest, the bank pays half of it (50¢) at the end of 6 mo. and the other half at the end of the year.

Exercises

1. Mr. Brett has \$800 in a bank that pays interest at 4% semiannually. How much interest does he receive each half year?

Interest "at 4%" means at 4% *for one year*. For 6 mo. the interest is half as much. The interest on \$800 for a whole year at 4% is \$32, while for 6 mo. it is $\frac{1}{2}$ of \$32, or \$16.

Hence Mr. Brett receives \$16 interest every 6 mo.

$$\begin{array}{r} 800 \\ .04 \\ \hline 2\overline{)32.00} \\ 16.00 \end{array}$$

2. Belle has \$35 in a savings bank that pays 4%. How much interest does she get every 6 mo.?

3. Paul borrowed \$600 and paid interest on it semiannually at 6%. How much did he pay every 6 mo.?

Tell how much interest is paid semiannually on:

4. \$150 at 6%	**8.** \$65 at 4%	**12.** \$42 at 4%
5. \$280 at 6%	**9.** \$30 at 4%	**13.** \$15 at 4%
6. \$700 at 4%	**10.** \$75 at 8%	**14.** \$75 at 6%
7. \$950 at 6%	**11.** \$90 at 7%	**15.** \$50 at 5%

BORROWING MONEY TO GO TO COLLEGE

1. George borrowed \$800 at 6% to enable him to go to college. He kept the money for 4 yr. How much interest did he pay in that time?

The interest for 1 yr. is 6% of \$800, or \$48.
Hence the total interest for 4 yr. is 4×\$48, or \$192.
You see that in 4 yr. George paid \$192 interest.
Arrange the work as shown at the right.

800
.06
48.00
4
192.00

2. Myra borrowed \$700 for $2\frac{1}{2}$ yr. to help her finish her college course. At 5%, how much interest did she have to pay during that time?

3. A student who borrowed \$1250 at 6% kept the money for 3 yr. Find the total interest.

4. When he started to go to college, Glenn borrowed \$600 at 7% interest. He repaid the loan 5 yr. later. Find the total interest he paid during that time.

Find, to the nearest cent, the interest on:

5. \$200 for 2 yr. at 5%
6. \$250 for 5 yr. at 5%
7. \$780 for 3 yr. at 4%
8. \$150 for 3 yr. at 6%
9. \$900 for 4 yr. at 7%
10. \$650 for 2 yr. at 7%
11. \$340 for 5 yr. at 4%
12. \$980 for 4 yr. at 6%
13. \$1500 for 6 yr. at 5%
14. \$3850 for 5 yr. at 3%
15. \$6275 for 4 yr. at 5%
16. \$500 for $2\frac{3}{4}$ yr. at 6%
17. \$600 for $3\frac{1}{2}$ yr. at 5%
18. \$350 for $1\frac{1}{4}$ yr. at 4%
19. \$980 for $4\frac{1}{2}$ yr. at 6%
20. \$740 for $3\frac{3}{4}$ yr. at 3%
21. \$670 for $3\frac{1}{2}$ yr. at 7%
22. \$350 for $1\frac{1}{4}$ yr. at 7%
23. \$495 for $5\frac{1}{2}$ yr. at 6%
24. \$876 for $2\frac{1}{4}$ yr. at 5%
25. \$235 for $3\frac{1}{2}$ yr. at 7%
26. \$913 for $1\frac{3}{4}$ yr. at 6%

FINDING INTEREST FOR PARTS OF A YEAR

1. What part of a year is 1 mo.? 5 mo.? 7 mo.? 4 mo.? 6 mo.? 3 mo.? 9 mo.? 8 mo.?

2. If I borrow \$400 for 1 yr. from Mr. Mead, he charges me \$24 interest. How much should he charge me for 6 mo.? for 1 mo.? 5 mo.? 3 mo.? 8 mo.? 9 mo.?

3. Mr. Long lends John \$150 at 5%. If John keeps the money for 7 mo., how much interest does he pay?

The interest for 1 yr. is 5% of \$150, or \$7.50. Since 7 mo. is $\frac{7}{12}$ of 1 yr., the interest for 7 mo. is $\frac{7}{12} \times \$7.50$, or \$4.38, to the nearest cent. The product \$4.38 is found by multiplying \$7.50 by 7 and dividing the result by 12, as shown at the right.

$$12\overline{)52.50}\quad 7 \quad 7.50 \quad 4.37\tfrac{1}{2}$$

4. Mr. Parker borrows \$1500, which he pays back at the end of 8 mo. with interest at 6%. How much interest does he pay? What is the total amount due at the end of 8 mo.?

Find the interest, to the nearest cent, on:

5. \$200 for 6 mo. at 4%
6. \$350 for 3 mo. at 6%
7. \$700 for 9 mo. at 5%
8. \$550 for 4 mo. at 7%
9. \$840 for 7 mo. at 6%
10. \$920 for 8 mo. at 5%
11. \$175 for 6 mo. at 5%
12. \$300 for 4 mo. at 6%
13. \$500 for 1 mo. at 6%
14. \$890 for 2 mo. at 6%
15. \$450 for 5 mo. at 6%
16. \$950 for 2 mo. at 6%
17. \$700 for 7 mo. at 4%
18. \$1000 for 3 mo. at 5%
19. \$1250 for 8 mo. at 6%
20. \$4500 for 1 mo. at 6%

21. Mr. Roberts borrows \$350, at 6% interest, to repair his house. He keeps the money 2 yr. 9 mo. How much interest does he pay during this time?

IMPROVEMENT TEST No. 8

Test 8 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
962	745	821	630	514
257	984	353	231	895
326	623	844	751	356
837	435	430	939	583
942	992	248	927	767
707	159	818	535	382
655	750	887	161	386
195	767	681	168	462
900	398	567	729	107
618	123	344	747	136

Test 8 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
745091	807842	609356	958300
480812	552085	234696	763970
5.	**6.**	**7.**	**8.**
977319	623525	924414	900000
679071	130257	380457	215309
9.	**10.**	**11.**	**12.**
664176	737811	615643	500003
372229	390244	478055	386418

Test 8 C — Multiplication (Time 4 min. after copying)

1.	2.	3.	4.	5.
483	418	576	863	475
928	763	725	518	438

Test 8 D — Division (Time 4 min. after copying)

1. $173\overline{)44807}$ 2. $748\overline{)433840}$ 3. $688\overline{)567945}$

MR. SMITH'S AUTOMOBILE EXPENSES

1. Mr. Smith paid \$1270 for an automobile. After using the car for 2 yr., he sold it for \$725. How much did it lose in value? Such a loss is called *depreciation* and is one of the expenses of owning a car.

2. If Mr. Smith had not bought the car, he could have received 6% interest on the \$1270 he paid for it. How much would the interest be for 2 yr.? This loss of interest is another one of the expenses of owning a car.

3. During the 2 yr. Mr. Smith used a total of 1367 gal. of gasoline, which cost an average of 18¢ a gallon. He also used 51 gal. of oil at 64¢ a gallon. How much did the fuel and oil together cost?

4. During the 2 yr. he also paid \$140 for tires, \$62.65 for taxes, \$64.74 for repairs, washing, etc., and he was in an automobile accident that cost him \$75.99. Find the total amount of these items.

5. He paid \$8 a month garage rent, \$7 each year for a car license, and \$3 each year for a driver's license. How much did these items amount to in 2 yr.?

6. Find the total cost of the car for the 2 yr. that Mr. Smith had it. (Add together the answers of ex. 1 to 5.)

7. Mr. Smith drove the car a total of 20,151 mi. during the 2 yr. he had it. Using the total cost found in ex. 6, find, to the nearest tenth of a cent, how much it cost him per mile to run his car.

8. Since Mr. Smith used 1367 gal. of gasoline to travel 20,151 mi. (see ex. 3 and 7), how many miles did he average per gallon of gasoline? Find the result to the nearest tenth of a mile.

DENOMINATE NUMBERS

Measuring Lengths

Last year you learned about important units of measure like the foot, the mile, the pound, and the quart. Numbers like 3 ft., $4\frac{1}{2}$ lb., and 7 qt. are called **denominate numbers.**

Review this table of *length:*

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards	= 1 rod (rd.)
5280 feet or 320 rods	= 1 mile (mi.)

Exercises

1. How many feet are there in 3 yd.? in 2 rd.? in $\frac{1}{4}$ mi.? in 100 in.? in 6 in.?

2. How many yards are there in 6 rd.? in $\frac{1}{2}$ mi.? in 96 ft.?

3. A boy's step measures $2\frac{1}{4}$ ft. In walking $\frac{3}{4}$ mi. how many steps will he take?

4. Mr. Archer finds that the distance around the deck of a steamship is 264 ft. How many times must he go around the deck to walk a mile?

5. Jack said that his uncle's collection of stamps was half a mile long. He meant that the stamps, put end to end, would cover $\frac{1}{2}$ mi. If the average length of a stamp is 1 in., how many stamps had Jack's uncle?

6. On field day, Jack ran the 200-yard dash. How far, to the nearest hundredth of a mile, did he run?

7. The deepest place in the Atlantic Ocean is 31,366 ft. deep. Express this to the nearest hundredth of a mile.

HOW FRUITS AND VEGETABLES ARE MEASURED

Fruits, vegetables, and grain usually are sold either by weight or by dry measure.

Review this table of *dry measure:*

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Review this table of *weight:*

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

Exercises

1. How many quarts are there in $1\frac{1}{2}$ pk.? in $\frac{1}{2}$ bu.? in 7 pt.? in $1\frac{1}{4}$ bu.? in $\frac{1}{4}$ pk.? in 1 pt.?

2. How many pounds are there in 8 oz.? in $\frac{1}{4}$ T.? in $1\frac{1}{2}$ cwt.? in 24 oz.? in 5 T.? in 10 cwt.?

3. John sold string beans from his garden at 10¢ a quart. How much did he receive for 4 pk.? for 7 qt.?

4. Mr. Ross feeds his horse $1\frac{1}{4}$ pk. of oats a day. How many days will 10 bu. of oats last?

5. Coal was delivered to a factory in truck loads. Each time that a load was delivered, the truck full of coal was weighed on the factory scales; the weight of the truck was then subtracted, which gave the weight of the coal. It was thus found that the number of pounds of coal delivered in the four loads was as follows: 6550, 6810, 6905, and 6135. How many tons of coal were delivered?

MEASURING LIQUIDS

1. Review this table of *liquid measure:*

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

2. How many quarts are there in $6\frac{1}{2}$ gal.? in 10 pt.? in $\frac{3}{4}$ gal.? in 1 pt.? in $1\frac{1}{2}$ gal.? in 20 gal.?

3. How many gallons are there in 30 qt.? in 18 pt.?

4. Mr. Low gets \$3.20 per 100 lb. for milk. How much will he get for 1250 lb. of milk?

5. How many gallons of ice cream are needed for 88 children if $\frac{1}{2}$ pt. is allowed for each child?

MEASURING TIME

1. Review this table of *time:*

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days or 12 months (mo.) = 1 year (yr.)
366 days = 1 leap year
100 years = 1 century

2. If your heart beats 80 times a minute, how many times does it beat in a day? how many times in a week? in a year (52 weeks)? Do you see why you should take good care of your heart?

3. Beginning at 6.08 A.M., the trolley passes our house every 12 min. Make a time-table showing every time the trolley passes from 6.08 A.M. until noon.

HOW TO REDUCE COMPOUND NUMBERS

1. Virginia Henderson is 4 ft. 10 in. tall. How many inches tall is Virginia?

Doing the work as at the right, you see that 4 ft. 10 in. = 58 in. Hence Virginia is 58 in. tall.

1 ft. = 12 in.
4 ft. = 48 in.
48 in. + 10 in. = 58 in.

2. James finds that his step is 27 in. long. How many feet and inches does 27 in. make?

There are 12 in. in 1 ft. To find the number of feet there are in 27 in., you must find how many times 12 in. is contained in 27 in. Dividing 27 in. by 12 in., you get 2, with 3 in. left over.

Hence 27 in. equals 2 ft. 3 in.

$$12\overline{)27}\quad 2,\ 24,\ 3$$

Numbers like 2 ft. 3 in., which contain two or more units of measure, are called **compound denominate numbers.**

Exercises

1. Mary bought some cloth 54 in. wide. How many yards and inches wide was it?

2. At school, Irving has 135 min. of shop work each week. How many hours and minutes is this?

3. Sam is 5 ft. 2 in. tall. How many inches tall is he?

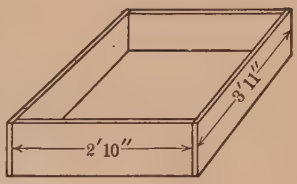
4. Mrs. Jackson bought 62 qt. of milk last month. How many gallons and quarts did she buy?

Tell what numbers should be put in place of the dots:

5. 7 yd. 2 ft. = . . . ft.
6. 4 lb. 2 oz. = . . . oz.
7. 4 gal. 3 qt. = . . . qt.
8. 5 bu. 3 pk. = . . . pk.
9. 2 hr. 7 min. = . . . min.
10. 70 in. = . . ft. . . . in.
11. 40 oz. = . . lb. . . . oz.
12. 19 qt. = . . pk. . . . qt.
13. 45 in. = . . yd. . . . in.
14. 11,220 ft. = . . mi. . . . ft.

HOW TO ADD FEET AND INCHES

To make the sides of this box, John needs two pieces of board each 3 ft. 11 in. long, and two pieces each 2 ft. 10 in. long. If John cuts the four pieces from one board, how long must the board be?



To add the lengths of the four pieces, first add the inches, which gives 42 in., or 3 ft. 6 in. Write 6 in. and carry 3 ft. to the next column. Then add the feet, which gives 13 ft.

John needs a board 13 ft. 6 in. long.

3 ft. 11 in.
3 ft. 11 in.
2 ft. 10 in.
2 ft. 10 in.
13 ft. 6 in.

Exercises

1. Mrs. Nelson's dining-room table is 5 ft. 6 in. long. If she puts in 2 extra leaves, each 1 ft. 4 in. wide, how long will the table be?

Add. Check the work by going over it again:

	ft.	in.		ft.	in.		yd.	ft.		ft.	in.
2.	1	2	**3.**	2	6	**4.**	1	2	**5.**	4	0
	7	9		9	3		7	2		3	11
	3	4		7	4		8	1		2	8
	5	8		8	10		9	2		4	1

	ft.	in.		ft.	in.		yd.	ft.		yd.	in.
6.	1	2	**7.**	1	7	**8.**	2	1	**9.**	2	3
	9	11		2	3		4	1		1	20
	6	10		4	11		5	0			6
	4	8		5	0		7	1		9	18
	9	9		8	8		8	2		5	31

ADDING HOURS AND MINUTES

Dick worked 3 hr. 20 min. on Monday, 2 hr. 45 min. on Tuesday, and 1 hr. 45 min. on Friday. Find his total time.

Adding the minutes, you get 110 min., which must be changed to 1 hr. 50 min. Write 50 min. and carry 1 hr. to the *hour* column. Adding the hours, you get 7 hr.

Hence Dick worked 7 hr. 50 min. in all.

3 hr. 20 min.
2 hr. 45 min.
1 hr. 45 min.
7 hr. 50 min.

Exercises

1. These children were paid by the hour for addressing envelopes. Find the total time each child worked.

	John	*Ellen*	*Nancy*
Mon.	1 hr. 20 min.	2 hr. 10 min.	1 hr. 35 min.
Tues.	2 hr. 5 min.	1 hr. 45 min.	45 min.
Wed.	55 min.	1 hr. 50 min.	2 hr. 15 min.
Thurs.	1 hr. 50 min.	2 hr. 30 min.	3 hr. 5 min.

2. Alfred helped last Saturday at the Sweets Shop. He sold 2 gal. 1 qt. of vanilla ice cream, 3 gal. 2 qt. of chocolate ice cream, and 4 gal. 3 qt. of peach ice cream. How much ice cream did he sell in all?

3. Robert orders articles of the following weights from a mail-order store: suit, 3 lb. 8 oz.; socks, 8 oz.; hat, 14 oz.; shoes, 1 lb. 6 oz.; gloves, 4 oz.; skates, 3 lb. 10 oz.; boots, 3 lb. 12 oz. Find the total weight of these articles.

Add. Check the work by going over it again:

	hr.	min.		hr.	min.		bu.	pk.		lb.	oz.
4.	1	10	**5.**	7	10	**6.**	2	1	**7.**	9	15
	3	45		5	35		1	3			4
	2	30		4	20		5	2		8	3
	2	18		3	25		3	1		2	10

SUBTRACTING FEET AND INCHES

John throws a basket ball 36 ft. 9 in. Walter throws it 41 ft. 5 in. By how much does Walter beat John?

Since you cannot take 9 in. from 5 in., think of 41 ft. 5 in. as 40 ft. 17 in. Then 17 in. − 9 in. = 8 in. and 40 ft. − 36 ft. = 4 ft.

Hence Walter beats John by 4 ft. 8 in.

41 ft. 5 in.
36 ft. 9 in.
4 ft. 8 in.

Exercises

1. Michael Donnelly jumps 5 ft. 5 in. in the standing broad jump. Tell by how much Michael beats each of the following boys:

Clyde,	5 ft. 2½ in.	Will,	5 ft. 0 in.
Henry,	4 ft. 10 in.	Hugh,	4 ft. 9 in.
Elmer,	4 ft. 7 in.	Dick,	4 ft. 6 in.

2. Mr. Hall is 6 ft. 2 in. tall and Ray is 4 ft. 10 in. tall. Find the difference in their heights.

3. A package sent by parcel post must not weigh more than 70 lb. What is the greatest number of pounds of books that you can ship in a box weighing 6 lb. 5 oz.? in a box weighing 8 lb. 3 oz.?

4. Arthur started on a hiking trip at 1.45 P.M. and got back at 5.10 P.M. How long was he gone?

5.10 P.M. means 5 hr. 10 min. after noon. What does 1.45 P.M. mean? Since you cannot subtract 45 from 10, think of 5 hr. 10 min. as 4 hr. 70 min. and then subtract as shown at the right.

5 hr. 10 min.
1 hr. 45 min.
3 hr. 25 min.

Subtract. Check the work by going over it again:

	ft.	in.		yd.	ft.		lb.	oz.		hr.	min.
5.	8	2	**6.**	7	1	**7.**	6	10	**8.**	7	15
	4	7		5	2		3	15		5	30

HOW FAR CAN YOU JUMP?

Healthy boys and girls should be able to make these records in running, jumping, and throwing.

Age	Standing Broad Jump		Running Broad Jump		50-Yard Dash		Ball Throw Free Style	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
11	4′ 10″	4′ 2″	8′ 3″	6′ 6″	$8\frac{3}{5}$ sec.	$9\frac{1}{5}$ sec.	38′	30′
12	5′ 0″	4′ 4″	9′ 3″	7′ 0″	$8\frac{1}{5}$ sec.	9 sec.	46′	32′
13	5′ 2″	4′ 6″	10′ 0″	7′ 2″	8 sec.	$8\frac{4}{5}$ sec.	54′	32′
14	5′ 6″	4′ 8″	10′ 6″	7′ 6″	$7\frac{4}{5}$ sec.	$8\frac{3}{5}$ sec.	60′	38′

You see that a boy of 11 should be able to jump 4 ft. 10 in. in the standing broad jump, to run 50 yd. in $8\frac{3}{5}$ sec., and to throw a ball 38 ft. These numbers are called the " age standards " for each event.

1. From the above table, tell how far a girl of 12 should jump in the standing broad jump; in the running broad jump. Tell how long it would take her to run 50 yd.

2. How much further should a boy of 11 jump in the standing broad jump than a girl of 11? Answer the same question for boys and girls of 12; of 13; of 14.

COMPARING ATHLETIC RECORDS

In working these problems, use the table on page 120.

1. How much farther should a boy of each age jump in a running broad jump than a girl of the same age?

2. In the 50-yard dash a boy of each age should run how much faster than a girl of the same age?

3. Compare each of these records in the running broad jump with the standards in the table:

Gene, age 11, 9′ 1″	Alice, age 12, 8′ 4″
Will, age 14, 8′ 8″	Fanny, age 13, 6′ 9″
Bert, age 13, 8′ 9″	Julia, age 11, 7′ 2″

4. How much farther did Gene jump than each of the other children in ex. 3?

5. Irving, who is 11, ran 50 yd. in $8\frac{1}{10}$ sec. How much less time was that than the standard for his age?

6. Elinor, who is 13, did 5 ft. 1 in. in a standing broad jump. How much is that above the girls' standard? How close is it to the boys' standard?

7. Find how far you can jump and throw a ball and how long it takes you to run 50 yd. Then compare your records with the standards for your age.

8. Barbara is 13 yr. old. She can throw a ball 40 ft. 3 in. By how much does she beat her standard?

9. In ex. 8, how much farther would Barbara have to throw the ball to reach the boys' standard for 13 yr.?

10. Edwin is 13 and does the 50-yard dash in $7\frac{3}{5}$ sec. How much better than the standard is this?

11. Kenneth is 12 yr. old. He threw a ball 42 ft. 8 in. How close did he come to the 12-year-old record?

PREPARING FOR CAMP

1. Edna is taking 4 blankets to camp. Each weighs 3 lb. 5 oz. What is the total weight of the blankets?

First multiply 5 oz. by 4, which gives 20 oz., or 1 lb. 4 oz. Write 4 oz. and carry 1 lb. Then multiply 3 lb. by 4 and add 1 lb. (carried), which gives 13 lb. The 4 blankets weigh 13 lb. 4 oz.

3 lb. 5 oz.
4
13 lb. 4 oz.

2. Edna is also taking to camp 3 pairs of tweed knickers weighing 1 lb. 2 oz. each. How much do the knickers weigh in all?

3. Find the total weight, in pounds and ounces, of 6 middy blouses weighing 9 oz. each.

4. Edna is taking 2 serge skirts weighing 1 lb. 2 oz. each and 2 sweaters weighing 1 lb. 9 oz. each. How much do all of these weigh?

5. How heavy will 5 books on nature study be if each book weighs 1 lb. 4 oz.?

Practice in Multiplying Compound Numbers

1. Elizabeth found this problem in her grandfather's arithmetic: Multiply 2 mi. 17 rd. 4 yd. 2 ft. 7 in. by 6. Can you do it, even though it isn't a practical problem?

Multiply. Check the work by going over it:

2.	4 ft. 3 in. 7	**5.**	9 yd. 2 ft. 7	**8.**	7 pk. 4 qt. 4
3.	6 gal. 2 qt. 5	**6.**	7 lb. 6 oz. 3	**9.**	2 yd. 8 in. 5
4.	8 hr. 15 min. 2	**7.**	2 ft. 8 in. 9	**10.**	2 ft. 5 in. 4

3 1 9

FINDING THE LENGTH OF YOUR STEP

1. The cement walk to the door of Mark's house is 33 ft. 9 in. long. Mark takes 15 steps in walking it. Find the average length of his step.

Dividing 33 ft. by 15, you get 2 ft. and a remainder of 3 ft. Write 2 ft. Then change 3 ft. to 36 in. and add it to 9 in., which gives 45 in. Dividing 45 in. by 15, you get 3 in.

The average length of Mark's step is 2 ft. 3 in.

15)33 ft. 9 in.
2 ft. 3 in.

2. Mark's sister walks only 31 ft. 3 in. in 15 steps. What is the average length of her step?

3. Little Hilda can go across a garden 17 ft. 4 in. wide in 13 steps. How long is her step?

4. Count the number of steps you take to walk 100 ft. What is the average length of your step?

Other Problems

1. In 6 days Jane spent 7 hr. 30 min. practicing on the piano. Find her average time per day.

2. Polly picks 3 baskets of cherries in 5 hr. 45 min. What is her average time per basket?

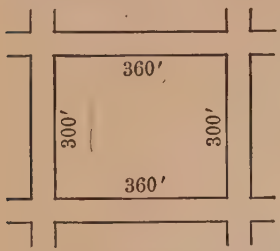
3. A catalogue says that 1 doz. scout axes weigh 27 lb. 12 oz. Find the weight of each axe.

4. An ocean liner made these records on 5 different trips between New York and England: 6 da. 3 hr., 5 da. 22 hr., 5 da. 20 hr., 6 da. 7 hr., and 5 da. 15 hr. Find the average time for a trip.

5. Sam tried the standing broad jump 6 times. These are his records: 4 ft. 10 in., 4 ft. 8 in., 4 ft. 9 in., 5 ft. 0 in., 4 ft. 8 in., 5 ft. 1 in. Find the average length of his jump.

FINDING THE PERIMETER OF A RECTANGLE

1. How often must Paul ride around a city block 360 ft. long and 300 ft. wide to go a mile?



The distance around the block is 300 ft. + 360 ft. + 300 ft. + 360 ft., or 1320 ft. Since 5280 ft. make a mile, divide 5280 by 1320 to find how many times Paul must ride around the block to go a mile.

The distance around any rectangle is called its **perimeter.** How do you find the perimeter of a rectangle?

2. How much string will you need to go around a tulip bed which is $3\frac{1}{2}$ ft. wide and $7\frac{1}{4}$ ft. long?

3. A baseball diamond is a square 90 ft. on a side. How many yards does a boy run in making a home run?

4. Janet is bordering with lace a table scarf $1\frac{1}{2}$ ft. by $3\frac{1}{4}$ ft. How many yards of lace does she need if she allows 6 in. extra in all for turning the corners?

5. Here are the sizes of six different gardens:

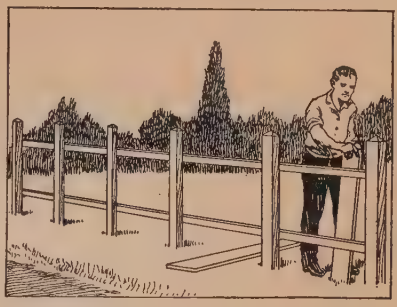
Fred's,	30 ft. × 30 ft.	Mary's,	12 ft. × 75 ft.
Jane's,	18 ft. × 50 ft.	Dick's,	10 ft. × 90 ft.
Will's,	20 ft. × 45 ft.	Dora's,	15 ft. × 60 ft.

How many feet of wire netting will be needed for a fence for each garden? Which garden will need the least netting?

6. Compare the areas of the gardens in ex. 5. Draw each garden to scale, letting $\frac{1}{8}$ in. = 1 ft. What is the shape of the garden that takes the least amount of wire netting?

FENCING AND GARDEN PROBLEMS

1. Mr. Drew is putting a fence 25 ft. long across the front of his lot. He sets the posts 5 ft. apart. Roger says that he needs 5 posts because 25 ft. ÷ 5 ft. = 5. Hilda says that he needs 6 posts because he must allow 1 extra post for the end of the row. Who is right?



2. In her garden Alice is setting tomato plants 2 ft. apart. How many plants will there be in each row if the rows are 16 ft. long? if the rows are 12 ft. long? 20 ft. long? 24 ft. long?

3. Louise is planting tulip bulbs in her garden in rows 6 ft. long. How many bulbs will there be in each row if she sets them 6 in. apart? if she sets them 4 in. apart? 3 in. apart? 8 in. apart?

4. William is putting a wire netting around a garden 30 ft. long and 20 ft. wide. If he fastens the netting to posts which are set 5 ft. apart, how many posts will it take to go all around the garden? Does he have to allow an extra post, as in ex. 1? Prove that your answer is right by making a diagram showing how the posts are set.

In your garden, how many plants can you set:

5. In a row 15 ft. long, with the plants $1\frac{1}{4}$ ft. apart?

6. In a row 12 ft. long, with the plants $1\frac{1}{2}$ ft. apart?

7. In a row 20 ft. long, with the plants $2\frac{1}{2}$ ft. apart?

MEASURING RECTANGLES

In a square, all the sides have the same length; in a rectangle, two sides may be longer than the other two.

Every square or rectangle has four square corners, or **right angles,** which look like this: ⌞ ⌝ ⌟

A square foot is a square having each side 1 ft. long. What is a square inch? a square yard? a square mile?

To measure the length of a room, you find how many feet or yards it contains. To measure the area of a rectangle, like a floor, you find how many *square feet* or *square yards* it contains.

This figure represents a rectangle 4 ft. long and 3 ft. wide. Dividing it into small squares of 1 sq. ft. each, you see that it contains (3×4) sq. ft., or 12 sq. ft. Hence its area is 12 sq. ft.

To find the area of a rectangle, multiply its length by its width.

In finding the area of a rectangle, express the length and the width *both* as inches, *both* as feet, or *both* as yards, etc.

Exercises

Find the areas of the following rectangles:

1. 6 ft.×4 ft.
2. 7 yd.×3 yd.
3. 8 ft.×$2\frac{1}{2}$ ft.
4. 12″×5″
5. $9\frac{1}{2}$″×6″
6. $6\frac{1}{2}$″×$6\frac{1}{2}$″
7. $8\frac{1}{2}$ mi.×8 mi.
8. 90 ft.×$40\frac{1}{2}$ ft.
9. 180 yd.×1 yd.

FINDING AREAS IN EVERYDAY LIFE

1. Agnes can knit 1 sq. in. in 5 min. Which will it take her longer to knit, a scarf 44 in. long and 10 in. wide or one 48 in. long and 8 in. wide?

2. Mabel has a piece of gold paper 27 in. long and 6 in. wide. She is making gold stars for a party dress and needs a 3-inch square of paper for each star. How many stars can she make?

3. Cut an old newspaper into squares, each measuring 1 sq. ft. How many of these squares will it take to cover the top of a table 6 ft. by 4 ft.? How could you find the answer without using the paper squares?

4. A hotel is buying some velvet carpet to cover a room 32 yd. long and 28 yd. wide. How much will the carpet cost at $6.25 a square yard?

5. Mr. Mann is varnishing a floor 30 ft. by 25 ft. If 1 gal. of varnish covers 300 sq. ft., how many gallons of varnish does he need?

6. A garage is 9 yd. long and 6 yd. 2 ft. wide. How much will a cement floor for it cost, at $2.40 a square yard?

7. The area of California is 158,297 sq. mi., while the area of Delaware is 2370 sq. mi. California is how many times as large as Delaware? Carry your result to two decimal places.

Find the areas of the following rectangles. Express both dimensions in the same unit before multiplying:

8. 2 ft. $\times$ 6 in.

9. 7 ft. $\times$ 1 yd.

10. $1\frac{1}{3}$ ft. $\times$ 3 in.

11. $\frac{3}{4}$ yd. $\times 1\frac{1}{2}$ ft.

12. $6\frac{1}{2}$ ft. $\times$ 27 in.

13. $4\frac{3}{4}$ ft. $\times$ 30 in.

14. 2′ 4″ $\times$ 6′

15. 7′ 9″ $\times$ 8′

16. 2′ 3″ $\times$ 1′4″

TABLE OF SQUARE MEASURE

Review this table of *square measure:*

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
43,560 sq. ft. or 160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Exercises

1. How many square feet of concrete are needed for a sidewalk $2\frac{1}{2}$ ft. wide and 30 ft. long? How many square yards are needed? Find the cost at $1.95 a square yard.

2. Some boys are making a baseball diamond 90 ft. square in a field. How many square yards of sod must they remove? If Mr. Jones gives the boys 10¢ a square yard for the sod they remove, how much will they get for it?

3. Frank's mother wishes to put linoleum on her kitchen floor, which is 15 ft. by 12 ft. How much will it cost at $1.85 per square yard?

4. William has a garden 121 ft. long and 45 ft. wide. How much fertilizer should he buy for it if it takes 1000 lb. of this fertilizer per acre?

Tell what numbers should be put in place of the dots:

5. 5 acres = . . sq. rd.

6. 2 sq. rd. = . . sq. yd.

7. 6.5 sq. yd. = . . sq. ft.

8. $1\frac{1}{2}$ sq. ft. = . . sq. in.

9. 0.6 acre = . . sq. ft.

10. 720 sq. rd. = . . A.

11. 2178 sq. ft. = . . A.

12. 80 acres = . . sq. mi.

13. $60\frac{1}{2}$ sq. yd. = . . sq. rd.

14. 360 sq. ft. = . . sq. yd.

IMPROVEMENT TEST No. 9

Test 9 A — Reduction (Time 4 min. after copying)

Change to decimals, correct to the nearest hundredth:

1. $\frac{6}{7}$	**2.** $\frac{2}{3}$	**3.** $\frac{29}{30}$	**4.** $\frac{11}{64}$
5. $\frac{5}{9}$	**6.** $\frac{11}{12}$	**7.** $\frac{13}{15}$	**8.** $\frac{5}{16}$

Test 9 B — Multiplication (Time 4 min. after copying)

1. 809 × .27	**2.** 3.45 × .57	**3.** .082 × 8	**4.** 94.2 × 700
5. 350 × 2.5	**6.** 4.52 × .09	**7.** .483 × 1.9	**8.** 8.5 × 6000
9. 400 × .55	**10.** .007 × .006	**11.** .134 × .08	**12.** .0011 × 900

Test 9 C — Division (Time 4 min. after copying)

Divide, annexing enough zeros to each dividend to make the quotient come out exact:

1. $24\overline{)114}$ **2.** $56\overline{)35}$ **3.** $450\overline{)769.5}$

Test 9 D — Division (Time 4 min. after copying)

Find each answer correct to the nearest hundredth:

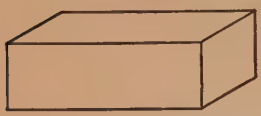
1. $22\overline{)11.9}$ **2.** $55\overline{)432}$ **3.** $67\overline{)318.9}$

Test 9 E — Division (Time 4 min. after copying)

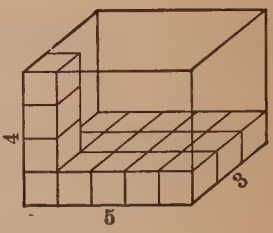
1. $.94\overline{)11.656}$ **2.** $.0092\overline{)36.708}$ **3.** $7.2\overline{).17352}$

THE VOLUME OF A RECTANGULAR SOLID

A solid shaped like a box is called a **rectangular solid** because its six faces are all rectangles. If its six faces are all squares, it is called a **cube.** A cubic foot is a cube 1 ft. long, 1 ft. wide, and 1 ft. high.



The picture represents a box 5 ft. long, 3 ft. wide, and 4 ft. high. It takes 5×3, or 15, cu. ft., to cover the bottom of the box and 4×15 cu. ft., or 60 cu. ft., to fill the box. Hence its **volume** is $(5 \times 3 \times 4)$ cu. ft., or 60 cu. ft.

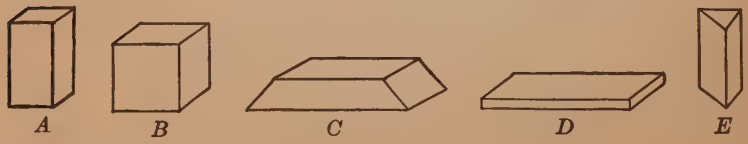


Volume of solid = length × width × height.

Before finding the volume of a solid, express all its dimensions as inches, all as feet, or all as yards, etc.

Exercises

1. *Which of these are rectangular solids or cubes?*



Find the volumes of these rectangular solids:

2. 5 in. × 8 in. × 8 in.

3. 8 ft. × 7 ft. × 4 ft.

4. 9 yd. × 5 yd. × 9 ft.

5. 8 ft. × 7 ft. × 18 in.

6. $7\frac{1}{2}$ ft. × 5 ft. × 32 in.

7. $4\frac{2}{3}$ ft. × $11\frac{1}{4}$ ft. × 9 ft.

8. $9\frac{1}{3}$ ft. × $5\frac{1}{4}$ ft. × 36 in.

9. $10\frac{1}{2}$ ft. × $5\frac{1}{2}$ ft. × 4 ft.

FINDING VOLUMES

Review this table of *cubic measure:*

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Problems

1. Our swimming pool is 60 ft. long and 32 ft. wide. When it is filled to a depth of $5\frac{3}{4}$ ft., how many cubic feet of water does it hold?

2. Find the cost of digging a cellar $18' \times 15' \times 7\frac{1}{2}'$ at 65¢ a cubic yard.

3. How many cubic feet of air space has a schoolroom 35 ft. by 30 ft. by 12 ft.? If 200 cu. ft. are needed per pupil, this room is large enough for how many pupils?

4. Last night Jennie dreamed that she had 1,000,000 pieces of chocolate fudge, which filled her room from the floor to the ceiling. If the floor of Jennie's room is 8 ft. 4 in. square, how high would the room have to be to hold exactly 1,000,000 pieces of fudge, each of which is an inch cube?

5. Mabel's room is 10 ft. 5 in. long and 8 ft. 4 in. wide. If she could pile the 1,000,000 pieces of fudge in her room so that they would completely cover the floor, how high would the pile be?

6. Roy said that he would like to pile the 1,000,000 pieces of fudge on a table 8 ft. 4 in. by 6 ft. 8 in. at the School Fair. How high would the pile be?

MEASURING BINS

Memorize these useful facts in *dry measure:*

1 bushel = about 2150 cu. in. or $1\frac{1}{4}$ cu. ft.
1 cubic foot = about .8 bu.
1 ton hard coal = about 35 cu. ft.

A bushel contains exactly 2150.42 cu. in., but in most calculations it would be a waste of time to use this exact value. In practical life, we cannot measure out a bushel of grain so closely as 2150.42 cu. in.

Grain and vegetables are often bought by the weight-bushel instead of by the measured bushel. Thus, a bag of wheat or potatoes is called a bushel if it weighs 60 lb.

Exercises

In ex. 1 *to* 8, *use* 1 *cu. ft.* $= \frac{4}{5}$ *bu. or* 1 *cu. ft.* $= .8$ *bu.*

1. Mr. Fenton stored his wheat in a bin 10 ft. by 8 ft. by 6 ft. How many bushels did the bin hold?

Find the number of cubic feet and multiply by $\frac{4}{5}$ or by .8.

2. Mr. Nugent builds a bin $6' \times 5' \times 3\frac{1}{2}'$ to hold feed for his horses. How many bushels will it hold?

Tell how many bushels each of these bins will hold:

3. $12' \times 8' \times 5'$ **5.** $9' \times 7' \times 5'$ **7.** $10' \times 8\frac{1}{3}' \times 7\frac{1}{2}'$

4. $8\frac{1}{3}' \times 6' \times 5'$ **6.** $9\frac{2}{3}' \times 6\frac{1}{4}' \times 6'$ **8.** $7\frac{1}{2}' \times 5\frac{1}{4}' \times 2\frac{2}{3}'$

9. A bin 5 ft. wide and 9 ft. long is filled 7 ft. deep with hard coal. How many tons of coal is that?

10. A farmer wishes to make a box that will hold $\frac{1}{2}$ bu. for measuring seed. Tell which of these boxes will most nearly contain $\frac{1}{2}$ bu.:

(*a*) $11'' \times 11'' \times 8\frac{7}{8}''$ (*c*) $11\frac{1}{2}'' \times 11\frac{1}{2}'' \times 8''$

(*b*) $10'' \times 10'' \times 10\frac{3}{4}''$ (*d*) $10\frac{1}{2}'' \times 10\frac{1}{2}'' \times 9\frac{3}{4}''$

329

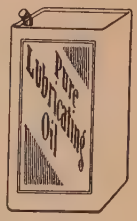
MEASURING WATER AND OTHER LIQUIDS

Learn these useful facts in *liquid measure:*

1 gallon = 231 cu. in.
1 cubic foot = about $7\frac{1}{2}$ gal.

Problems

1. Cora has an aquarium 1 ft. 10 in. long, 14 in. wide, and 16 in. deep. How many gallons of water are needed to fill it $\frac{3}{4}$ full?



2. Andrew bought a can of automobile oil which measured 6 in. by 7 in. by 11 in. How many gallons of oil did it hold?

3. Mary asked the grocer for a pint of olive oil. He gave her a can that measured 5 in. $\times 3\frac{1}{8}$ in. $\times 1\frac{7}{8}$ in. Did this can hold a pint?

4. Using 231 cu. in. = 1 gal., find, to the nearest hundredth of a gallon, how many gallons there are in 1 cu. ft. Is 1 cu. ft. about $7\frac{1}{2}$ gal.?

5. How many gallons of water will fill a swimming pool 40 ft. long and 18 ft. wide to a depth of $5\frac{1}{4}$ ft.?

Use 1 cu. ft. = $7\frac{1}{2}$ gal.

6. A broken pipe flooded a cellar to a depth of 5 ft. 3 in. If the cellar is 28 ft. long and 20 ft. wide, how many gallons of water had to be pumped out?

7. During a storm 1 in. of rain fell. How many cubic feet of rain fell on a garden 72 ft. by 40 ft.? How much would it have cost to sprinkle the same amount of water on the garden at 28¢ per 100 cu. ft.?

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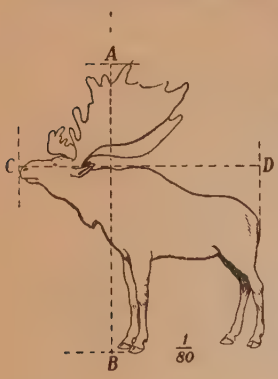
HOW TALL IS THE MOOSE?

Under the picture of a moose in the dictionary, Agnes finds the fraction $\frac{1}{80}$. This tells the scale to which the picture is drawn, and means that each measurement in the picture is $\frac{1}{80}$ as great as the same measurement in a real moose. Agnes measures the line AB and finds it to be $1\frac{1}{2}$ in. long. Hence the real height of the moose is $80 \times 1\frac{1}{2}$ in., which equals 120 in., or 10 ft. Measure the line CD and find the real length of the moose.



Exercises

1. In a large dictionary see whether there are pictures with the words *ostrich*, *zebra*, *lion*, and *tiger*. If there are, find the scale of each picture. On the picture measure, to the nearest $\frac{1}{16}$ in., the length and the height of each animal; then find its real length and height.

Agnes made these measurements on some pictures of animals. Find the real height and length of each:

	NAME	SCALE	HEIGHT	LENGTH
2.	Giraffe	$\frac{1}{100}$	$1\frac{3}{4}$ in.	1 in.
3.	Buffalo	$\frac{1}{60}$	$1\frac{3}{8}$ in.	$1\frac{7}{8}$ in.
4.	Kangaroo	$\frac{1}{48}$	$\frac{7}{8}$ in.	$2\frac{3}{8}$ in.
5.	Squirrel	$\frac{1}{7}$	$\frac{3}{4}$ in.	$1\frac{1}{4}$ in.
6.	Large bear	$\frac{1}{48}$	1 in.	$1\frac{1}{2}$ in.
7.	Elephant	$\frac{1}{108}$	$1\frac{1}{16}$ in.	$1\frac{3}{8}$ in.
8.	Catbird	$\frac{1}{8}$	$\frac{5}{8}$ in.	$1\frac{1}{2}$ in.

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IMPROVEMENT TEST No. 10

Test 10 A — Addition (Time 4 min.)

1.	2.	3.	4.
37120	47457	98598	98724
93837	58486	65871	55965
84778	49305	26389	53794
96363	54875	15108	24036
5.	**6.**	**7.**	**8.**
46975	82149	64238	39146
41698	97741	81874	92779
34062	47332	77676	58989
12528	18385	44529	17336

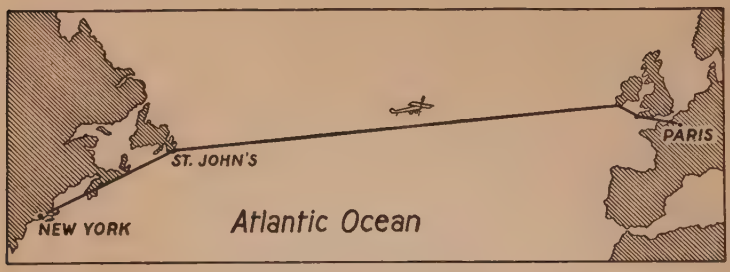
Test 10 B — Addition (Time 4 min.)

1.	2.	3.	4.	5.
8133	1956	5436	5039	6455
4145	1731	2393	9564	4099
5676	5694	8722	4456	9853
6909	2859	1100	5009	7878
4739	9087	6687	8785	9760
7243	8750	8917	6984	7502
7846	3628	6478	8260	3759
3805	7263	2233	2136	1446

Test 10 C — Subtraction (Time 3 min.)

1.	2.	3.	4.	5.
9525	7384	8406	9618	8123
3856	5188	4537	4790	2268
6.	**7.**	**8.**	**9.**	**10.**
6714	9306	6657	8213	5352
5240	1429	4168	3457	3379
11.	**12.**	**13.**	**14.**	**15.**
8130	4420	4197	8401	6049
1690	2676	1209	6415	2699

LINDBERGH FLIES ACROSS THE ATLANTIC



Find all answers to the nearest tenth of a unit.

1. Clyde read that Lindbergh, on his non-stop flight across the Atlantic Ocean from New York to Paris, flew 3647 mi. in 33.5 hr. He flew at an average speed of how many miles an hour?

2. Lindbergh had 450 gal. of gasoline at the beginning of the flight. He used 432 gal. in traveling the 3647 mi. How many miles did he fly, on an average, on 1 gal. of gasoline?

3. Clyde read that Lindbergh traveled alone. He asked his father why Lindbergh did not take a man along to help him. His father said, "A gallon of gasoline weighs 6.12 lb. Hence for every 6.12 lb. of the weight of a companion, a gallon of gasoline would have to be sacrificed." How many gallons of gasoline would Lindbergh have had to sacrifice to make up for the weight of a 160-pound man?

4. Lindbergh had 18 gal. of gasoline left when he reached Paris. Could he have reached Paris if he had taken the man instead of the extra gasoline?

Use the answer in ex. 3 in working this problem.

5. How many miles farther than Paris would the 18 gal. of gasoline he had left have taken Lindbergh if each gallon had carried him the same number of miles that he averaged during the trip?

Use the number of miles per gallon found in ex. 2.

6. When Lindbergh left New York, his empty plane with its instruments weighed 2150 lb. It carried as its load 450 gal. of gasoline weighing 6.12 lb. a gallon, 25 gal. of oil weighing 7 lb. per gallon, 45 lb. of miscellaneous load, and Lindbergh, who weighed 170 lb. How much did the plane with its entire load weigh?

7. How many gallons of gasoline per hour, on an average, did Lindbergh's plane consume if it used 432 gal. in 33.5 hr.?

8. Lindbergh left New York at 7.52 A.M. and passed over St. John's, Newfoundland, at 7.15 P.M. Show that this part of the trip took about 11.4 hr.

9. If he used 12.9 gal. of gasoline per hour, how much lighter was his plane when he passed over St. John's than when he left New York? (Use facts in ex. 6 and 8.)

10. How many passengers averaging 150 lb. each could Lindbergh have carried from St. John's to Paris to make up for the loss in weight of gasoline between New York and St. John's? (Use the answer to ex. 9.)

11. Ex. 6 tells how much the loaded plane weighed when Lindbergh left New York. If on the entire trip he used 432 gal. of gasoline and 11.8 gal. of oil, how much lighter was his plane when he landed at Paris than it was when he started?

FRACTIONS AND MEASURES

Tell what numbers should be put in place of the dots:

1. $2\frac{1}{2}$ yd. = . . . in.
2. $2\frac{3}{4}$ mi. = . . . ft.
3. 64 rd. = . . . mi.
4. 72 oz. = . . . lb.
5. 5 min. = . . . hr.
6. $3\frac{1}{2}$ yr. = . . . mo.
7. 96 pt. = . . . bu.
8. $5\frac{1}{4}$ gal. = . . . pt.
9. 9 ft. 3 in. = . . . ft.
10. 2 lb. 7 oz. = . . . oz.
11. 2 ft. 3 in. = . . . yd.
12. $2\frac{1}{2}$ sq. mi. = . . . A.
13. 486 sq. ft. = . . . sq. yd.
14. $18\frac{3}{4}$ cu. yd. = . . . cu. ft.
15. 108 sq. in. = . . . sq. ft.
16. 450 cu. ft. = . . . cu. yd.

Add the following and check the work:

17. $4\frac{5}{8}$ lb. + $2\frac{3}{4}$ lb. + $7\frac{1}{2}$ lb. + $6\frac{5}{16}$ lb.
18. $1\frac{1}{3}$ hr. + $5\frac{3}{4}$ hr. + $6\frac{1}{2}$ hr. + $\frac{5}{6}$ hr.
19. $7\frac{1}{2}$ yd. + $2\frac{1}{4}$ yd. + $6\frac{5}{8}$ yd. + $3\frac{7}{8}$ yd.
20. $6\frac{1}{5}$ sec. + $8\frac{1}{2}$ sec. + $7\frac{7}{10}$ sec. + $6\frac{9}{10}$ sec.

Subtract the following and check the work:

21. $8\frac{3}{4}$ yd. − $2\frac{7}{8}$ yd.
22. 6 lb. − $3\frac{5}{16}$ lb.
23. $19\frac{7}{10}$ mi. − $18\frac{1}{2}$ mi.
24. $5\frac{1}{12}$ ft. − $2\frac{7}{12}$ ft.

Multiply or divide, as the problem indicates:

25. $6\times\frac{7}{8}$
26. $\frac{2}{3}\times\frac{9}{10}$
27. $\frac{3}{4}\times64$
28. $\frac{7}{8}\times50$
29. $11\times\frac{3}{4}$
30. $5\times18\frac{1}{4}$
31. $18\times\frac{1}{16}$
32. $14\times5\frac{1}{2}$
33. $12\times1\frac{1}{8}$
34. $2\frac{2}{3}\times10$
35. $5\frac{1}{3}\times4\frac{1}{8}$
36. $3\frac{5}{8}\times240$
37. $16\div3\frac{1}{2}$
38. $1\frac{1}{2}\div4\frac{1}{2}$
39. $3\frac{3}{8}\div\frac{3}{4}$
40. $21\frac{1}{4}\div2\frac{1}{2}$
41. $5\div\frac{1}{8}$
42. $\frac{3}{4}\div\frac{7}{8}$
43. $\frac{3}{4}\div\frac{3}{4}$
44. $\frac{3}{16}\div\frac{1}{4}$
45. $2\frac{1}{2}\div5$
46. $3\frac{1}{8}\div5$
47. $11\div3\frac{1}{7}$
48. $10\div2\frac{1}{2}$

CAN YOU SOLVE ALL THESE PROBLEMS?

1. A store advertises hats "at $33\frac{1}{3}\%$ off." How much does Ruth pay for a hat originally marked $7.50?

2. In a test of 50 words Edwin's mark was 70%. How many words had he right? How many had he wrong?

3. In a test of 75 words Ann had 72 words correct. Her paper was marked 96%. Was that right?

4. In May, Dan spent 68% of his wages and saved the rest. What per cent of his wages did he save?

5. Ann has $150 in a bank which pays 4% interest. How much interest does Ann get each half year?

6. An agent sold Mr. Faraday's car for $600, charging him 6% commission. How much of the $600 did Mr. Faraday receive?

7. Harry's father borrowed $2000 at 6% to purchase supplies for his new store. He repaid the money 1 yr. 6 mo. later. How much interest did he pay in all?

8. The men in a shoe factory have had their wages increased 5%. If Mr. Cox earned $6 a day before the increase, how much more per day does he earn now?

9. Last year the population of our city was 12,600. Now it is 40% greater. What is the population now?

Say these in percentage language:

10. Only $\frac{3}{4}$ of the Lake City pupils took part in the Field Day exercises.

11. Ella's dress is guaranteed to be all wool.

12. Ada gives $\frac{3}{5}$ of her salary to her mother.

13. Mr. Young sold his house on Washington Street for $\frac{2}{3}$ of what he paid for it.

PROBLEMS FOR A GOOD THINKER

1. If Bernard knows how many feet high Victory Hill is, how can he find, to the nearest hundredth of a mile, what decimal part of a mile its height is?

2. Claudia Mitchell is selling soap this summer at 35% commission. If she has a record of each day's sales for a week, how can Claudia find what her commission for that week should be?

In ex. 3, 4, *and* 5, *a necessary fact has been left out. Supply the fact and work the problem:*

3. Margery received a mark of 90% on a spelling test. How many words did she have right?

4. On a trip to Florida and return, Mr. Waterman drove his car a total of 4356 mi. How many miles a day did Mr. Waterman average?

5. If Mr. Gainsborough averages 106.3 mi. an hour in his airplane, how long will it take him to reach Spokane, Wash., from his home?

In ex. 6, 7, *and* 8, *make up a question that uses all the given facts, and answer it:*

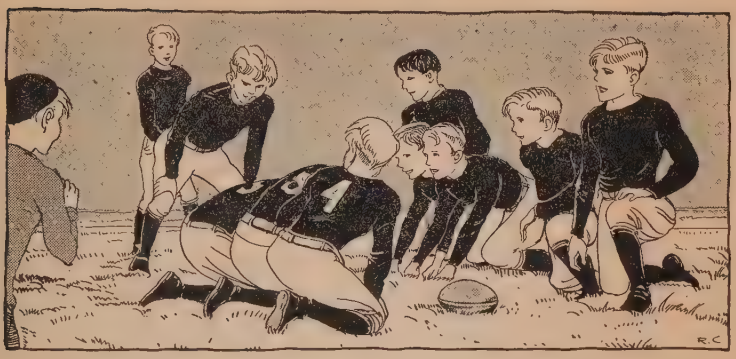
6. In making some bookshelves, Charles cut a board into 4 pieces, each piece being 3 ft. 5 in. long.

7. Yesterday Agnes made 8 lb. of fudge. She sold $2\frac{1}{2}$ lb. of the fudge to Mary, $\frac{5}{8}$ lb. to Doris, $1\frac{3}{4}$ lb. to Mrs. Peck, and 2 lb. to Mrs. Dean.

8. Edward had \$450 in a savings bank for one year at 4% interest. If Edward had lent the money to Mrs. Eldridge for a year instead of keeping it in the bank, she would have paid him 6% interest.

CHAPTER III

GETTING INTO PRACTICE AGAIN



Every fall, when the boys begin to play football after the long vacation, they find that they must work hard to get their game into good form because they are "out of practice."

In the same way, if you have not had to add or subtract during the summer, you now find that in computing you are "out of practice." You must get your arithmetic back into good form as soon as possible. You can do this by faithfully practicing the Improvement Tests in this book.

Last year you probably kept a careful record (as suggested on page 5) of your progress in the Improvement Tests. Look up this record and see how well you did in Improvement Test 10 A (page 135) at the end of last year. Try the same test now and see how well you can do it. Your score will probably be below your former records, showing that *you need more practice.*

To gain skill in computing, *always try to improve on your best previous record.* Always try also to keep above the average or median score of your class.

The median score is explained on pages 144 to 147.

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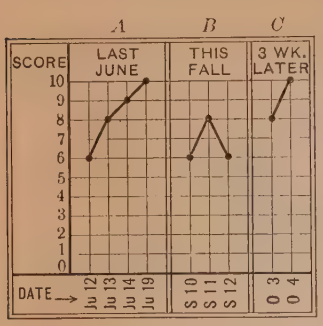
HOW EDNA IMPROVED IN ADDITION

Edna looked up her last year's Graphic Record Book and found that last June her scores in Improvement Test 10 A (page 135) were as shown in Graph *A* at the left.



To see if she could do just as well this fall, Edna tried Test 10 A again the first week of school and was disappointed to find that her scores were much lower than last year, as shown in *B*. She therefore practiced some of the other Improvement Tests in addition, such as Tests 1 A, 2 A, 4 A, and 8 A. Then she practiced Test 11 A on page 143. After three weeks she tried Test 10 A again and made the scores shown in Graph *C*. Did Edna do better this time than she did last June?

Reading Edna's Record

1. What was Edna's lowest score last June? Did she do better each time she took the test in June?

2. How many times did Edna try the test in June before she made a score of 10?

3. What was Edna's score on Sept. 10? Sept. 11? Sept. 12? How could she tell that she needed more practice on this test?

4. When Edna took the test in October, how many trials did she have to make to get a score of 10? Was this record, as given in Graph *C*, better than her record in June?

NOTE. Each pupil should get a notebook of squared paper and keep a graphic record of his scores in all Improvement Tests. See pages 2, 4, and 5 for further directions for taking and scoring tests, and for keeping records.

IMPROVEMENT TEST No. 11

Test 11 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
954	765	891	378	687
718	962	121	882	939
617	963	684	302	254
828	300	807	922	436
970	113	318	341	186
341	932	263	207	600
295	328	877	890	758
451	517	104	447	675
850	657	760	258	492
546	196	929	859	109
727	704	822	875	923

Test 11 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
16441	37324	81605	51000
12053	28842	67945	33265
5.	**6.**	**7.**	**8.**
38311	96028	96530	74254
22361	38122	42792	64634
9.	**10.**	**11.**	**12.**
96442	73006	30409	73879
89764	15595	21831	71780

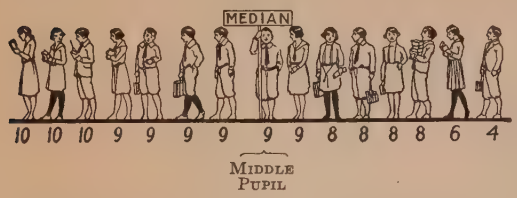
Test 11 C — Subtraction (Time 3 min.)

1.	2.	3.	4.
293709	192510	596302	436000
79489	18970	18770	30842
5.	**6.**	**7.**	**8.**
658967	317296	951246	274600
31980	29027	812089	265725
9.	**10.**	**11.**	**12.**
474000	697934	760721	525241
364320	248049	414343	327861

HOW TO FIND THE MEDIAN SCORE

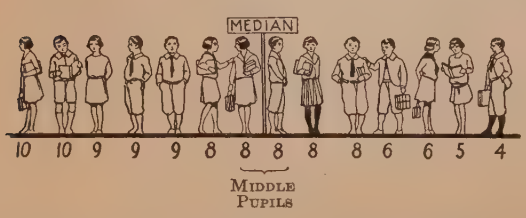
Every time your class takes an Improvement Test, the median score of the class should be found and put on the board. The class usually repeats each test several days until it reaches a median score of 10; then a new test is taken up.

This picture shows the members of a class arranged according to their marks on Improvement Test 11 B. The numbers under them show their scores. The pupils with the highest scores come first; those with the lowest come last. The middle pupil has the middle, or *median, score* of the class.



There are 15 pupils in the above class. The middle pupil is the 8th, there being 7 pupils before him and 7 pupils after him. His mark of 9 is the median score of the class. Pupils with less than 9 must work hard to reach the median next time.

The class below has 14 pupils. Here there are 2 middle pupils who both have a mark of 8; hence the middle mark of the class is 8.



Below are the marks of another class of 14 pupils. The two middle pupils have marks of 9 and 8. The median score is half way between the two

10 10 9 9 9 9 9 ↑ 8 8 8 8 8 6 6
MEDIAN

median scores, or half way between 9 and 8. This gives a median of $8\frac{1}{2}$, which is called 9.

WHICH SIDE WILL YOU BE ON?

The median divides a class into two equal groups, representing the upper and lower halves of the class. In which half do you want to be? Will you be before the median flag or behind it? The median is often a high mark; but since at least half of your class has reached it, you also can reach it.

Always try to be among those whose mark is above the median, so that you will be in the upper half of the class.

A Short Way to Find the Class Median

Here is a short way to find the class median:

Consider the first class of 15 pupils, which is pictured on page 144. After the papers are scored, the teacher need not take the time to arrange the pupils in line. She simply asks how many have a score of 10, and 3 pupils raise their hands. She then asks how many have 9, and 6 pupils raise their hands. She writes on the board the number of pupils having each score, as shown above.

Score	*Number of Pupils*	
10	3	
9	6	← Median score in this group
8	4	
6	1	
4	1	
	15	

Since the 8th score is the middle score, the teacher counts down in the column marked "Number of Pupils" until she reaches the 8th pupil. In the first group there are 3 pupils; to these she adds 5 pupils from the next group in order to reach the 8th pupil. Since the 8th pupil belongs to the group of 6 pupils who received a mark of 9, the median score is 9.

NOTE. To find which is the middle score, add 1 to the total number of scores and divide by 2. In the example above, add 1 to 15, which gives 16. Dividing 16 by 2, you get the 8th as the middle score. For a class of 14 pupils, add 1 to 14, which gives 15. Since $15 \div 2 = 7\frac{1}{2}$, the $7\frac{1}{2}$th score, which is half way between the 7th and the 8th score, is the middle score.

FINDING CLASS MEDIANS

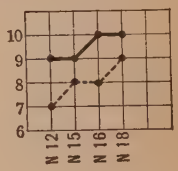
1. In a class, 4 pupils had a score of 10 on an Improvement Test, 7 pupils had 9, 7 had 8, 4 had 7, 2 had 6, and 1 had 3. How many pupils took the test? Arrange the marks in a table, and find the median by the short method given on page 145.

2. On Monday, in Amy's class, 5 pupils had a score of 10 on an Improvement Test, 6 had 9, 9 had 8, 4 had 7, 2 had 6, and 2 had 5. Find the class median.

3. On Tuesday Amy's class repeated the test. This time 7 pupils had a score of 10, 7 had 9, 9 had 8, 3 had 7, 1 had 6, and 1 had 5. Find the class median for Tuesday. Had it gone up since Monday?

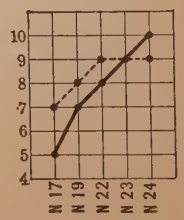
Comparing Your Work with That of Your Class

In this graph the heavy line shows Anna's record on a test; the dotted line shows the median scores of the entire class on that test. What was the median on Nov. 12? on Nov. 18? What were Anna's scores on those days? Was Anna's work always satisfactory?



The second graph shows Joe's record by a heavy line and that of his class by a dotted line. Joe was far below the class on Nov. 17. On Nov. 19 he did better, but the class median also rose. What was the class score on Nov. 22? What was Joe's score?

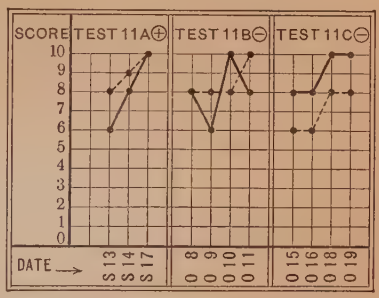
Joe worked hard and continued to improve. How did his score compare with the class median on Nov. 23? on Nov. 24?



For every Improvement Test your class medians will be kept on the board. In your record book, make a graph of the class medians in red and of your own scores in black. Try to keep your line above the class line.

COMPARING RECORDS WITH THE CLASS RECORD

This graph shows Philip's scores (heavy lines) and the medians of his class (dotted lines) on three Improvement Tests.



1. Was Philip above or below the class median when he tried Test 11 A on Sept. 13? Did he improve on Sept. 14? Did the class also improve on Sept. 14? On what day did Philip come up to the class median on Test 11 A?

2. On Test 11 B, how did Philip's score compare with the class median on Oct. 8? on Oct. 9? on Oct. 10? on Oct. 11? Was his work always satisfactory on Test 11 B? Does he need more practice on this test?

3. Compare Philip's score with the class score on each of the days Test 11 C was tried. Did he always make a satisfactory score on Test 11 C? Which needs more practice on this test, Philip or the rest of the class?

4. On Test 11 A, Nora's marks were 8 on Sept. 21, 6 on Sept. 24, 8 on Sept. 26, and 10 on Sept. 28. The medians of her class were 8 on Sept. 21, 8 on Sept. 24, 9 on Sept. 26, and 9 on Sept. 28. Draw a graph of Nora's scores in black and of the class medians in red. On which days was Nora's score below the class median? On which days was her score equal to the class median? above the median?

5. Draw a graph showing Bertha's scores on a test, if her score was 8 on Oct. 5, 10 on Oct. 8, 9 on Oct. 26, and 10 on Oct. 29. On the same graph show, in red, that the class medians on that test were 7 on Oct. 5, 9 on Oct. 8, 8 on Oct. 26, and 10 on Oct. 29. Compare Bertha's graph with the class graph.

SAVING TIME IN ADDITION

In adding a column of figures, you can often save time by grouping the figures. Thus, in adding the columns at the right, look ahead quickly as you go up each column and group mentally the figures whose sum is 10 or less. In adding Column *A*, for example, think 10, 15, 21. In Column *B*, think 7, 17, 23. Always take the figures in their regular order in grouping; thus, in Column *C*, do *not* try to group 5 and 5 or 8 and 2. If you skip around to form a group, you may easily forget the figures you skip.

A	*B*	*C*
6	2	3
3	4	5
2	5	2
4	5	5
6	7	8

In Improvement Tests in addition, you will soon find it much quicker to group the figures than to add one figure at a time. Bookkeepers often group three and four figures at a time.

CHECKING ADDITION

Since an error in adding up a bill or an account may mean the loss of many dollars, the wise business man always checks his computations. Hence it is important for you to form, now, the habit of checking your work.

In an example like that at the right, begin at the bottom of each column and add upwards. *When all the columns have been added, check the work by adding downwards in each column.* You should get the same sum each way.

697
341
258
201
905
217
2619

In an Improvement Test in addition or subtraction, like Test 11 A, the teacher will allow twice the given time (8 minutes in all for Test 11 A) if she wishes you to do and check the entire test. If she allows only the given time, do not begin to check your work until you have finished the test. Then, if there is time left before the signal to stop, check as many of your answers as you can.

CHECKING ADDITION

Add and check the following:

1.	2.	3.	4.	5.
5589	8198	3749	3246	6934
6776	5566	5496	2297	1878
7792	9859	1865	7472	8910
8649	7084	8586	3493	9545
6935	8724	5649	6619	3834

6.	7.	8.	9.	10.
9687	5445	6296	8792	9025
4196	8453	1758	1128	2845
7837	9417	8845	6789	4871
8641	5778	7257	2867	9632
9923	4561	9389	6643	9363
4735	5647	3569	4924	5656

CHECKING SUBTRACTION

The best way to check subtraction is by adding the remainder and the subtrahend; the result should equal the minuend. In checking subtraction, always begin with the remainder and add up in each column.

8052 Minuend
6397 Subtrahend
1655 Remainder

To check the above problem, think as follows:

$5 + 7 = 12$. The 2 of 12 checks with the 2 of 8052.
1 (carried) $+ 5 + 9 = 15$. The 5 of 15 checks with the 5 of 8052.
1 (carried) $+ 6 + 3 = 10$. The 0 of 10 checks with the 0 of 8052.
1 (carried) $+ 1 + 6 = 8$. The 8 checks with the 8 of 8052.

Subtract and check the following:

1.	3.	5.	7.	9.
5171	7260	6485	2843	3111
2090	5459	3892	1756	2999

2.	4.	6.	8.	10.
8643	9276	9065	6422	8343
1916	8579	438	2466	2965

USING SUBTRACTION TO SOLVE PROBLEMS

Solve these problems and check the work.

1. Raymond had a collection of 3500 stamps. He sold 1275 stamps to Edgar and gave 92 stamps to Thomas. How many stamps did Raymond have left?

2. Bert has saved $23.85. He wants to buy a radio set costing $45.50. How much more must he save?

3. At the school picnic Grace sold 123 bags of popcorn, while Edna sold 94 bags. How many more bags of popcorn did Grace sell than Edna?

4. Mr. Adams started on a business trip with $200 in his pocket. When he returned home he had $27.85 left. How much in all did he spend on the trip?

5. Harriet is treasurer of the Science Club. Her account book shows that during the first month of school she collected $8.75 in dues and spent $5.93. How much money should she have left at the end of the month? When Harriet counted the money on hand, she found that she had $2.77. Give several possible reasons why her account did not balance.

6. When Mr. Bates started on a trip, the speedometer on his car read 7284 mi. At the end of the trip it read 8493 mi. How far did he drive?

7. An automobile Route Book gives these distances for a trip from Washington, D. C., to Indianapolis, Ind.

Miles from Washington	
0	Washington, D. C.
138	Cumberland, Md.
267	Wheeling, W. Va.
394	Columbus, Ohio
462	Dayton, Ohio
567	Indianapolis, Ind.

How far is it from Cumberland to Wheeling? from Cumberland to Columbus? from Wheeling to Columbus? from Columbus to Dayton? from Dayton to Indianapolis?

HOW CHANGE IS MADE

Albert buys a notebook for 13¢ and gives the clerk $1.00. The clerk makes the change very quickly.

Albert thinks that the clerk first subtracts 13¢ from $1.00, which leaves 87¢, and then picks up 87¢ in coins.

Jane says that the clerk does not subtract at all, but makes the change by adding, thus: He thinks 13¢, then he picks up 2 cents and thinks 15, then he takes a dime and thinks 25, next a quarter and thinks 50, and then a half dollar and thinks $1.00. As he puts the change into Albert's hand he counts again, saying, "13, 15, 25, 50, $1."

Jane is right. In making change the clerk does not subtract; he starts with the amount of your purchase and adds coins to that amount until he reaches a dollar. Then he hands you the coins. When you receive the change, you should check it by adding in the same way.

Exercises

1. Marie buys a book for $1.29 and gives the clerk $2. He gives her 1 cent, 2 dimes, and a half dollar. Count the change as the clerk does. Is it right?

2. If Thomas bought a battery from you for 58¢ and gave you $1, what coins would you give him as change? How would you count them out?

Tell what coins you would give as change from $1 *on each of these purchases, and count out the change:*

3. 45¢ **4.** 25¢ **5.** 28¢ **6.** 83¢ **7.** 69¢ **8.** 92¢

Make change from $5 *for each purchase and count it out:*

9. $2.50 **11.** $3.70 **13.** $4.12 **15.** $1.80 **17.** $3.39
10. $2.08 **12.** $1.05 **14.** $0.46 **16.** $1.99 **18.** $4.35

HOW TO CHECK MULTIPLICATION

Fred has multiplied 273 by 812. To find out if his answer is correct, he may check it by any of these methods:

1. *By doing the work over.* If he does the example again and gets the same result, his work is probably correct. This is the best check when the multiplier has only one figure.

2. *By reversing the factors.* He may exchange the positions of the numbers and multiply again. For example, after multiplying 273 by 812, he can check the work by multiplying 812 by 273. The product should be the same either way.

3. *By casting out 9's.* This method, discovered by the Arabs many centuries ago, is the shortest check for multiplication when the multiplier contains two or more figures.

Checking Multiplication by Casting Out 9's

You check multiplication by "casting out 9's" like this:

First add the digits of 273, thus: $2 + 7 + 3 = 12$. There is one 9 in 12. If you subtract, or "cast out," this 9, you have 3 left. This 3 is the "check number" of 273.

Find the check number of 812 thus: $8 + 1 + 2 = 11$. $11 - 9 = 2$. The check number of 812 is 2.

	9's Check
273	3
812	2
546	(6) ←
273	
2184	
221676 →	(6) ←

These numbers should agree

Now multiply the first check number, 3, by the second, 2. This gives 6 as the final check number. Write 6 in parentheses.

Finally, find the check number of the product, 221,676, thus: $2 + 2 + 1 + 6 + 7 + 6 = 24$. Casting out both of the 9's in 24, you have 6 as the check number. Since this is the same as the final check number, 6, obtained above, the answer is correct.

Multiply the following and check by casting out 9's:

1.	346	**2.**	813	**3.**	2422	**4.**	1805	**5.**	2612
	119		344		71		667		993

A QUICKER WAY TO FIND CHECK NUMBERS

In the example at the right, the sum of the digits of 296 is $2 + 9 + 6$, or 17. Instead of subtracting 9 from 17 to get 8 as the check number, you can find 8 more quickly by adding the digits of 17, thus: $1 + 7 = 8$. Similarly, the sum of the digits of 626 is 14, and $1 + 4 = 5$.

```
  296   8
  626   5
 1776  40 (4)
  592
1776
185296 (4)
```

Multiplying the check number 8 by the check number 5, you get 40. Adding the digits of 40, you get 4 as the final check number, which must always be less than 9.

Next add the digits of 185,296, which gives 31. Adding $3 + 1$, you get 4 as the check number. Since this 4 agrees with the final check number, 4, the work is considered correct.

To find a check number by adding the digits, you may skip a 9 or any digits whose sum is 9. In finding the check number of 185,296, you need add only 5, 2, and 6. The check number of 7245 and of 4320 is 0.

Study these examples, which are checked by casting out 9's:

```
  429   6
  174   3
 1716  18 (0)
3003
429
74646 (0)
```

This work is considered correct because the final check numbers agree.

```
  554   5
  168   6
 4432  30 (3)
3224
554
92072 (2)
```

This work is incorrect because the final check numbers do *not* agree.

Exercises

Multiply the following and check by casting out 9's:

1.	2.	3.	4.	5.
3298	793	2409	3298	4577
364	512	673	819	2736

6. In multiplying 888 by 666, Frank got 491,408, Harry got 393,407, and Arthur got 591,408. Who was right?

WATCHING THE PARTIAL PRODUCTS

1. Robert checked this example by casting out 9's. His final check numbers were both 5; but when he came to school, he found that his answer was not correct. Can you tell what is wrong?

```
   557   8
    67   4
  ----  --
  3899  32 (5)
3342
------
338099 (5)
```

Robert wrote the partial product 3342 one place too far to the left. Even though this example checks by 9's, the work is incorrect.

2. In doing this example, Mary put the partial product 876 one place too far to the right. Does her example check by 9's in spite of the error?

```
   438   6
   205   7
  ----  --
  2190  42 (6)
  876
 -----
 10950 (6)
```

Since the check of 9's does not show when a partial product has been misplaced, be sure that each partial product is in the right place before you cast out 9's.

It also rarely happens that two errors in a problem counteract each other so that the check of 9's does not disclose the errors. These exceptions are of minor importance, however, since the check of 9's detects 95% of all errors that occur and is by far the quickest check for multiplication and division. It is not recommended for addition and subtraction.

Exercises

Multiply. Check each answer by casting out 9's:

1. 724	**2.** 829	**3.** 457	**4.** 466	**5.** 715	**6.** 235
386	297	928	639	452	869

If the partial products are in the right place, check by casting out 9's. Work again and check the incorrect examples:

7. 843	**8.** 265	**9.** 764	**10.** 682
97	18	309	501
5901	2120	6776	682
7587	265	2292	3410
81771	28620	29696	341682

USING MULTIPLICATION TO SOLVE PROBLEMS

Solve these problems and check the work.

1. In the summer Clinton works 7 hr. a day in Mr. Foster's garden. How many hours does he work in 25 da.? If he is paid 25¢ an hour, how much does he earn in 25 da.?

2. Julia has 18 rows of tomato plants with 15 plants in each row. How many plants has she in all?

3. Mr. Campbell averages 28 mi. an hour in his car. If he drives 8 hr. a day for 9 da., how far can he drive in all?

4. Mr. Monroe says that in his new car he averages 21 mi. on each gallon of gasoline. About how many miles did he drive in August if he used 72 gal. of gasoline that month?

5. There are 1760 yd. in 1 mile. How many feet are there in 1 mile? How many inches are there in 1 mile?

6. The Outdoor Shop bought 110 tennis rackets at $2.25 each and sold them at $3.85 each. How much profit did the Outdoor Shop make on all these tennis rackets?

7. Mr. Leonard is laying a cement walk in front of his store. If the walk is 18 yd. long and 2 yd. wide, how much will it cost to lay it at $2.35 per square yard?

8. Dorothy is making chocolate creams to sell at the School Fair. If she puts 24 chocolate creams in each box, how many must she make to fill 15 boxes? If she sells the candy at 40¢ a box, how much in all will she get for it?

9. If it costs $17.50 a week at a summer camp, how much will it cost for 12 weeks?

10. The area of Connecticut is 4820 square miles. Find the area of Texas if it is about 54 times as large as the area of Connecticut.

11. Hilda Jenson earns $3.25 a week helping Mrs. Ross and $2.50 a week helping Mrs. Ferris. How much does she earn in 48 wk.?

HOW TO CHECK DIVISION

There are two good ways of checking long division:

1. *By multiplication.* In dividing 10,498 by 23, the quotient is 456 and the remainder is 10. To check, multiply 456 by 23 and add the remainder, 10. Since this gives 10,498, which equals the dividend, the work is correct.

```
      456        Check
23)10498          456
    92             23
    ---           ----
    129           1368
    115           912
    ---          -----
     148         10488
     138            10
     ---         -----
      10         10498
```

Short division should be checked by this method or by doing the work over again.

2. *By casting out 9's.* You may check the above example much more quickly by casting out 9's.

```
        (6)
(5)    456          9's Check
23)10498 (4)          5
    92                6
    ---              --
    129              30
    115               1
    ---              --
     148             31 (4)
     138
     ---
      10 (1)
```

Find the check numbers for 23, 456, 10, and 10,498, as in checking multiplication. Then multiply the check number 5 by the check number 6, which gives 30; to this product add 1 (the check number of the remainder), getting 31. Adding the digits of 31 gives 4 as the final check number. Since this 4 agrees with the check number of 10,498, which is also 4, you may consider the work correct.

Exercises

Divide. Check by multiplication and by casting out 9's:

1. $53\overline{)7314}$ **3.** $27\overline{)66459}$ **5.** $48\overline{)11856}$ **7.** $53\overline{)18323}$

2. $47\overline{)9114}$ **4.** $257\overline{)66439}$ **6.** $29\overline{)5351}$ **8.** $62\overline{)27975}$

9. Henry divided 373,062 by 487 and got a quotient of 745 and a remainder of 247. Frank got 766 as quotient and 20 as remainder. Which, if either, was correct?

IMPROVEMENT TEST No. 12

Test 12 A — Multiplication (Time 4 min. after copying)

1. 62×351	**2.** 73×173	**3.** 19×587
4. 84×902	**5.** 26×694	**6.** 26×278
7. 59×326	**8.** 38×173	**9.** 85×894

Test 12 B — Multiplication (Time 4 min. after copying)

1. 769×253	**2.** 947×847	**3.** 533×813
4. 918×964	**5.** 562×657	**6.** 475×929

Test 12 C — Multiplication (Time 3 min. after copying)

1. 957×8146	**2.** 508×3527
3. 610×9057	**4.** 324×4081

Test 12 D — Division (Time 4 min. after copying)

Express remainders, if any, as fractions in the quotient:

1. $78\overline{)24736}$	**2.** $92\overline{)42059}$	**3.** $61\overline{)43249}$
4. $47\overline{)10293}$	**5.** $38\overline{)32137}$	**6.** $85\overline{)30435}$

Test 12 E — Division (Time 4 min. after copying)

Express remainders, if any, as fractions in the quotient:

1. $64\overline{)41440}$	**2.** $56\overline{)50579}$	**3.** $71\overline{)37133}$
4. $31\overline{)34435}$	**5.** $39\overline{)29172}$	**6.** $58\overline{)33263}$

NOTE. In an Improvement Test in multiplication or division, first copy all the examples. The time allowance does not include the time required for copying. If 1 min. extra is allowed on each test, all the examples can be checked by casting out 9's.

USING DIVISION TO SOLVE PROBLEMS

Solve these problems and check your computations.

1. It cost $24 to hire busses to carry 160 pupils to the school picnic. What was each pupil's share of the expense?

2. Helen's marks in arithmetic for 5 days were 94, 88, 100, 90, 93. What was her average mark? What effect would it have had on her average if her last mark of 93 had been 73? if her last mark had been 63?

3. Mr. Allen drove his car 140 mi. in 5 hr. How many miles per hour did he average?

4. If Mr. Allen used 8 gal. of gasoline for his trip of 140 mi., how many miles did he average on each gallon?

5. Mr. Johnson bought a radio set for $100. He paid $25 cash and the rest at the rate of $15 a month. How many months did it take him to pay for it?

6. Last week Madge picked 180 roses from her garden. If she sold them at $2 a dozen, how much in all did she receive for them?

7. Mr. Davis drives his car at an average of 24 mi. per hour. How long will it take him to drive 204 mi.?

8. Arthur has a fish line 54 ft. long. How many yards long is it? How many inches long is it?

9. Lucy has saved $15. Margaret has saved $195. Margaret has saved how many times as much as Lucy?

10. It cost James $18.75 for lumber to make 5 bookcases each of the same size. How much did the lumber cost for 1 bookcase?

11. Alfred has a piece of rope 25 ft. long with which he wants to measure off a distance of 600 ft. How can he do it?

12. The distance around the school yard is 660 ft. How many times must Jack run around the yard to run a mile?

AN IMPORTANT PRINCIPLE OF FRACTIONS

You have learned that you can change a fraction like $\frac{5}{8}$ into another fraction with the same value, like $\frac{10}{16}$, by using this fundamental principle:

$$\frac{5}{8} = \frac{2 \times 5}{2 \times 8} = \frac{10}{16}$$

If you multiply or divide both terms of a fraction by the same number, you get another fraction having the same value.

The examples below show how this principle is used.

1. Add $\frac{1}{2}$ and $\frac{3}{4}$.

$$\frac{1}{2} = \frac{4}{8}$$
$$\frac{3}{4} = \frac{6}{8}$$
$$\frac{10}{8} = 1\frac{1}{4}$$

First change $\frac{1}{2}$ to $\frac{4}{8}$ by multiplying both terms of $\frac{1}{2}$ by 4. Next change $\frac{3}{4}$ to $\frac{6}{8}$ by multiplying both terms of $\frac{3}{4}$ by 2. You see that you use the fundamental principle twice in this example.

2. Reduce $\frac{45}{105}$ to lowest terms.

$$\frac{\cancel{45}^{\,9}}{\cancel{105}_{\,21}} = \frac{3}{7}$$

You can do this work quickly by *cancellation*. When you *cancel*, you *divide* both terms of the fraction by the same number. First divide both terms of $\frac{45}{105}$ by 5, which gives $\frac{9}{21}$. Then divide both terms of $\frac{9}{21}$ by 3, which gives $\frac{3}{7}$. Hence $\frac{45}{105} = \frac{3}{7}$.

3. Multiply $\frac{5}{6}$ by $\frac{9}{10}$.

$$\frac{9}{10} \times \frac{5}{6} = \frac{\cancel{9}^{\,3} \times \cancel{5}^{\,1}}{\cancel{10}_{\,2} \times \cancel{6}_{\,2}} = \frac{3}{4}$$

Here you use the fundamental principle twice, first dividing both 9 and 6 by 3 and then dividing both 5 and 10 by 5.

To check work in common fractions, go over it again.

Exercises

Tell what word or number should be put in place of the dots:

1. To change $\frac{2}{8}$ to $\frac{1}{4}$, . . . both terms of $\frac{2}{8}$ by . . .

2. To change $\frac{1}{4}$ to $\frac{25}{100}$, . . . both terms of $\frac{1}{4}$ by . . .

3. To change $\frac{75}{100}$ to $\frac{3}{4}$, . . . both terms of . . . by . . .

4. How do you change $\frac{7}{8}$ to $\frac{28}{32}$? $\frac{48}{56}$ to $\frac{6}{7}$? $\frac{90}{100}$ to $\frac{9}{10}$? $\frac{80}{100}$ to $\frac{4}{5}$? $\frac{150}{300}$ to $\frac{1}{2}$? $\frac{1}{2}$ to $\frac{50}{100}$? $\frac{1}{25}$ to $\frac{4}{100}$?

MARY HAS A BIRTHDAY PARTY

Check all the work in these problems.

1. Mary Dexter is having a birthday party. She needs 3 qt. of ice cream, which she can buy at 70¢ a quart. If she makes the ice cream herself, she uses 8 eggs at 66¢ a dozen, $1\frac{1}{2}$ qt. of milk at 19¢ a quart, 1 pt. of cream at 42¢ a pint, 1 lb. of sugar at 10¢ a pound, and 3 tbsp. of vanilla costing 15¢. The freezing requires $\frac{1}{2}$ bag of rock salt at 20¢ a bag and 10 lb. of ice at 60¢ per 100 lb. How much does Mary save by making the ice cream instead of buying it?

2. Mary's recipe for mint lemonade calls for the following:

4 cups water	12 mint sprigs
$1\frac{1}{2}$ cups sugar	1 cup strawberry juice
2 cups boiling water	1 cup raspberry juice

juice of 8 lemons

This recipe is sufficient for 10 persons. Change the recipe to serve 5 persons; to serve 15 persons; to serve 25 persons.

3. A recipe for cookies requires the following:

$\frac{1}{2}$ cup butter	$\frac{1}{4}$ cup raisins
$\frac{1}{3}$ cup sugar	$\frac{3}{4}$ cup flour

1 tsp. vanilla

This recipe makes 24 cookies. How much of each ingredient would Mary's mother need for 36 cookies? for 48 cookies?

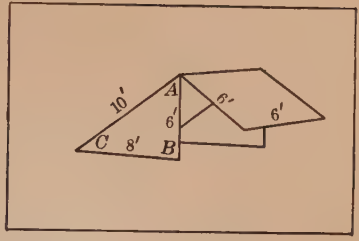
4. Mary's mother uses these ingredients for sponge cake:

yolks of 6 eggs	1 tbsp. lemon juice
1 cup sugar	$\frac{1}{2}$ lemon rind grated
1 cup flour	whites of 6 eggs

Find the total cost if eggs cost 54¢ a dozen, sugar 9¢ a pound, flour 8¢ a pound, lemons 40¢ a dozen (1 lemon is used).

In finding the cost of recipes, use these equivalents:

1 lb. sugar = 2 cups	1 pt. milk = 2 cups
1 lb. flour = 4 cups	1 lb. butter = 32 tbsp.



CAMPING PROBLEMS

1. How many yards of duck 72 in. wide must Ted and Donald buy to make the tent shown above? How much will the duck cost at \$1.10 per running yard?

The two triangular sides, like *ABC*, can both be cut from one rectangular piece of duck 6 ft. wide and 8 ft. long. Show how. The roof and flap together make a rectangle 6 ft. wide and 16 ft. long.

2. How much will the duck for the tent weigh if each running yard of duck weighs $22\frac{1}{2}$ oz.?

3. How many boards each 16 ft. long and 10 in. wide do the boys need to make a platform 8 ft. by 15 ft. for the tent?

4. How many square feet of duck are there in the tent? Use the dimensions given in the picture at the top of the page.

5. To make their tent waterproof, the boys want to cover it with a waterproof liquid which is sold only by the whole quart or by the whole gallon. If 1 qt. of liquid covers 25 sq. ft. of duck, how many quarts must they buy? A quart can costs 60¢ and a gallon can costs \$2. Find the cost of the liquid needed if it is bought in the most economical way.

6. On a hike the boys found a tall pine tree that had been uprooted by a storm. Since they had no ruler, Donald used a stick to measure the tree. The tree was 39 times as long as the stick. When they reached camp, they found that the stick was $3\frac{1}{3}$ ft. long. How tall was the tree?

BUYING CLOTH

1. Jane needs 20 in. of silk. The salesman tells her that this silk is sold only in yards and in halves, quarters, or eighths of a yard. What part of a yard should Jane buy to have at least 20 in., with as little as possible left over?

Since $\frac{1}{2}$ yd. contains 18 in., $\frac{1}{2}$ yd. is not enough. How many inches are there in $\frac{5}{8}$ yd.? Is this enough to serve Jane's purpose?

2. Complete the table at the right, showing how many inches there are in each eighth of a yard. Use this table for reference in the problems below and also on page 163.

In each of the problems below, assume that parts of a yard can be bought only in halves, quarters, or eighths of a yard.

Yards	*Inches*
$\frac{1}{8}$	$4\frac{1}{2}$
$\frac{1}{4}$	9
$\frac{3}{8}$	$13\frac{1}{2}$
$\frac{1}{2}$	
$\frac{5}{8}$	
$\frac{3}{4}$	
$\frac{7}{8}$	
1	

3. Margery needs 30 in. of wide ribbon. Find the least fraction of a yard that she can buy to give her 30 in.

Tell what part of a yard you must buy to get the following:

4. 9 in. **6.** 15 in. **8.** 10 in. **10.** 19 in.

5. 8 in. **7.** 13 in. **9.** 25 in. **11.** 33 in.

12. Ethel needs 49 in. of silk. How much material should she ask the clerk for?

To change 49 in. to yards, divide 49 by 36. The quotient is 1 and the remainder is 13. Hence 49 in. = 1 yd. 13 in. The table above shows that 13 in. is nearly $\frac{3}{8}$ of a yard.

Ethel must ask for $1\frac{3}{8}$ yd.

$$\begin{array}{r} 1 \\ 36\overline{)49} \\ 36 \\ \hline 13 \end{array}$$

13. How much silk must Amy buy to get 53 in. of silk?

Tell how much material you must buy to get these lengths:

14. 38 in. **16.** 45 in. **18.** 48 in. **20.** 110 in.

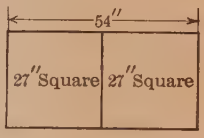
15. 55 in. **17.** 70 in. **19.** 54 in. **21.** 191 in.

SEWING FOR THE HOME

1. Elsie's couch is 6 ft. 6 in. long. How many inches long is that? The couch cover is to hang over 12 in. on each end, with 2 in. extra allowed at each end for a hem. How many yards of cretonne must Elsie buy for the cover?

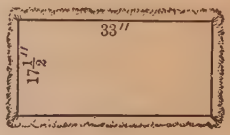
In this problem and in the problems below, assume that the cloth can be bought only in yards, and in halves, quarters, or eighths of a yard.

2. The cretonne is 54 in. wide. To cover the sofa pillow, Elsie needs 2 pieces each 27 in. square. How much cretonne must she buy for the pillow?



3. How much cretonne must she buy in all for the couch cover and pillow? Find the cost at 75¢ a yard.

4. Elsie is using for a dresser cover a piece of linen 33 in. by $17\frac{1}{2}$ in. How many yards and eighths of a yard of lace must she buy to put lace all around the edge, allowing $2\frac{1}{2}$ in. extra at each corner for turning? Find the cost at 25¢ a yard.



5. How many yards of toweling will Elsie need for 6 towels which are to be 27 in. long when finished and are to have $\frac{3}{8}$ in. allowed at each end for a hem? Find the cost at 45¢ a yard.

6. Elsie has a circular table cover 20 in. in diameter. She wants to edge it with lace $\frac{1}{2}$ in. wide. The lace should be as long as the circumference of the *completed* cover (including the lace). The circumference of a circle is the distance around it, and is about $\frac{22}{7}$ times the length of the diameter. How much lace should Elsie buy?

7. Elsie decides to buy more of the 54-inch cretonne (ex. 2) to make 2 curtains. If each curtain is to be 27 in. wide and 31 in. long, how much more cretonne must she buy?

MULTIPLYING AND DIVIDING DECIMALS

1. Mr. Adams drove for 9.25 hours at the rate of 26.3 miles an hour. How far did he go?

Explain how this example is worked. The shortest way to check the multiplication is by casting out 9's.

```
   26.3
   9.25
  -----
  1 315
  5 26
236 7
-------
243.275
```

In multiplying decimals, multiply as with whole numbers; then, beginning at the right, point off as many decimal places in the product as there are in the multiplier and the multiplicand together.

2. A store paid $22 for 24 caps. Find the cost per cap.

To avoid a fraction in the quotient, annex 0's and divide, placing the decimal point in the quotient directly above that in $22.000. The result, to three decimal places, is $.916, which is called $.92 correct to the *nearest hundredth* (or to the *nearest cent*) because $.916 is closer to $.92 than to $.91. Hence each cap cost $.92.

If the answer had been $.913, the result correct to the nearest hundredth would be $.91.

Check the work by multiplication or by casting out 9's.

```
      $.916
24)$22.000
   21 6
   ----
     40
     24
     ---
     160
     144
     ---
      16
```

Carry the quotient to three decimal places to get a result correct to the nearest hundredth or to two decimal places for a result correct to the nearest tenth.

If the last figure is less than 5, drop it; if the last figure is 5 or more, make the figure before it 1 larger.

Exercises

Do the following and check the work. If the division is not exact, carry quotients to the nearest hundredth:

1. .607×.09	**6.** 527 ÷ 248	**11.** 639.24 ÷ 84
2. 26.5×2.03	**7.** 730.58 ÷ 96	**12.** 23.008 ÷ 53
3. 9.82×7.2	**8.** 2863.8 ÷ 74	**13.** 3.2396 ÷ 52
4. 5000×5.8	**9.** 133.93 ÷ 57	**14.** 420.85 ÷ 81
5. .700×.305	**10.** 291.51 ÷ 79	**15.** 896.05 ÷ 94

IMPROVEMENT TEST No. 13

Test 13 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$3\frac{3}{4}$	$1\frac{2}{5}$	$4\frac{3}{4}$	$5\frac{1}{2}$	$7\frac{1}{2}$
$1\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{3}{8}$	$3\frac{1}{3}$	$4\frac{3}{4}$
$8\frac{1}{4}$	$5\frac{3}{5}$	$1\frac{7}{16}$	$7\frac{5}{6}$	$2\frac{1}{3}$
$2\frac{1}{8}$	$3\frac{3}{10}$	$2\frac{5}{16}$	$2\frac{1}{4}$	$3\frac{2}{3}$

Test 13 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$4\frac{5}{8}$	4	$7\frac{1}{2}$	$2\frac{9}{10}$
$1\frac{1}{2}$	$3\frac{7}{8}$	$5\frac{1}{3}$	$2\frac{3}{10}$
5.	**6.**	**7.**	**8.**
$5\frac{1}{4}$	7	$8\frac{1}{16}$	$9\frac{3}{4}$
$3\frac{7}{8}$	$2\frac{5}{12}$	$6\frac{9}{16}$	$7\frac{1}{3}$
9.	**10.**	**11.**	**12.**
$3\frac{7}{10}$	8	$5\frac{1}{2}$	$6\frac{3}{4}$
$1\frac{4}{5}$	$4\frac{2}{3}$	$2\frac{2}{5}$	$5\frac{11}{16}$

Test 13 C — Multiplication (Time 4 min. after copying)

1. $\frac{2}{3}$ of 39	**2.** $6\times\frac{7}{8}$	**3.** $4\frac{1}{2}\times15$
4. $\frac{1}{4}$ of 19	**5.** $3\frac{3}{5}\times3\frac{1}{3}$	**6.** $8\frac{3}{4}\times202$
7. $\frac{5}{8}$ of 28	**8.** $2\frac{2}{3}\times2\frac{5}{8}$	**9.** $136\times7\frac{3}{8}$

Test 13 D — Division (Time 3 min. after copying)

1. $6\div\frac{3}{8}$	**2.** $3\frac{1}{8}\div5$	**3.** $8\frac{2}{3}\div13$
4. $8\div3\frac{1}{7}$	**5.** $2\frac{1}{5}\div\frac{3}{10}$	**6.** $3\frac{3}{4}\div3\frac{1}{3}$
7. $\frac{11}{16}\div1\frac{3}{8}$	**8.** $3\frac{3}{8}\div\frac{3}{4}$	**9.** $7\frac{1}{2}\div1\frac{1}{3}$

MAKING RAFTS

1. Since a pine board will float on water, you can see that pine wood is lighter than water. Pine weighs only .4 as much as water. If 1 cu. ft. of water weighs 62.5 lb., what is the weight of 1 cu. ft. of pine? of 5 cu. ft. of pine? How much does a pine beam 12 ft.$\times\frac{3}{4}$ ft.$\times$1 ft. weigh?

2. If 1 cu. ft. of pine weighs 25 lb. and 1 cu. ft. of water weighs 62.5 lb., 1 cu. ft. of pine is 37.5 lb. lighter than 1 cu. ft. of water. Then 1 cu. ft. of pine, floating in the water, could carry 37.5 lb. without sinking. How many pounds could 10 cu. ft. of pine wood carry?

3. Dave and Jack made a raft of 6 pine beams, each beam measuring 8 ft.$\times\frac{3}{4}$ ft.$\times\frac{1}{4}$ ft. How many cubic feet of pine did the raft contain? How many pounds of load could the raft carry without sinking?

Use any figures you need from ex. 2.

4. If Dave weighs 90 lb. and Jack weighs 105 lb., could both of them ride together on the raft in ex. 3 without making it sink entirely?

5. Sam made a raft of 8 pine boards, each 12 ft.$\times\frac{3}{4}$ ft.$\times\frac{1}{12}$ ft. How much weight would this raft carry?

6. Little Mary weighs 40 lb. Will a pine board that is 5 ft.$\times$2 ft.$\times\frac{1}{8}$ ft. support her weight on the water?

CHANGING COMMON FRACTIONS TO DECIMALS

1. What decimal part of a ton is $\frac{7}{8}$ of a ton?

Since $\frac{7}{8}$ means $7 \div 8$, divide 7 by 8. To do this, put a decimal point after 7, annex 0's, and divide.

You find that $\frac{7}{8}$ ton = .875 ton.

$$8\overline{)7.000}$$
$$.875$$

To change a fraction to a decimal, put a decimal point after the numerator, annex 0's, and divide by the denominator.

Change to decimals, correct to the nearest thousandth:

2. $\frac{1}{7}$ **3.** $\frac{4}{9}$ **4.** $\frac{3}{8}$ **5.** $\frac{4}{7}$ **6.** $\frac{15}{16}$ **7.** $\frac{29}{45}$ **8.** $\frac{5}{12}$ **9.** $\frac{9}{16}$ **10.** $\frac{5}{8}$ **11.** $\frac{5}{32}$ **12.** $\frac{13}{15}$ **13.** $\frac{41}{111}$

DIVIDING BY A DECIMAL

1. Mr. Carter drove his car 233.75 mi. in 8.5 hr. How many miles an hour did he average?

In dividing 233.75 by 8.5, first multiply both numbers by 10 in order to change the divisor, 8.5, to a whole number. This moves each decimal point 1 place to the right. Instead of erasing the decimal points or rewriting the numbers, you may use carets to indicate the new positions of the points, as shown here. Place the decimal point in the quotient above the caret in the dividend.

$$\begin{array}{r} 27.5 \\ 8.5_\wedge\overline{)233.7_\wedge5} \\ 170 \\ \hline 637 \\ 595 \\ \hline 425 \\ 425 \\ \hline \end{array}$$

Mr. Carter averaged 27.5 mi. an hour.

Check the computation. There should be as many decimal places in the divisor and quotient together as there are in the dividend.

In dividing by a decimal, place a caret as many places to the right of the decimal point in both divisor and dividend as there are decimal places in the divisor. Place the decimal point in the quotient over the caret in the dividend.

Divide. Carry results to the nearest hundredth, if necessary:

2. $271.56 \div 6.2$
3. $.20007 \div 3.9$
4. $4.2739 \div .79$
5. $135.08 \div .022$
6. $16269 \div .51$
7. $3.1119 \div .53$
8. $.51968 \div 6.4$
9. $39.264 \div .96$
10. $24.476 \div 4.6$
11. $45.341 \div 9.2$
12. $.19304 \div 7.6$
13. $157.44 \div .384$

MULTIPLYING BY 10, 100, AND 1000

To multiply a number by 10, 100, *or* 1000, *move the decimal point as many places to the right as there are* 0*'s in the multiplier.*

Thus, $10\times24 = 240$ and $100\times\$3.65 = \$365$. In multiplying a number by 10, 100, or 1000, you sometimes have to annex 0's at the right of it. Thus, $100\times3.5 = 350$.

Multiply the following at sight:

1. 10×43
2. 10×58
3. 10×5.4
4. 10×205
5. 10×98.7
6. 100×50
7. 100×3.4
8. 100×66.2
9. 100×0.63
10. 100×0.05
11. 100×44.4
12. 1000×7.5
13. 1000×0.6
14. 1000×2.66
15. 1000×5.625

DIVIDING BY 10, 100, AND 1000

To divide by 10, 100, *or* 1000, *move the decimal point as many places to the left as there are* 0*'s in the divisor.*

Thus, $1450 \div 10 = 145$ and $62.8 \div 100 = .628$. In moving the decimal point to the left, it is sometimes necessary to prefix 0's. Thus, $14 \div 10 = .014$ and $.5 \div 100 = .005$.

1. Divide 8768 by 400.

Since $400 = 4\times100$, first divide 8768 by 100 by moving the decimal point two places to the left, and then divide 87.68 by 4.

$$4\overline{)87.68}$$
$$21.92$$

If a divisor ends in 0*'s, cross off the* 0*'s before dividing and move the decimal point in the dividend one place to the left for each* 0 *crossed off.*

Divide the following at sight:

2. $570 \div 10$
3. $88.9 \div 10$
4. $1069 \div 10$
5. $25.8 \div 100$
6. $9.8 \div 20$
7. $2.5 \div 100$
8. $261 \div 300$
9. $4.32 \div 100$
10. $0.08 \div 10$
11. $396 \div 600$
12. $4.51 \div 1000$
13. $29.8 \div 1000$

SHORT CUTS IN MULTIPLICATION

1. Find the number of seats in a theater having 25 rows of seats with 42 seats in each row.

Frank does the work as shown in *A*, actually multiplying 42 by 25.

Mildred uses a different method, as shown in *B*. She says that 25 is the same as $\frac{100}{4}$; therefore she first multiplies 42 by 100 by annexing two 0's and then divides the result by 4.

Whose way is quicker, Frank's or Mildred's?

A

42
25
210
84
1050

B

4)4200
1050

2. Reduce $\frac{100}{3}$ to a mixed number. Is $\frac{100}{3}$ the same as $33\frac{1}{3}$? Then what is a short way to multiply by $33\frac{1}{3}$?

3. Learn these short cuts in multiplication:

To multiply by 5, *multiply by* 10 *and divide by* 2.
To multiply by 25, *multiply by* 100 *and divide by* 4.
To multiply by 50, *multiply by* 100 *and divide by* 2.
To multiply by $12\frac{1}{2}$, *multiply by* 100 *and divide by* 8.
To multiply by $33\frac{1}{3}$, *multiply by* 100 *and divide by* 3.
To multiply by $16\frac{2}{3}$, *multiply by* 100 *and divide by* 6.
To multiply by 125, *multiply by* 1000 *and divide by* 8.

Numbers like 25, 50, and $33\frac{1}{3}$ are called **aliquot parts** of 100 because they divide 100 exactly.

Exercises

Multiply, using the short methods:

1. 5×62
2. 5×87
3. 50×36
4. 25×96
5. $33\frac{1}{3}\times78$
6. $16\frac{2}{3}\times42$
7. 25×408
8. 50×377
9. 125×632
10. 960×125
11. $12\frac{1}{2}\times743$
12. $33\frac{1}{3}\times570$
13. 5×2447
14. 25×739
15. 25×2648
16. $726\times16\frac{2}{3}$
17. 125×360
18. 610×100

SHORT CUTS IN DIVISION

1. Divide 1576 by 25.

John did the work the long way, as in *A*. Alice got the same answer by using a short cut, as in *B*. This is the way Alice explained her method: To divide 1576 by 25 is the same as to find $\frac{1}{25}$ of 1576, or $\frac{4}{100}$ of 1576. Hence to divide 1576 by 25, multiply by 4, which gives 6304; then divide 6304 by 100 by moving the decimal point two places to the left, as in *B*.

A

```
  63.04
25)1576.00
  150
   76
   75
  100
  100
```

B

```
1576
   4
63.04
```

2. Learn the following short cuts in division:

To divide by 25, *multiply by* 4 *and divide by* 100.
To divide by 50, *multiply by* 2 *and divide by* 100.
To divide by $12\frac{1}{2}$, *multiply by* 8 *and divide by* 100.
To divide by $33\frac{1}{3}$, *multiply by* 3 *and divide by* 100.
To divide by 125, *multiply by* 8 *and divide by* 1000.

Divide the following, using the short methods:

3. $300 \div 50$
4. $1300 \div 25$
5. $2140 \div 25$
6. $7000 \div 125$
7. $650 \div 25$
8. $980 \div 12\frac{1}{2}$
9. $455 \div 12\frac{1}{2}$
10. $800 \div 33\frac{1}{3}$
11. $752 \div 12\frac{1}{2}$
12. $6060 \div 50$
13. $1100 \div 33\frac{1}{3}$
14. $5102 \div 25$

A RACE IN MULTIPLYING AND DIVIDING

Work these by short cuts. When one pupil finishes, all stop working. Whoever then has the most answers right wins:

1. 5×66
2. 25×22
3. 50×7.4
4. 25×332
5. 430×50
6. $12\frac{1}{2} \times 696$
7. $764 \div 100$
8. 25×2800
9. $39 \div 1000$
10. 10×45.63
11. $33\frac{1}{3} \times 976$
12. 100×4.8
13. $5100 \div 25$
14. 1000×2.8
15. $10 \times .001$
16. $.624 \div 10$
17. $100 \times .064$
18. 100×8800

MEASURING ELECTRICITY

Roger found "50 W" marked on the electric lamp bulb in his room. His father explained that a 50-watt (50 W) lamp is one which uses 50 *watt hours* of electricity in 1 hr. A *watt hour* is a unit by which we measure the amount of electricity that has been used.

If Roger has his 50-watt lamp lighted for 4 hr., it uses 4×50 watt hours, or 200 watt hours, of electricity. In $\frac{1}{2}$ hr. it requires $\frac{1}{2}$ of 50 watt hours, or 25 watt hours.

If Roger's lamp is lighted 20 hr., it uses 20×50 watt hours, or 1000 watt hours in all. 1000 watt hours make a *kilowatt hour;* the prefix *kilo* means 1000. The electricity used in our homes is measured by the *kilowatt hour* (K.W.H.).

To change from watt hours to kilowatt hours, divide by 1000.

Thus, when a 50-watt lamp is lighted for 35 hr., it uses 35×50 watt hours, or 1750 watt hours, of electricity. 1750 watt hours equal 1.750 kilowatt hours.

Exercises

1. How many watt hours of electricity would a 40-watt lamp use in 1 hr.? in 3 hr.? in 4 hr.? in $2\frac{1}{2}$ hr.?

2. John found electric bulbs of these sizes in his home: 15 W, 25 W, 50 W, 60 W. How many watt hours does each use in 1 hr.? in 5 hr.? in $7\frac{1}{2}$ hr.? in 12 hr. 30 min.?

3. If you burn a 60-watt lamp for 20 hr. in one week, how many watt hours do you use? how many kilowatt hours?

4. How many K.W.H. does the light in ex. 3 need in 25 hr.? in 28 hr.? in 32 hr.? in 78 hr.? in 16 hr. 40 min.?

5. What decimal part of a kilowatt hour would each of the bulbs in ex. 2 require in 1 hr.? in 6 hr.? in 10 hr.?

6. How many K.W.H. does a 615-watt electric heater use in 2 hr.? in 3 hr.? in 1 hr. 40 min.?

THE PRICE OF ELECTRICITY

Find all answers correct to the nearest tenth of a cent.

1. Mr. Hill bought a new 75-watt lamp. How much a month will it cost him for current if the lamp is lighted 60 hr. a month? He pays 8¢ per K.W.H. for electricity.

In one month the lamp consumes 60×75 watt hours, or 4500 watt hours, which equals 4.5 kilowatt hours. At 8¢ per K.W.H., the monthly cost of using the lamp is 4.5×8¢, or 36¢.

In some cities electricity is sold at 10¢ or more per K.W.H.; other cities have lower rates, like 7¢ or 8¢ per K.W.H.

2. A family used 34 K.W.H. during August. The rate where they lived was 12¢ per K.W.H. What was their bill for August?

3. At 9¢ per K.W.H., what is the monthly bill for electricity of a family that uses 26 K.W.H.? 29 K.W.H.? 37 K.W.H.?

4. Mrs. Todd's washing machine has a 300-watt motor. If she operates it 8 hr. in June, how many K.W.H. does it use? Find the cost of the current at 10¢ per K.W.H.

5. If electricity costs 7¢ per K.W.H., find the cost of burning a 60-watt light for 50 hr.; for 30 hr.; for 25 hr.

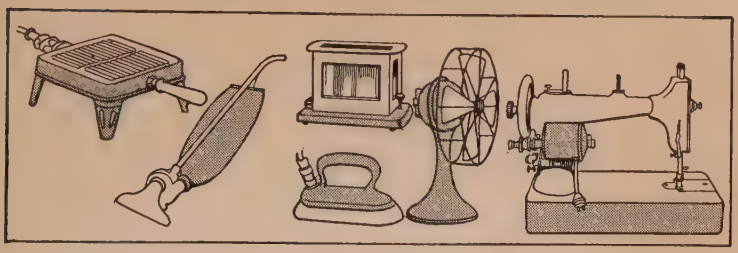
6. Mr. Adams left a 75-watt lamp burning in the basement of his house for 72 hr. while he was away on a trip. If he has to pay 10¢ per K.W.H. for current, how much did this carelessness cost him?

7. Calvin finds that the 300-watt motor of their electric refrigerator runs $5\frac{1}{2}$ hr. out of every 24 hr. Find the daily cost for current at 9¢ per K.W.H.

8. On a certain night Mr. Lee's family used one 75-watt lamp for 4 hr., three 40-watt lamps for 5 hr., one 10-watt lamp for 8 hr., and one 60-watt lamp for $2\frac{1}{2}$ hr. Find the total cost for the current at 9¢ per K.W.H.

9. If Mr. Lee paid $7\frac{1}{2}$¢ per K.W.H., instead of 9¢, find the cost of the current used in ex. 8.

MRS. BAKER BUYS ELECTRIC SERVANTS



Find all answers to the nearest tenth of a cent.

Mrs. Baker heard that she could do her housework with less labor and slight additional expense by installing electrical appliances. She pays 7¢ per K.W.H. for current.

1. She bought an electric flatiron that used 550 watts. How much did it cost her to operate it for 1 hr.?

2. Mrs. Baker needs 20 lb. of coal at $14 a ton to do the same ironing that she can do in $2\frac{1}{2}$ hr. with her electric iron. Find the cost of using coal; of using electricity.

3. It would take 80 cu. ft. of gas at $1.25 per 1000 cu. ft. to do the ironing in ex. 2. Is it cheaper to use gas or electricity? Does using electricity make ironing easier?

4. How much would it cost Mrs. Baker to use a 500-watt toaster for 12 min. to make toast for breakfast?

5. If she uses a vacuum cleaner with a 150-watt motor for 1 hr. 40 min., how much does the current for cleaning cost?

6. Find the cost of operating a 40-watt electric fan for $3\frac{1}{2}$ hr.; of using a 50-watt electric sewing machine for $2\frac{1}{2}$ hr.

7. An electric stove needs 660 watts. How much would it cost to cook a meal on it, if it took 1 hr.? if it took $1\frac{1}{2}$ hr.?

8. How much does each electric appliance in ex. 1 to 7 cost per hour? Which costs the most? the least? Do those which produce heat cost more or less than the others?

CAN YOU READ YOUR ELECTRIC METER?

If you use electricity, there is in your house an electric meter which measures the number of kilowatt hours of electricity that are used.

In reading the electric meter at the left, read it like a four-figure number in which dial A represents thousands, dial B represents hundreds, dial C represents tens, and dial D represents units.

When the pointer on a dial is between two numbers, read the smaller number for that dial.

Thus, the dials of the above meter are read as follows:

On A the pointer is between 2 and 3. The reading is 2.
On B the pointer is between 1 and 2. The reading is 1.
On C the pointer is between 9 and 10. The reading is 9.
On D the pointer is between 1 and 2. The reading is 1.

Hence the reading of the above meter is 2191 K.W.H.

Note that the 0 on the dials stands for 10 when the pointer is between 9 and 0, as shown above on dial C.

Reading the Electric Meter

1. On the above meter, does the pointer move to the right or to the left on dial A? on dial C? on dial B? on dial D?

2. The first set of dials at the right shows the electric meter of a store on Oct. 31, while the second set of dials shows the same meter on Nov. 30. Read each set of dials. How many K.W.H. of electricity has the store used since Oct. 31?

THE RELATION OF THE DIALS OF THE METER

While the pointer on any dial of a meter makes a complete revolution, the pointer on the dial to the left of it moves slowly ahead one space, just as the hour hand on a watch moves ahead 1 hour for every revolution of the minute hand.

FIG. 1

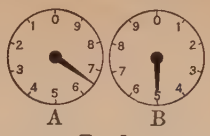


FIG. 2

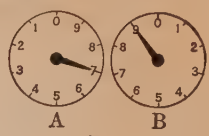


FIG. 3

Suppose that pointer B starts on 0 and pointer A on 6, as in Fig. 1. When pointer B reaches 5, pointer A is half way between 6 and 7, as shown in Fig. 2. When B reaches 9, A is nearly at 7, as shown in Fig. 3. When B reaches 10, where is pointer A?

1. Where is the pointer on dial D? From this reading, tell whether pointer C is just before or just after 5. Do these dials read 59 or 49?

2. Does the set of dials shown below at M read 81 or 91? Read the set of dials at N; the set of dials at R.

3. Draw sets of dials reading 70; 73; 76; 79; 80; 82.

4. If the first meter at the right shows Mrs. Grady's reading on Sept. 30 and the second shows her reading on Oct. 31, how many K.W.H. of electricity did she use in October?

COMPUTING BILLS FOR ELECTRICITY

1. Mrs. Hill's electric meter read 2965 K.W.H. on May 31 and 3021 K.W.H. on June 30. At 8¢ per K.W.H., find the amount of her bill for electricity for June.

Since $3021 - 2965 = 56$, Mrs. Hill used 56 K.W.H. during June. At 8¢ per K.W.H., her bill was $56 \times 8¢$, or \$4.48.

2. The two sets of dials at the right show the readings of Mr. Brown's electric meter on July 31 and on Aug. 31. At 9¢ per K.W.H., find the amount of Mr. Brown's bill for August.

3. In Mr. Shaw's city the charge is 7¢ per K.W.H. for the first 1000 K.W.H. of electricity he uses in a month, 6¢ per K.W.H. for the next 400 K.W.H., and 5¢ per K.W.H. for everything over 1400 K.W.H. If Mr. Shaw used 1750 K.W.H. in his store during July, find the amount of his bill for July.

Consider 1750 equal to $1000 + 400 + 350$.

4. How much does Mr. Shaw pay if he uses 1320 K.W.H. in a month? 2016 K.W.H.? 950 K.W.H.?

5. The dials at the right show the readings of the electric meter in Mr. Shaw's store on Nov. 30 and on Dec. 31. Find how much electricity he used in December and compute his bill for December. Give several possible reasons why Mr. Shaw's December bill for electricity is greater than his July bill.

HOW MUCH GAS HAS MRS. LOW USED?

The dials below show the readings of a gas meter on May 1 (A) and on June 1 (B).

CUBIC FEET

A

CUBIC FEET

B

In A, the dial at the left represents in all 100,000 cu. ft.; hence each of its 10 divisions stands for 10,000 cu. ft. Each division on the middle dial represents 1000 cu. ft. and each on the right-hand dial, 100 cu. ft.

The total reading on May 1 is (20,000 + 1000 + 200) cu. ft., or 21,200 cu. ft. The reading on June 1 is 32,400 cu. ft. How many cubic feet of gas were used in May?

Exercises

In each problem read the gas meter on Sept. 1 and on Oct. 1; then tell how many cubic feet of gas were used during September:

SEPTEMBER 1 OCTOBER 1

1.

2.

3.

MAKING OUT GAS BILLS

This gas bill was sent to Mrs. Low at the beginning of June.

To UNION GAS COMPANY, DR.

Mrs. R. C. Low, South Lakeville

For Gas Supplied		
To June 1, 1928. Index 32,400 cu. ft.		
From May 1, 1928. Index 21,200 cu. ft.		
Gas consumed . . . . 11,200 cu. ft. at $1.25 per M	14	00

The *index* for June 1 means the *reading* of the meter on that day. Subtracting the index for May 1 from the index for June 1, you get 11,200 cu. ft. as the amount of gas consumed between the two dates. The cost of the gas is $1.25 per M, or $1.25 for each 1000 cu. ft. of gas. Dividing 11,200 by 1000 by moving the decimal point three places to the left, you get 11.2. Multiplying $1.25 by 11.2, you get $14.00 as the amount of the bill.

Exercises

Make out gas bills, at $1.05 *per* M, *for customers whose readings, in cubic feet, were as follows. Supply names and places:*

	July 1	August 1		July 1	August 1
1.	44,200	46,000	**5.**	30,400	36,100
2.	69,300	72,100	**6.**	81,100	89,300
3.	20,200	21,100	**7.**	88,400	91,300
4.	37,700	40,300	**8.**	16,500	21,000

9. At $1.15 per M, compute the amounts of the September bills for each of the customers in ex. 1 to 3 on page 177.

10. Bring gas and electric bills from home. Have the class check the amounts to see if they are correct.

IMPROVEMENT TEST No. 14

Test 14 A — Reduction (Time 4 min.)

Change to common fractions in lowest terms:

1. .58 **2.** .65 **3.** $.16\frac{2}{3}$ **4.** .125 **5.** .008

6. .90 **7.** .25 **8.** $.62\frac{1}{2}$ **9.** .050 **10.** .0625

Change to exact decimals:

11. $\frac{12}{25}$ **12.** $\frac{7}{8}$ **13.** $\frac{11}{20}$ **14.** $\frac{37}{50}$ **15.** $\frac{117}{500}$

Test 14 B — Reduction (Time 4 min. after copying)

Change to decimals, correct to the nearest hundredth:

1. $\frac{3}{7}$ **2.** $\frac{5}{14}$ **3.** $\frac{14}{15}$ **4.** $\frac{9}{16}$

5. $\frac{5}{9}$ **6.** $\frac{7}{32}$ **7.** $\frac{1}{12}$ **8.** $\frac{27}{64}$

Test 14 C — Multiplication (Time 4 min. after copying)

1. 403 × .39 **2.** .79 × 8.2 **3.** .005 × .009 **4.** 6100 × .0087

5. 600 × .69 **6.** .586 × .04 **7.** 4.58 × .27 **8.** 2.095 × .0001

9. 980 × 6.5 **10.** .079 × 6 **11.** 8.26 × .08 **12.** 81.9 × 12000

Test 14 D — Division (Time 3 min. after copying)

1. $6.3\overline{).52101}$ **2.** $.28\overline{)12796}$ **3.** $.97\overline{)7.68531}$

PERCENTAGE

How the Business Man Writes Hundredths

You remember from your study of decimals that $\frac{6}{100}$ may be written as .06. A more common way of writing $\frac{6}{100}$ is 6%, which is read *6 per cent.* *Per cent* means *hundredths.* The sign % is used in writing to stand for *per cent.*

You now have these four ways of writing hundredths:

$\frac{6}{100} = .06 = 6\% = 6$ per cent

Likewise, $\frac{93}{100} = .93 = 93\% = 93$ per cent

It is important to remember that each of the four ways of writing hundredths expresses the same value. For example, you may at any time change 93% to $\frac{93}{100}$ or to .93; or you may change .93 to $\frac{93}{100}$ or to 93%.

This new way of writing hundredths is called **percentage.** It is much used in business and in newspapers and magazines.

Exercises

Copy, writing the correct numbers in place of the dots:

1. .08 = $\frac{\cdot\cdot}{100}$ = . . per cent .89 = $\frac{\cdot\cdot}{100}$ = . .%

2. .28 = $\frac{\cdot\cdot}{100}$ = . . per cent .97 = $\frac{\cdot\cdot}{100}$ = . .%

3. .73 = $\frac{\cdot\cdot}{100}$ = . . per cent .03 = $\frac{\cdot\cdot}{100}$ = . .%

4. .05 = $\frac{\cdot\cdot}{100}$ = . . per cent .61 = $\frac{\cdot\cdot}{100}$ = . .%

Write each of the following as a decimal:

5. $\frac{4}{100}$	**8.** 1%	**11.** 4%	**14.** 9 per cent
6. $\frac{1}{100}$	**9.** 21%	**12.** 99%	**15.** 2 per cent
7. $\frac{38}{100}$	**10.** 65%	**13.** 78%	**16.** 48 per cent

Write the following as per cents, using the sign %:

17. .61	**20.** .02	**23.** .54	**26.** .33	**29.** .24
18. .07	**21.** .18	**24.** .93	**27.** .05	**30.** .20
19. $\frac{44}{100}$	**22.** $\frac{81}{100}$	**25.** $\frac{7}{100}$	**28.** $\frac{19}{100}$	**31.** $\frac{87}{100}$

PERCENTAGE AS A NEW LANGUAGE

In percentage you need not learn any new rules for addition, subtraction, multiplication, or division. Since *per cent* means *hundredths,* you may solve any problem in percentage by using decimals. Percentage is like a new language in which the words "per cent" show the old idea of hundredths. You should become thoroughly familiar with this new language.

Exercises

1. The city of New York contains over .05 of the population of the United States. Write this by using the sign %; by using a common fraction.

Write ex. 2 to 7 in two other ways, so that each number appears as a common fraction, a decimal, and a per cent:

2. Susan had 78% of the arithmetic test correct.

3. 18% of Tom's class failed on the state examinations.

4. From 28% to 40% of the contents of cream is butter fat.

5. John spent 62% of his salary for room and board.

6. Anna tries to save 15% of her allowance each week.

7. Last year the Midgets lost only $\frac{8}{100}$ of the baseball games they played.

8. The label at the right tells you that 98% of the contents of the can is pure olive oil. Write the label another way, using a common fraction instead of the per cent sign. If the label read "42% pure," what would that mean? Would you buy that kind of olive oil?

9. Express each of the marks at the right as a decimal. If there were 100 words on the test, how many did each child have right?

Spelling	Apr. 3
Maud Allison	98%
Arthur Baker	78%
Doris Bradley	92%

USING DECIMALS IN FINDING PER CENTS

George says that 98% of the 150 boys in his school are members of the Athletic Association. This is the same as saying that .98 of 150 boys are members. .98 *of* 150 means .98 *times* 150. Since .98×150 = 147, 98% of 150 = 147.

Check the work by casting out 9's or by going over it again.

```
 150
 .98
------
12 00
135 0
------
147.00
```

To find a per cent of a number, change the per cent to a decimal and multiply the number by the decimal.

Exercises

In ex. 1 *to* 8, *tell what numbers should be put in place of the dots:*

1. 48% of 150 is the same as .48×150, which equals . . .

2. 37% of 200 is the same as . . . ×200, which equals . . .

3. 56% of 750 is the same as . . . ×750, which equals . . .

4. 6% of 250 is the same as .06×250, which equals . . .

5. 5% of 540 is the same as . . . ×540, which equals . . .

6. 7% of $350 is the same as . . . ×. . ., which equals . . .

7. 12% of $890 is the same as . . . ×. . ., which equals . . .

8. 41% of $263 is the same as . . . ×. . ., which equals . . .

9. On a spelling test of 50 words, Alice was marked 94%. How many words did she have right?

10. Louise was marked 85% on an arithmetic test of 20 examples. Find how many examples she had correct.

11. In a school of 250 pupils, 92% were present on a certain day. How many pupils were present that day?

12. Last spring the Bedford baseball team played 25 games and won 56% of them, while the Cadets played 15 games and won 60% of them. How many games did each team win?

HOW MUCH DID THESE MOTHERS SAVE?

The prices in Mr. Morgan's dry-goods store were 15% less in January than before Christmas. Many women saved money by buying at the January sale.

1. Mrs. Adams bought a pair of curtains which regularly cost $2.20. How much did she save if the price was reduced 15%?

15% of $2.20 = $.33. Hence Mrs. Adams saved $.33.

2. Mrs. Adams bought Joe a sweater regularly priced at $8.60. How much did she save on the sweater?

3. Mrs. Smith bought a pair of woolen gloves that had previously cost $1.80, a scarf that had cost $4, a tablecloth that had cost $10.40, and a set of napkins that had cost $3.20. How much did she save on each? How much did she save in all?

4. Mrs. Lewis bought a bedspread originally marked $6, a house dress that had sold at $2.80, a piece of cretonne that had sold at $3.60, and a scarf that had sold at $2. At "15% off," find her savings on each and her total savings.

5. At another sale, Mrs. Lewis got 35% off on a coat that had cost $65. How much was the price lowered?

Finding Per Cents

Find the following and check your results:

1. 5% of 240
2. 2% of 450
3. 5% of 840
4. 1% of 600
5. 8% of 250
6. 7% of 15
7. 3% of 600
8. 9% of 350
9. 4% of 450
10. 8% of $4450
11. 1% of $8106
12. 6% of $9900
13. 22% of $1500
14. 35% of $7602
15. 81% of $3950
16. 92% of $1830
17. 58% of $6240
18. 74% of $2304
19. 7% of 1000 boys
20. 14% of 350 men
21. 26% of 150 dogs
22. 35% of 660 pens
23. 45% of 180 hats
24. 98% of 750 girls
25. 55% of 120 days
26. 68% of 225 eggs
27. 65% of 260 rugs

SAVING TIME IN FINDING PER CENTS

1. Paul sold magazines for a publisher who paid him 25% of what he collected. On Monday Paul collected \$2.80. To find how much of this was his, he multiplied \$2.80 by .25 and got \$.70.

The publisher finds Paul's pay by a much quicker method. He knows that $25\% = \frac{25}{100}$, or $\frac{1}{4}$. Hence the publisher finds $\frac{1}{4}$ of \$2.80. This gives \$.70, the same answer that Paul got.

Long Method

\$2.80
.25
1400
560
\$.7000

Short Method

4)\$2.80
\$ .70

25% of any number equals $\frac{1}{4}$ of that number.

2. Similarly, $50\% = \frac{50}{100} = \frac{1}{2}$, and $75\% = \frac{75}{100} = \frac{3}{4}$.

3. *Remember these important facts:*

25% of anything is the same as $\frac{1}{4}$ of it.
50% of anything is the same as $\frac{1}{2}$ of it.
75% of anything is the same as $\frac{3}{4}$ of it.

Exercises

1. Two boys went to Orchard Lake on a picnic. They divided the expenses "fifty-fifty." What does this mean?

Use the short methods to find the following. Check the work by going over it again:

2. 25% of 64
3. 50% of 90
4. 75% of 72
5. 25% of 180
6. 75% of 964
7. 50% of 170
8. 25% of \$2.40
9. 75% of \$5.60
10. 50% of \$5.00
11. 75% of \$9.12
12. 50% of \$7.30
13. 25% of \$3.80
14. 75% of \$13.76
15. 50% of \$22.25
16. 25% of \$11.84
17. 75% of \$25.00
18. 50% of \$19.91
19. 25% of \$26.01

20. Alice was marked 75% on an arithmetic test. If there were 12 examples in the test, how many did she get right?

FRACTIONAL EQUIVALENTS OF PER CENTS

1. Dick read that $33\frac{1}{3}\%$ of the 450 students in the high school worked on farms last summer. How many of the students worked on farms?

You could find $33\frac{1}{3}\%$ of 450 by multiplying 450 by $.33\frac{1}{3}$, as shown in *A*.

A shorter way to find $33\frac{1}{3}\%$ of 450 is to take $\frac{1}{3}$ of 450, as shown in *B*. Does this give the same answer that you got in *A*?

A.

$$\begin{array}{r} 450 \\ .33\frac{1}{3} \\ \hline 1\,50 \\ 13\,50 \\ 135\,0 \\ \hline 150.00 \end{array}$$

B. $3\overline{)450}$

150

You can show that $33\frac{1}{3}\% = \frac{1}{3}$ like this:

$$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{3\times 33\frac{1}{3}}{3\times 100} = \frac{100}{300} = \frac{1}{3}$$

2. Similarly, you may change 60% and $12\frac{1}{2}\%$ to fractions:

$$60\% = \frac{60}{100} = \frac{3}{5} \qquad 12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{2\times 12\frac{1}{2}}{2\times 100} = \frac{25}{200} = \frac{1}{8}$$

3. *Memorize the following equivalents:*

$25\% = \frac{1}{4}$	$20\% = \frac{1}{5}$	$12\frac{1}{2}\% = \frac{1}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$50\% = \frac{1}{2}$	$40\% = \frac{2}{5}$	$37\frac{1}{2}\% = \frac{3}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$75\% = \frac{3}{4}$	$60\% = \frac{3}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$10\% = \frac{1}{10}$	$80\% = \frac{4}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$

Exercises

Show how the following are found to be true:

1. $20\% = \frac{1}{5}$ **2.** $37\frac{1}{2}\% = \frac{3}{8}$ **3.** $66\frac{2}{3}\% = \frac{2}{3}$

Use the short cuts to find the following:

4. 20% of 35
5. 10% of 590
6. 40% of 125
7. 60% of 395
8. 75% of 136
9. $16\frac{2}{3}\%$ of 96
10. $62\frac{1}{2}\%$ of 752
11. $12\frac{1}{2}\%$ of 960
12. $83\frac{1}{3}\%$ of 258
13. $33\frac{1}{3}\%$ of 690
14. 50% of \$987.50
15. 10% of \$893.50
16. $37\frac{1}{2}\%$ of \$64.40
17. $87\frac{1}{2}\%$ of \$560.80
18. $66\frac{2}{3}\%$ of \$477.75

FINDING PER CENTS

Find the following, using short cuts wherever possible:

1. 18% of 95	**7.** 40% of \$550	**13.** 35% of \$38.80
2. 20% of 755	**8.** 50% of \$417	**14.** 79% of \$45.00
3. 19% of 471	**9.** 25% of \$124	**15.** $16\frac{2}{3}$% of \$96.90
4. $37\frac{1}{2}$% of 112	**10.** 75% of \$625	**16.** $66\frac{2}{3}$% of \$37.20
5. $33\frac{1}{3}$% of 528	**11.** 39% of \$813	**17.** $87\frac{1}{2}$% of \$63.20
6. $62\frac{1}{2}$% of 104	**12.** $12\frac{1}{2}$% of \$152	**18.** $12\frac{1}{2}$% of \$89.36

By changing the fractions to per cents, tell which is greater:

19. $\frac{1}{4}$ or 20%	**21.** $\frac{1}{10}$ or 10%	**23.** $\frac{5}{8}$ or $\frac{3}{5}$	**25.** $\frac{2}{5}$ or $\frac{3}{8}$
20. $\frac{1}{8}$ or 15%	**22.** $\frac{5}{6}$ or 80%	**24.** $\frac{2}{3}$ or $\frac{5}{8}$	**26.** $\frac{7}{8}$ or $\frac{4}{5}$

27. Of the 288 girls in the Walton School, $62\frac{1}{2}$% are Girl Scouts. How many are Girl Scouts? How many are not?

Computing to the Nearest Cent

1. Donald can buy a bicycle on the installment plan for \$29.35. If he pays the entire amount in cash, he gets the bicycle for $12\frac{1}{2}$% less. How much does he save by paying cash?

To find $12\frac{1}{2}$% of \$29.35, take $\frac{1}{8}$ of it. Since the answer is not exact, annex a 0 after \$29.35 and carry the quotient to three decimal places. \$3.668 is called \$3.67 to the nearest cent because \$3.668 is closer to \$3.67 than to \$3.66.

8)\$29.350
\$ 3.668

When an answer represents money, a figure of 5 *or more in the third decimal place is regarded as another cent.*

Find the following correct to the nearest cent, and check:

2. 1% of \$266.43	**7.** 21% of \$1.20	**12.** 46% of \$44.40
3. 12% of \$141.60	**8.** 10% of \$4.22	**13.** 93% of \$75.20
4. 60% of \$865.36	**9.** 25% of \$8.65	**14.** 25% of \$950.33
5. $33\frac{1}{3}$% of \$11.51	**10.** 75% of \$5.83	**15.** $16\frac{2}{3}$% of \$11.00
6. $12\frac{1}{2}$% of \$91.15	**11.** 90% of \$2.71	**16.** $66\frac{2}{3}$% of \$143.11

USING SHORT CUTS TO ESTIMATE RESULTS

1. The school board reported that 52% of the 750 children in Roxville were transferred to the new school. Estimate quickly about how many children were transferred.

52% is a little over 50%, which equals $\frac{1}{2}$. Hence about $\frac{1}{2}$ of 750, or 375, children, were transferred. Is 375 more or less than the exact answer?

2. The Grant school raised \$880, of which 73% was to go to the Athletic Association. In finding 73% of \$880, Donald got \$64.24. Was his answer approximately right?

Since 73% is almost 75%, estimate the answer mentally by taking $\frac{3}{4}$ of \$880, which is \$660. Hence Donald's answer, \$64.24, was much too small.

Find the exact answer. What mistake did Donald make?

You can quickly estimate a per cent of a number if it is close to a per cent like 25% or $33\frac{1}{3}$%, which is found by a short cut.

Thus, you can estimate 11% of a number because 11% is close to 10%, which equals $\frac{1}{10}$. Similarly, 15% is close to $16\frac{2}{3}$%, which equals $\frac{1}{6}$.

Exercises

Find both the approximate and the exact value of each:

1. 49% of \$800
2. 26% of \$544
3. 13% of \$992
4. 34% of \$1590
5. 19% of \$3805
6. 51% of \$1877
7. 67% of \$7227
8. 76% of \$42,000
9. 12% of \$65,200
10. 84% of \$56,100
11. 11% of \$481.59
12. 24% of \$727.68
13. 59% of \$396.20
14. 21% of \$222.65
15. 88% of \$554.08

By estimating, tell which of these answers are approximately right and which are much too large or too small:

16. 19% of 280 = 53.2
17. 41% of 350 = 14.35
18. 17% of 486 = 826.2
19. 9% of \$763 = \$6.87
20. 73% of \$2000 = \$1460
21. 24% of \$735 = \$176.40
22. 35% of \$75.00 = \$26.25
23. 51% of \$699.20 = \$49.15

THE MEANING OF 100%

The Lincoln School has a 100% membership in the Junior Red Cross. If there are 49 children in the school, how many belong to the Junior Red Cross? How much is 100% of 49?

Since per cent means hundredths, $100\% = \frac{100}{100}$, or 1. Hence 100% of 49 children = 1×49 children, or 49 children. You see that a 100% membership means that all the children are members.

Likewise, 100% of any number means $1 \times$ that number, which equals the number itself.

100% of anything means the whole of it, or all of it.

$$100\% = \frac{100}{100} = 1 = 1.00 = \text{all} = \text{the whole}$$

Everyday Uses of 100%

State ex. 1 *to* 8 *in another way.*

1. The storage warehouse is 100% fireproof.
2. 100% of the boys on the team were promoted.
3. Last week our class had a 100% record in attendance.
4. James had all the examples on an arithmetic test right.
5. This powdered sugar is guaranteed to be 100% pure.
6. Frank had 100% of the words on a spelling test right.
7. It is almost 100% safe to deposit money in that bank.
8. All the inhabitants of Concord went to the county fair.
9. Mary has 100% of her money in the School Savings Bank. If she has $34.20, how much of it is in the School Savings Bank?
10. Jack picked $18\frac{1}{2}$ quarts of strawberries and sold them all. What per cent of them did he sell?
11. If you were a manufacturer of cloth, how could you label a piece of goods to show that it is *all wool?*
12. Henry saved $46 and spent it all for a canoe. What per cent of his savings did he spend?
13. What is 100% of $11? 100% of 15 boys? 100% of $1?

FINDING PARTS OF A WHOLE

Jay read that the Giants won 64% of all the baseball games they played last summer. What per cent of all the games did they lose?

All the games the Giants played equals 100% of them. If they won 64% of them, they lost 100% − 64%, or 36%, of them.

When all the parts of a whole are expressed as per cents, their sum is 100%. If you know what per cent of the whole one part is, you can find the other part by subtracting the given per cent from 100%.

Exercises

1. What per cent of the examples on a test are all the examples? If Mary had 85% of the examples on a test right, what per cent of them had she wrong?

2. If $33\frac{1}{3}$% of our baseball squad are graduated and the rest remain at school, what per cent of the squad remain at school?

3. If 72% of the graduates of the Grange School go to college, what per cent of the graduates do not go to college?

4. Will bought a tennis racket for 20% less than the regular price. What per cent of the regular price did he pay?

5. Japan produces 39% of the world's raw silk and China produces 31% of it. What per cent of it do they produce together? What per cent of it is produced by all other countries?

6. Of all the coal mined in the United States, 40% comes from Pennsylvania and 16% from West Virginia. What per cent of the coal comes from all the other states?

7. Mr. Frank spent 60% of his salary of $55 and saved the rest. What per cent of his salary did he save?

SAYING "3 TIMES AS MANY" BY PER CENTS

So far all the per cents you have studied have been 100% or less. Sometimes you find an expression like 300%. Since per cent means hundredths, 300% means $\frac{300}{100}$, or 3.

300% of a number equals 3 times that number.

Thus, 300% of $120 = 3 \times 120 = 360$
Likewise, 200% of $198 = \frac{200}{100} \times 198 = 2 \times 198 = 396$
and 1000% of $525 = \frac{1000}{100} \times 525 = 10 \times 525 = 5250$

Exercises

1. Tell the meaning of 400% of a number; of 500%, 800%, 1100%, 3000%, and 3500% of a number.

Find the following per cents:

2. 10% of 10
3. 300% of 6
4. 400% of 87
5. 200% of 290
6. 700% of 465
7. 800% of 980
8. 100% of 980
9. 1000% of 10
10. 50% of \$3000
11. 500% of \$3000
12. 200% of \$9580
13. 900% of \$6450

Restate each of the following in per cent language:

14. 8×55
15. 2×96
16. 7×32
17. $8 \times \$700$
18. $5 \times \$250$
19. $4 \times \$990$
20. 1 times 18 girls
21. 10 times 15 boys
22. 12 times 50 pupils

23. In 1900 the population of Seaville was 800. It is now 200% as great. How many people are there in Seaville now?

24. The population of Cleveland was 3 times as large in 1920 as it was in 1890. Say this as a per cent.

25. Mr. Roe started business with \$1000. He now has 400% as much money as when he started. How much has he now?

26. Mr. Stillman, who also started with \$1000, now has 400% *more than* when he began. How much more has he than when he started? How much has he all together?

OTHER PER CENTS

1. A man who earned \$2854 last year earned 145% as much this year. How much did he earn this year?

You must find 145% of 2854. Since *per cent* means hundredths, $145\% = \frac{145}{100}$, or 1.45. Therefore 145% of \$2854 = 1.45×\$2854, or \$4138.30.

\$2854
1.45
142 70
1141 6
2854
\$4138.30

2. Express as decimals: 326%; 45%; 5%.

$326\% = \frac{326}{100} = 3.26$ $45\% = \frac{45}{100} = .45$ $5\% = \frac{5}{100} = .05$

To change a per cent to a decimal, drop the per cent sign and move the decimal point two places to the left.

3. Express as per cents: 2.73; .45; 2; .08.

2.73 = 273% .45 = 45% 2 = 2.00 = 200% .08 = 8%

To change a decimal to a per cent, move the decimal point two places to the right and annex the per cent sign.

Exercises

Change to decimals:

1. 89%	**3.** 9%	**5.** 125%	**7.** 150%	**9.** 1000%
2. 43%	**4.** 7%	**6.** 300%	**8.** 215%	**10.** 1150%

Change to per cents:

11. .75	**12.** 1.60	**13.** 2.00	**14.** 1.75	**15.** 12.50

Find answers correct to the nearest cent. Check the work:

16. 106% of \$841	**19.** 160% of \$375	**22.** 112% of \$66.49
17. 210% of \$326	**20.** 425% of \$966	**23.** 110% of \$50.55
18. 150% of \$980	**21.** 275% of \$870	**24.** 965% of \$80.75

25. Margery had \$71.20 saved last September. She now has 125% of that amount. Find how much she has at present.

26. Last year Lewis had \$300. Now he has 120% more. How much have his savings increased? How much has he now?

NEW EXPRESSIONS IN PERCENTAGE

Fred bought an old bicycle for \$8 and sold it for 50% *more than* it cost him. How much did he receive for it?

You may work this problem in either of two ways:

First Method. Fred's gain was 50% of \$8, or \$4. Hence Fred received \$8 + \$4, or \$12, for the bicycle.

50% of \$8 = \$ 4
\$8 + \$4 = \$12

Second Method. Fred received \$8 + 50% of \$8. Since the whole of the \$8 is 100% of it, Fred got 100% + 50%, or 150%, of \$8 in all. Since 150% = 1.50, you see that he received 1.50×\$8, or \$12. This is the same answer that you got above.

1.50×\$8 = \$12

If Fred had sold the bicycle for 10% *less than* the cost, he would have sold it for 100% − 10%, or 90%, of the cost.

50% more than a number = 150% of the number.
10% less than a number = 90% of the number.

Exercises

Tell what numbers should be put in place of the dots:

1. 65% more than \$20 = . . .% of \$20 = \$. . .
2. 6% more than \$220 = . . .% of \$220 = \$. . .
3. 18% less than \$450 = . . .% of \$450 = \$. . .
4. 72% less than \$950 = . . .% of \$950 = \$. . .
5. 100% more than \$748 = . . .% of \$748 = \$. . .
6. 180% more than \$384 = . . .% of \$384 = \$. . .
7. 300% more than \$469 = . . .% of \$469 = \$. . .
8. 103% of \$900 = . . .% more than \$900 = \$. . .
9. 420% of \$115 = . . .% more than \$115 = \$. . .

10. Would it be better to sell a bat for 65% of its cost or for 65% more than its cost?

11. The population of Harwood, which was 8400 in 1915, is now 24% less than that. What per cent of its former population has Harwood now? What is its present population?

OTHER NEW PHRASES IN PERCENTAGE

You have just learned the meaning of expressions like 45% *more than* and 10% *less than*. Several other phrases express the same idea. For example, you may say that the membership of a Library Club *has increased* 45% since last year.

When the membership of a club has increased 45%, or increased by 45% of itself, it is 45% more than it was before. Similarly, if the membership has decreased 45%, or decreased by 45% of itself, or shown a 45% decrease, it is 45% less than it was before.

Exercises

State each in at least two other ways, using per cents:

1. The cost of living has doubled in 5 years.

2. The prices of automobile tires have decreased 10%.

3. Our school now has 115% more pupils than in 1924.

4. The population of Higsby has increased 68% since 1920.

5. Alice bought a tennis racket and sold it for half as much again as she paid for it.

Find the following:

6. \$7000 increased by 200% of itself.

7. \$200 increased by 38% of itself.

8. \$475 decreased by 95% of itself.

9. An amount equal to 150% more than \$7000.

10. The expenses of the United States Navy in 1920 showed an increase of 1800% over the expenses in 1900, which were \$56,000,000. Find the amount of increase.

11. Here are the populations of certain cities in 1920:

Philadelphia . .	1,823,000	Detroit . .	993,700
San Francisco .	506,700	Atlanta . .	200,600

What would be their present populations if each had increased 22% since 1920? if each had showed a 15% decrease since 1920?

THE AUTOMOBILE INDUSTRY

1. In a recent year Canada produced $3\frac{1}{2}\%$ of the 4,800,400 automobiles manufactured in the world. How many automobiles were made in Canada?

$$\begin{array}{r} 4800400 \\ .03\frac{1}{2} \\ \hline 24002\ 00 \\ 144012\ 00 \\ \hline 168014.00 \end{array}$$

To find $3\frac{1}{2}\%$ of 4,800,400, first change $3\frac{1}{2}\%$ to $.03\frac{1}{2}$. Since $.03\frac{1}{2} \times 4{,}800{,}400 = 168{,}014$, Canada produced 168,014 automobiles that year.

2. The United States produced $87\frac{1}{4}\%$ of the world's automobiles in the same year. How many of the 4,800,400 automobiles did the United States produce?

3. If Canada made $3\frac{1}{2}\%$ of the world's automobiles and the United States made $87\frac{1}{4}\%$ of them, what per cent of the cars that year were made by the rest of the world? How many of the 4,800,400 cars did the rest of the world make?

4. Iron is necessary in making automobiles. Of the 40,000,000 tons of pig iron produced yearly in this country, Pennsylvania produces about $38\frac{1}{2}\%$; Ohio, $22\frac{1}{2}\%$; Michigan, $9\frac{3}{4}\%$; Illinois, $9\frac{1}{2}\%$; New York, $7\frac{1}{2}\%$; and Alabama, $7\frac{1}{4}\%$. How many tons of pig iron does each of these states produce?

5. Gasoline comes from petroleum, about 774,000,000 barrels of which are produced yearly. Of these, the United States produces $66\frac{1}{2}\%$; Mexico, $20\frac{1}{2}\%$; Russia, $4\frac{1}{3}\%$; the Dutch East Indies, $2\frac{1}{4}\%$; Rumania, $1\frac{2}{3}\%$; and India, 1%. Find the yearly production of petroleum for each country. Find the per cent of the yearly output and the number of barrels produced by all other countries.

Find the following per cents and check the work:

6. $3\frac{1}{2}\%$ of 480
7. $5\frac{1}{2}\%$ of 466
8. $6\frac{1}{2}\%$ of 322
9. $3\frac{1}{4}\%$ of 944
10. $4\frac{1}{4}\%$ of \$988
11. $6\frac{1}{4}\%$ of \$534
12. $10\frac{1}{2}\%$ of \$285
13. $22\frac{1}{2}\%$ of \$720
14. $7\frac{1}{2}\%$ of \$2334
15. $4\frac{1}{2}\%$ of \$6287
16. $4\frac{3}{4}\%$ of \$7440
17. $27\frac{1}{2}\%$ of \$8056

FINDING WHAT PART ONE NUMBER IS OF ANOTHER

You can compare a number with a larger one by asking this question: The smaller number is what fractional part of the larger number? To compare \$10 with \$15, you ask: \$10 is what part of \$15? You find that \$10 is $\frac{10}{15}$, or $\frac{2}{3}$, of \$15.

To find what fractional part one number is of a larger one, write the smaller number over the larger one to make a fraction. Then reduce the fraction to lowest terms.

You must now review this work to prepare for a new method of comparing two numbers by using per cents.

Exercises

1. Tom bought 3 oranges. What part of a dozen is that?

2. What part of a pound is 1 oz.? 3 oz.? 5 oz.? 12 oz.?

3. What part of a yard is 1 in.? 11 in.? 18 in.? 27 in.?

4. What part of a dollar is a dime? a cent? 17¢? 35¢?

5. Jack can walk a mile in 18 min. What part of an hour is that? What part of an hour is 27 min.? 35 min.? 48 min.?

6. How many feet are there in a mile? If a school running track is 660 ft. long, what part of a mile long is it?

7. 25 min. is what part of 1 hr.? of 40 min.? of 55 min.?

8. On a test of 150 words, Alice had 144 words right, Jenny had 138, and Marie had 125. What part of the test did each have right?

9. Of \$1280 worth of tickets, Bill sold \$320; Lucy, \$480; Ted, \$256; May, \$160; and John, \$64 worth. What part of the total did each child sell?

10. If these cities each had an increase of 50,000 people, what part of the given population would the increase be?

Paris, 2,905,000	London, 7,475,000	Madrid, 755,000
Berlin, 3,800,000	Leningrad, 1,065,000	Warsaw, 930,000

FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER

The Cubs won 14 baseball games out of 25. What per cent of the games did they win?

To find what per cent 14 is of 25, you do three things:

1. Find what fractional part 14 is of 25.

If the Cubs won 14 games out of 25, they won $\frac{14}{25}$ of the games.

2. Express the fraction $\frac{14}{25}$ as a two-place decimal.

$\frac{14}{25}$ means 14 ÷ 25. Writing a decimal point and 0's after 14 and dividing by 25, you get .56. Therefore $\frac{14}{25}$ = .56. You could say that the team won .56 of all the games it played.

$$25\overline{)14.00} \quad .56 = 56\%$$
$$12\,5$$
$$1\,50$$
$$1\,50$$

3. Change .56 to a per cent.

You know that .56 may be written as 56%.
Hence the Cubs won 56% of the baseball games.
To check, take 56% of 25 to see if it gives 14.

To find what per cent one number is of another, find what fractional part the one number is of the other; change this fraction to a two-place decimal and express the decimal as a per cent.

Exercises

1. The Meadville football team won 13 games out of 20. What per cent of the games did they win? Check your answer.

Tell what numbers should be put in place of the dots, and check your results:

2. 38 is . . .% of 50
3. 76 is . . .% of 95
4. 51 is . . .% of 68
5. 18 is . . .% of 90
6. 11 is . . .% of 50
7. 23 is . . .% of 25
8. 11 is . . .% of 20
9. 78 is . . .% of 260

Express these first as decimals and then as per cents:

10. $\frac{21}{25}$ **11.** $\frac{7}{25}$ **12.** $\frac{9}{50}$ **13.** $\frac{18}{40}$ **14.** $\frac{24}{150}$

USING PER CENTS TO COMPARE NUMBERS

1. Of the 80 girls in the Glee Club, 7 have won prizes in singing. What per cent of the girls have won prizes?

$\frac{7}{80}$ is $\frac{7}{80}$ of 80. Changing $\frac{7}{80}$ to a two-place decimal, you get .08$\frac{3}{4}$, which equals 8$\frac{3}{4}$%. Hence 8$\frac{3}{4}$% of the girls have won prizes.

To check, take 8$\frac{3}{4}$% of 80 and you get 7.

In changing a fraction like $\frac{7}{80}$ to a per cent, find the decimal only to hundredths (two decimal places) because *per cent* means *hundredths*. If there is a remainder, write it in the quotient to form a fraction.

$$.08\tfrac{3}{4} = 8\tfrac{3}{4}\%$$

$$80\overline{)7.00}$$

$$\begin{array}{r} 6\,40 \\ \hline 60 \end{array}$$

$$\frac{60}{80} = \frac{3}{4}$$

2. What per cent of $250 is $85?

This is another way of saying "$85 is what per cent of $250?" $85 is $\frac{85}{250}$, or 34%, of $250.

In finding what per cent one number is of another, remember that the denominator of the fraction is always the number immediately following the word "of" in the example.

Comparing Numbers

What numbers should be put in place of the dots? Be sure to use the right numbers as denominators. Check the work:

1. 8 is . . .% of 9.
2. 1 is . . .% of 16.
3. 9 is . . .% of 40.
4. 15 is . . .% of 200.
5. 63 is . . .% of 400.
6. $36 is . . .% of $81.
7. $44 is . . .% of $800.
8. $28 is . . .% of $350.
9. . . .% of 16 is 5.
10. . . .% of 32 is 6.
11. . . .% of 80 is 3.
12. . . .% of 400 is 9.
13. . . .% of 120 is 3.
14. . . .% of 30 is 22.
15. . . .% of 40 is 29.
16. . . .% of 400 is 22.

17. Out of 40 problems, Frank had 32 right, Alice, 38, and Mollie, 39. What per cent of the problems did each have right?

FINDING PER CENTS OF INCREASE

1. This table shows the weights of ten pupils now and their weights 2 yr. ago. Irwin weighed 80 lb. then. As he now weighs 94 lb., he has gained 14 lb. To find the per cent of gain, find what per cent 14 lb. is of 80 lb.

Name	Weight in Pounds		Gain	
	2 Yr. Ago	Now	Pounds	As a %
Irwin	80	94	14	$17\frac{1}{2}\%$
Roger	70	84	—	——
Anna	66	88	—	——
Alice	72	90	—	——
Will	75	106	—	——
Grace	80	85	—	——
John	64	76	—	——
Isaac	80	92	—	——
Ralph	100	122	—	——
Mary	60	75	—	——

2. Copy and fill in the table, first finding the number of pounds each pupil has gained and then finding what per cent his gain is of his weight 2 yr. ago. Check all your answers.

3. Which pupil has gained the most weight? Who shows the greatest per cent of gain? the smallest per cent of gain?

Exercises

1. Here is the enrollment of four schools in 1923 and in 1928. Find what per cent the enrollment has increased in each.

	1923	1928		1923	1928
Dover . . .	420	441	Lincoln . . .	125	160
Baywood . .	740	851	Harrison . .	189	216

2. Mr. Hall earned $4000 one year and $3000 the next year. Find the amount and the per cent of decrease in his income.

Each problem shows a man's income for two successive years. Find the per cent of increase or decrease in each income:

3. $2000; $2500 **5.** $5000; $7800 **7.** $1000; $1120

4. $1800; $1500 **6.** $3000; $1600 **8.** $2500; $3200

FINDING THE INCREASE IN SAVINGS

1. Harold had \$150 in the savings bank on June 1 and \$300 on Sept. 1. What per cent did his savings increase?

The increase was \$300 − \$150, or \$150. You must find what per cent \$150 is of \$150. Since *any number is* 100% *of itself*, 150 is 100% of 150. Hence Harold's savings increased 100%.

You can also get this answer by using the rule on page 196. Thus, 150 is $\frac{150}{150}$ of 150. Since $\frac{150}{150} = 1 = 100\%$, you see that 150 is 100% of 150.

2. Ralph's savings amounted to \$150 on June 1 and to \$360 on Sept. 1. What per cent did his savings increase?

The increase is \$210. You must find what per cent \$210 is of \$150. Since 210 is $\frac{210}{150}$ of 150, you divide 210 by 150, which gives 1.40, or 140%. Hence Ralph's savings increased 140%. This answer is greater than 100% because $\frac{210}{150}$ is greater than 1.

Check by finding 140% of \$150.

3. This table shows the savings of each pupil in a certain class on June 1 and on Sept. 1. Find the per cent of increase or of decrease in the savings of each pupil:

Girls	June 1	Sept. 1	Boys	June 1	Sept. 1
Agnes	\$ 8	\$14	John	\$90	\$270
Laura	35	25	Louis	10	50
Sarah	80	30	Fred	20	5
May	66	154	Dan	28	42
Anne	31	62	Bob	24	60
Rae	100	101	Alvin	80	4
Leah	12	14	Bill	20	60
Josie	4	6	Jacob	12	72
Doris	84	77	Carl	16	28

4. What were the total savings of the girls on June 1? on Sept. 1? Find the amount and the per cent of increase.

5. Find the amount and the per cent of increase of the boys' total savings. Was their per cent better than that of the girls?

FINDING THE WHOLE WHEN A PER CENT IS GIVEN

Donald received \$30 for selling a radio set. This was 15% of the price of the set. For how much did he sell it?

15% of the price = \$30
Hence 1% of the price = \$30 ÷ 15, or \$2
and 100% of the price = 100 × \$2, or \$200

Since 100% of the price is the entire price, Donald sold the radio set for \$200.

To check, see if 15% of \$200 is \$30.

To find a number when a per cent of it is given, first find 1% *of the number, then multiply the result by* 100 *to find* 100% *of the number.*

Exercises

Check all your answers in these problems.

1. Mr. Lloyd's rent, which is \$720 a year, uses up 30% of his salary. How much a year does he earn?

2. Mr. Arnold failed in business and could pay each man only 20% of what he owed him. He paid Mr. Curt \$400, Mr. Steel \$60, and Mr. Voss \$1000. How much had he owed each?

Tell what numbers should be put in place of the dots:

3. 8 is 4% of . .
4. 24 is 6% of . .
5. 76 is 2% of . .
6. 91 is 1% of . .
7. 27 is 3% of . .
8. 45 is 9% of . .
9. 87 is 12% of . .
10. 45 is 45% of . .
11. 63 is 21% of . .
12. 78 is 13% of . .
13. 45 is 10% of . .
14. 21 is 100% of . .

15. Kate earns 15% of what she collects for selling seeds. How much worth of seeds must she sell to earn \$45?

16. Mr. Cox received \$40 for selling a piano. This \$40 was 5% of the amount he got for the piano. At what price did he sell it?

17. Ed Miller made 6 home runs. If 12% of all his hits were home runs, how many hits did he make?

IMPROVEMENT TEST No. 15

Test 15 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.	6.
31	32	39	84	14	75
74	18	72	26	21	53
87	35	93	99	48	78
23	87	64	39	87	18
59	61	11	90	76	90
87	59	61	57	40	49
56	29	69	19	65	45
10	50	80	64	24	76
21	71	98	85	87	41
83	14	56	20	70	98
62	60	12	83	71	29
78	47	97	64	47	98
57	88	33	36	64	57

Test 15 B — Subtraction (Time 3 min.)

1. 670754 − 45176	**2.** 351397 − 25439	**3.** 766621 − 432873	**4.** 644346 − 127738
5. 757502 − 28327	**6.** 625840 − 46850	**7.** 289300 − 157511	**8.** 305291 − 168961
9. 475821 − 26632	**10.** 265511 − 75177	**11.** 493811 − 269384	**12.** 784192 − 116503

Test 15 C — Multiplication (Time 4 min. after copying)

1. 275×769	**2.** 813×276	**3.** 915×567
4. 836×984	**5.** 542×432	**6.** 746×847

Test 15 D — Division (Time 4 min. after copying)

If there is a remainder, carry result to the nearest tenth:

1. $437\overline{)27589}$	**2.** $238\overline{)143990}$	**3.** $785\overline{)46378}$

COMMISSION

Frank Wiley sold \$22 worth of vegetables for his father, who gave him 10% of the money. Hence Frank got 10% of \$22, or \$2.20, for his work. This amount was his *commission.*

The **commission** is the amount a person receives for selling goods for some one else. It is often expressed as a per cent of the selling price. This per cent is the **rate of commission.** The person who sells the goods is called the **agent.** If you sell magazines and are paid 20% of the price of each, your rate of commission is 20%. On a \$3.00 subscription your commission is 20% of \$3.00, or \$.60.

Selling Goods on Commission

1. In June Frank sold \$36 worth of berries and received a 15% commission. How much was his commission?

2. In December Mr. Lewis sold aluminum kitchen ware at 35% commission. If his sales amounted to \$425, how much commission did he receive?

3. Agnes Black works in a dry-goods store. She gets a weekly salary of \$15 and also a 2% commission on all she sells. Find her total earnings in a week when her sales amount to \$175; to \$440; to \$260; to \$112.50.

Find the commissions on these sales at the given rates:

4. \$800, 2%
5. \$140, 6%
6. \$811, 3%
7. \$920, 12%
8. \$372, 25%
9. \$141, 10%
10. \$9220, 1%
11. \$333, 50%
12. \$800, $12\frac{1}{2}$%
13. \$160, $17\frac{1}{2}$%
14. \$760, 35%
15. \$154, $33\frac{1}{3}$%
16. \$17.56, $8\frac{1}{2}$%
17. \$42.20, $7\frac{1}{2}$%
18. \$62.60, $22\frac{1}{2}$%
19. \$90.65, 40%
20. \$93.60, $12\frac{1}{2}$%
21. \$24.69, $33\frac{1}{3}$%

22. If Maud receives $5\frac{1}{2}$% commission and her weekly sales average \$320, find her average weekly earnings.

FINDING THE NET PROCEEDS OF A SALE

Suppose that Mr. Stone has a lot to sell. He turns it over to a real estate agent, who sells it for \$600 and charges 5% commission. Hence the agent keeps 5% of \$600, or \$30, and sends Mr. Stone the remaining \$570. This \$570 is the *net proceeds* of the sale.

\$600	**\$600**	**Sales**
.05	**30**	**Commission**
\$30.00	**\$570**	**Net Proceeds**

The **net proceeds** are found by subtracting the commission from the selling price.

Exercises

1. A salesman sold \$3000 worth of cloth at 8% commission. Find the amount of commission and the net proceeds.

2. The Star Brush Company pays its salesmen $6\frac{1}{2}\%$ commission. Tell how much of each of the following sales goes to the salesman and how much goes to the company: \$2800; \$8600; \$17,300; \$1040; \$19,344.

Find the commission and the net proceeds on each of these sales:

3. \$300 at 6%
4. \$916 at 5%
5. \$130 at $7\frac{1}{2}\%$
6. \$233 at 20%
7. \$805 at 60%
8. \$242 at 40%
9. \$615 at 18%
10. \$120 at 1%
11. \$290 at $1\frac{1}{2}\%$
12. \$490 at 25%
13. \$580 at 35%
14. \$914 at 55%
15. \$765 at $33\frac{1}{3}\%$
16. \$217 at $12\frac{1}{2}\%$
17. \$82.64 at 2%
18. \$63.55 at 4%
19. \$35.75 at 10%
20. \$42.95 at 20%
21. \$50.00 at $37\frac{1}{2}\%$
22. \$85.11 at $66\frac{2}{3}\%$
23. \$36.30 at $16\frac{2}{3}\%$

24. Mr. Jarvis receives 9% commission for selling automobiles. His total sales were \$18,040 in 1923, \$26,355 in 1924, \$21,400 in 1925, \$48,532 in 1926, \$64,400 in 1927, and \$50,120 in 1928. How much did he earn each year? in the 6 yr.? What were the net proceeds each year?

BUYING SPORT GOODS AT A DISCOUNT

A new sport store had an "Opening Sale." Dick bought a bicycle marked \$30 at the sale at 20% less than the marked price. Hence the price was reduced 20% of \$30, or \$6.00. Dick paid \$30 − \$6, or \$24, for the bicycle.

\$30	**Marked Price**	**\$30.00**	**Marked Price**
.20	**Rate**	**6.00**	**Discount**
\$6.00	**Discount**	**\$24.00**	**Net Price**

The reduction of \$6 is called the **discount**. The \$24 which Dick paid after the discount was subtracted from the original price is called the **net price.**

Exercises

1. Why did the store have an "Opening Sale"?

2. Gilda bought a pair of skates, regularly priced at \$5, at a reduction of 20%. How much was the price reduced?

3. Emma bought a tennis racket, marked \$6, at 25% less than the marked price. How much did she pay for it?

4. Storekeepers offer discounts to attract customers, to sell goods that are out of season or slightly damaged, to use up an oversupply of goods, or to raise money in a hurry. For what reasons would a store have an "August Sale of Summer Dresses"? a "Fire Sale"? a "Bankrupt Sale"?

Find the net price on these articles if the marked price and the rate of discount are as follows:

5. Bat, \$1.20, 40%

6. Cot, \$6.87, $33\frac{1}{3}$%

7. Watch, \$2.80, 10%

8. Knife, \$1.60, $12\frac{1}{2}$%

9. Paddle, \$3.10, 30%

10. Camera, \$9.80, 20%

11. Coaster, \$5, 15%

12. Lantern, \$6, 25%

13. Tool Kit, \$1, 20%

14. Baseball, \$2, 25%

15. Blanket, \$9, $12\frac{1}{2}$%

16. Hatchet, \$2, $33\frac{1}{3}$%

TWO WAYS OF FINDING THE NET PRICE

A boat marked \$200 is sold at 15% off. Find the net price.

1. One way to find the net price is to find the discount, which is 15% of \$200, or \$30, and subtract it from the regular price. This gives \$170 as the net price.

\$200	\$200.00
.15	30.00
\$30.00	\$170.00

2. Here is a shorter way of doing this problem: The regular price is 100% of itself. Taking a discount of 15% from the regular price, you have 85% of it left. Hence the net price is 85% of \$200, or \$170.

Both the above methods give the same answer.

\$200
.85
\$170.00

Exercises

1. What per cent of the regular price is the net price when the discount is 12%? 10%? 30%? 45%? 50%? 25%?

Here are the regular prices of articles and the rates of discount. Find the net price of each in two ways:

2. \$30, 8%
3. \$90, 30%
4. \$600, 12%
5. \$700, 15%
6. \$480, 35%
7. \$260, 45%
8. \$740, 25%
9. \$440, $7\frac{1}{2}$%
10. \$956, 15%
11. \$895, 20%
12. \$660, 55%
13. \$564, $12\frac{1}{2}$%
14. \$794, $33\frac{1}{3}$%
15. \$465, $16\frac{2}{3}$%
16. \$764.90, 6%
17. \$436.98, 4%
18. \$585.20, 10%
19. \$910.60, 25%
20. \$812.75, 40%
21. \$464.00, $6\frac{1}{4}$%
22. \$932.25, 36%

23. Mrs. Dean's bill for electric current last month was \$3.40. If she pays the bill within 10 days, she receives a discount of 5%. How much does she save by paying promptly?

24. Mrs. Moore received a discount of 4% for paying a gas bill of \$2.50 promptly. How much did she save?

25. Mr. Carter can buy some furniture on the installment plan for \$175. If he pays cash, he gets a discount of 10%. How much will he save by paying cash?

CHOOSING THE BETTER BARGAIN

Frank buys for \$24 a suit which was marked \$32. Howard gets for \$16 a suit which was marked \$25. Which of them gets the higher rate of discount?

Frank's discount is \$32 − \$24, or \$8. You must find what per cent \$8 is of the regular price, \$32. \$8 is 25% of \$32.

Howard's discount amounts to \$9. You find that \$9 is 36% of \$25.

Since Howard's discount is 36% and Frank's is only 25%, Howard gets the higher discount.

Exercises

Tell on which the rate of discount is greater and how much greater:

1. On a boy's watch reduced from \$6 to \$4.50 or on one reduced from \$10 to \$8.

2. On a winter overcoat reduced from \$40 to \$35 or on one reduced from \$35 to \$28.70.

3. On a six-cylinder car reduced from \$1400 to \$1106 or on one reduced from \$1200 to \$936.

4. On a fountain pen reduced from \$1.80 to \$1.50 or on one reduced from \$2.50 to \$2.00.

5. On a floor lamp reduced from \$26 to \$18.40 or on one reduced from \$26 to \$23.40.

6. At a January sale girls' dresses are advertised like this: Formerly \$18.75, now \$14.25. What rate of discount do you get if you buy a dress at the sale?

7. At Murray's you can buy an Excelsior tennis racket, marked \$7, for $\frac{1}{3}$ off. The same kind of racket can be bought at the Athletic Shop at \$7 less 30% or at Taylor's Shop for \$5. Which store offers the best bargain?

8. The Wilson Fur Shop wants to sell a fur coat marked \$500 for \$425. What rate of discount is that?

9. The Sport Shop reduced sleds from \$6 to \$4.20 and skates from \$6.60 to \$4.95. What rate of discount was that on each?

TRADE DISCOUNT

Mr. Davis has a radio shop on Lincoln Avenue. He buys large quantities of supplies from Mr. Woodman, who manufactures radio parts. Mr. Davis sells the parts to the people in his neighborhood. He is a *retail merchant* because he sells the radio parts directly to the people who use them. Mr. Woodman is a *wholesale merchant,* which means that he sells supplies to retail merchants and not to the people who actually use them.

When Mr. Davis wants to order parts, he looks them up in a **catalogue** which contains a list of all the articles that Mr. Woodman sells. The price given in the catalogue for each article is called its **list price.**

From time to time Mr. Woodman sends Mr. Davis and other retail merchants a notice telling what discount he will allow on the prices in the catalogue. Such discount, given by wholesale merchants to retail dealers, is called **commercial,** or **trade, discount.**

Most wholesale merchants allow trade discounts on the prices in their catalogues. One reason for such discounts is this: It is so expensive to publish a catalogue that the wholesaler cannot print a new one every time a price changes. Therefore he makes the list price high and lowers it by a trade discount. It is easier to change the price by changing the rate of discount than to publish an entirely new catalogue. A discount may also be allowed for paying a bill promptly or for buying in very large quantities.

If Mr. Davis orders 8 radio cabinets listed at $5 each, their total list price is $40. If he gets a trade discount of 30%, his discount is 30% of $40, or $12.

Trade discount is found in the same way as a discount at a sale. Only the language of trade discount is new to you.

BUYING RADIO SUPPLIES AT A TRADE DISCOUNT

1. Mr. Sage bought for his radio store 10 loud speakers listed at \$16.50 each. The trade discount was 30%. How much did he pay for them?

2. He bought 15 head phones listed at \$3.40 each, discount 25%. How much did they cost him?

3. Mr. Burns ordered 5 radio cabinets listed at \$14.80 each and 5 stools listed at \$4.20 each. On all of these his trade discount was 40%. Find his discount on the entire order and the net amount of his bill.

The net amount of a bill is the amount to be paid after the discount has been deducted.

4. How much would a retail dealer pay for 14 batteries listed at \$12 each if the trade discount is 15%?

5. The Eureka Shop ordered 6 dials at 30¢ each, 8 coils at 98¢ each, 30 plugs at 24¢ each, 4 crystal sets at \$2.40 each, and 3 cabinets at \$22.50 each. Find the total bill for this order. If 6% is allowed for quick payment, how much, to the nearest cent, can be saved by paying at once?

6. The owner of a repair shop ordered 55 dry cells at 32¢ each. His commercial discount was 40%. What per cent of the list price was the net price? Find the net price in two different ways (see page 205).

7. The Manhattan Radio Store pays \$104 each for radio sets listed at \$160. Find the amount of discount and the rate of discount on the list price.

Find each net price to the nearest cent, using the following list prices and trade discounts. Check the work:

8. \$90, 20%	**11.** \$250, 30%	**14.** \$239.84, 40%
9. \$45, 25%	**12.** \$87, $33\frac{1}{3}$%	**15.** \$455.79, 50%
10. \$878, 15%	**13.** \$489, $16\frac{2}{3}$%	**16.** \$256.98, 35%

CHAIN DISCOUNTS

Mr. Ray bought 180 dresses listed at \$13.50 each. The trade discount was 30%, and he received an extra discount of 10% because the order was large. Find the total net price.

The list price of the 180 dresses is 180×\$13.50, or \$2430. The first discount is 30% of \$2430, or \$729. Subtracting \$729 from \$2430, you get \$1701.

The second discount is taken on \$1701. 10% of \$1701 = \$170.10. Subtracting \$170.10 from \$1701, you have \$1530.90 as the net price of the dresses.

\$2430	\$2430.00
.30	729.00
\$729.00	\$1701.00
	170.10
	\$1530.90

Two discounts like this, on the same bill, may be spoken of as a discount of 30%, 10%. They are called **successive** or **chain discounts.** A discount of 30%, 10% is not the same as a discount of 40% because the discounts of 30% and 10% are not deducted from the same amount; the second discount, 10%, is taken on the amount left after the first discount is deducted.

If Mr. Ray received an extra discount of 2% for prompt payment, you would take 2% of \$1530.90, which is \$30.62, and subtract it from \$1530.90, giving \$1500.28 as the net amount paid. In all, Mr. Ray would then receive a chain discount of 30%, 10%, 2%.

Exercises

Find the net price. Compute each discount to the nearest cent:

1. \$600, less 20%, 10%
2. \$800, less 30%, 10%
3. \$500, less 25%, $7\frac{1}{2}$%
4. \$208, less 75%, $7\frac{1}{2}$%
5. \$765, less 30%, 10%, 5%
6. \$148, less 25%, 5%, $2\frac{1}{2}$%
7. \$453, less $33\frac{1}{3}$%, 10%, 2%
8. \$500, less 20%, 5%, 5%, 2%

9. Mr. Marr offered suits listed at \$24 at a discount of 30%, 20%. Mr. Nye offered similar suits at 50% off. What net price did each ask? Which is better, a discount of 30%, 20% or one of 50%?

DID JOE MAKE A MISTAKE?

Joe King buys shoes for the Royal Shoe Stores. One day he wanted to buy 500 pairs of shoes listed at \$3 a pair. An agent from the Pilgrim Factory offered him the shoes at a discount of 30%, 10%, 2%. An agent from another factory offered him the same kind of shoes at a discount of 10%, 30%, 2%. Joe said to the second agent: "The Pilgrim Factory offers a better discount, because it gives the largest discount on the full amount." Joe therefore bought the shoes from the Pilgrim Factory.

Did the Pilgrim agent really give the better discount? To find out, deduct a discount of 30%, 10%, 2% from \$1500; then take a discount of 10%, 30%, 2% from \$1500 and compare the net prices. Also try 2%, 10%, 30%.

The order in which chain discounts are deducted does not make any difference in the net price.

Exercises

Find each net price, taking the discounts in the order given. Then change the order of the discounts, taking the last one first. Find each discount to the nearest cent. Are the results alike?

1. \$700; 35%, 5%
2. \$430; 5%, $2\frac{1}{2}$%
3. \$240; 20%, 5%
4. \$620; 30%, 2%
5. \$900; 45%, 10%
6. \$300; $33\frac{1}{3}$%, $7\frac{1}{2}$%
7. \$925.40; 25%, 2%
8. \$510.50; 20%, 10%
9. \$140.60; 5%, 5%, 2%
10. \$827.50; 10%, 5%, 2%
11. \$733.20; 25%, 5%, 3%
12. \$890.80; 30%, 10%, 5%

13. Glass worth \$1800 is needed for a new building. The Wilson Company offers 40%, 10%, 5% off. The Benson Company offers 55% off. The National Building Supply Company offers 30%, 20%, 5% off. What net price does each ask? Which discount is best?

BILLS AND INVOICES

Mr. Johnson is a retail bookseller. In August he ordered 140 copies of "Camping for Boys" and 75 copies of "Polly's Adventures." The following bill was sent him with the books:

C. H. SEAMAN and COMPANY
PUBLISHERS

Sold to W. R. Johnson
Kingston, N. Y.

Chicago, August 8, 19___

140	Copies Cook's "Camping for Boys" @ $1.05	147	00		
75	Copies Ward's "Polly's Adventures" @ .90	67	50	214	50
	Less 40%			85	80
				128	70

Received Payment
Sept. 7, 19—
C. H. Seaman and Company
per D. S.

A **bill** is a statement of what one person owes another. On the first of the month many stores send to customers bills for goods which they have had charged during the preceding month. A doctor sends to his patients bills for his services.

An **invoice** is the bill which a wholesale dealer sends with a shipment of goods. The above bill is an invoice. In bills and invoices the sign @ means "at." The numbers giving the total charge for each article, like $147.00, are called **extensions.**

Exercises

1. When was the above invoice sent? When was it paid?
2. Give the list price of each copy of "Camping for Boys"; the total price of 140 copies.
3. Give the list price of each copy of "Polly's Adventures"; the total price of 75 copies.
4. What rate and amount of discount did Mr. Johnson get?
5. How do you know that Mr. Johnson paid the bill?

CHAIN DISCOUNT ON AN INVOICE

Here is an invoice on which successive discounts were allowed:

E. F. TAYLOR and COMPANY

Sold to Mr. Frank Seton
Ridgewood, N.J.

Detroit, June 4, 19–

TERMS: 30 days net, 2% 10 days

40 Cord Tires	@ $10	400	00		
50 Inner Tubes	@ 4	200	00	600	00
	Discount 20%, 10%			168	00
				432	00
Received Payment					
June 9, 19–	2% off			8	64
E. F. Taylor				423	36

How many discounts are there on this invoice? Show the steps by which the discount of $168 was obtained. What is the net amount of the bill, after all the discounts have been deducted?

The heading of the bill contains a statement called *terms*. The **terms** of a bill tell in how many days it must be paid; they sometimes offer a special discount for quick payment. "30 days net, 2% 10 days" means that Mr. Seton must pay $432.00 within 30 days after June 4, the date of the bill; but that if he pays within 10 days after June 4, he gets an extra discount of 2%.

On what date did Mr. Seton pay the above bill? Did he pay it within 10 days after June 4? Did he get the 2% discount? How much discount did he receive in all from the list price of the goods he bought?

Exercise

Write an invoice for the following, inserting names and dates. Write on the bill each discount (to nearest cent) and the net amount:

8 watches @ $12.50; 24 pens @ $2.50; 4 bracelets @ $12.50. Discount 25%, 15%. Terms: 30 da. net, 2% 10 da. The bill is paid 8 days after its date.

WRITING INVOICES

Make out and receipt invoices, supplying names, dates, and places. Compute each discount to the nearest cent.

1. 40 boxes writing paper @ 50¢; 200 rulers @ 15¢; 450 notebooks @ 10¢. Discount 15%. Terms: 30 da. net.

2. 10 copies Woodman's "Essays" @ $.96; 25 copies Payne's "Primer of Art" @ $.85; 28 copies Hill's "Nature Poems" @ $1.15. Discount 20%. Terms: 60 da. net, 2% 30 da. The bill is paid 25 da. after its date.

3. 200 qt. strawberries @ 20¢; 45 bbl. apples @ $3.60; 80 baskets peaches @ 80¢; 360 qt. blackberries @ 18¢. 4% discount for cash. The bill is paid immediately.

4. 35 flannel middies @ $4.20; 28 pr. bloomers @ $2.75; 16 felt numerals @ 35¢; 2 trophy cups @ $9.50. 15%, 10% off. Terms: 60 da. net, 2% 30 da., 4% 10 da. Paid in 5 da.

5. 20 pr. soccer boots @ $9.50; 16 sweaters @ $6.75; 4 soccer balls @ $10.25; 1 pr. goal nets @ $24.50; 15 pr. shin guards @ $1.80. Discount 20%, 10%, 5%. Terms: 30 da. net, $2\frac{1}{2}$% 10 da. Paid in 18 da.

6. 6 banjos @ $9; 3 mandolins @ $14; 5 guitars @ $16.20; 2 violins @ $38.50; 4 phonographs @ $69.25. $12\frac{1}{2}$%, 3% off. Terms: 30 da. net, 2% 10 da. Paid in 7 da.

7. 6 baseballs @ $1.75; 24 uniforms @ $8.50; 3 bats @ $2; 6 fielder's gloves @ $3.20; 2 catcher's mitts @ $3.90; 18 pr. baseball shoes @ $3.50. Discount 10%, 5%. Terms: 30 da. net, 4% 10 da. Paid 12 da. after date.

8. 12 scout uniforms @ $12.75; 16 flashlights @ $1.25; 1 bugle @ $2.95; 5 knives @ 70¢; 3 camp stoves @ $6.40; 6 watches @ $3.50; 4 whistles @ 40¢. Discount $33\frac{1}{3}$%, 5%. Terms: 60 da. net, 2% 30 da. Paid 8 da. after date.

9. 45 dresses @ $12.75; 8 doz. gloves @ $21.00; 30 coats @ $18.50. Discount 25%. Terms: 30 da. net. Paid at once.

SELLING AT A PROFIT

Roderick bought a cow at the fair for \$80. He sold it an hour later for \$100. How much did he gain on it? What per cent of the cost was his profit?

The **cost** was \$80 and the **selling price** was \$100. Roderick sold the cow for \$20 more than it cost him. Since he had no expenses for it between the time he bought it and the time he sold it, the whole \$20 was gain or **profit** for Roderick. Since \$20 is $\frac{20}{80}$, or 25%, of \$80, his profit was 25% of the cost. Check by finding whether 25% of \$80 = \$20.

If he had sold the cow for \$70, he would have sold it at a **loss** of \$10. Then what would have been the per cent of loss on the cost?

Finding Profit and Loss

1. A man bought an automobile for \$900. A few days later he was forced to sell it for \$750. What was the cost of the car? the selling price? the amount of loss? Find what per cent the loss was of the cost.

2. Alice sold lemonade at the school fair. She bought 30 lemons at 3 for 5¢ and 15¢ worth of sugar. She sold 18 glassfuls of lemonade at 5¢ each and 9 glassfuls at 3¢ each. Find the per cent of profit on the cost.

3. If Mr. Roland buys a lot for \$600 and sells it the next day for \$750, what is the per cent of his profit on the cost?

The first number is the cost; the second, the selling price. Find what per cent the profit or the loss is of the cost:

4. \$800; \$900	**9.** \$1200; \$924	**14.** \$2760; \$3450
5. \$750; \$550	**10.** \$940; \$1081	**15.** \$1750; \$1505
6. \$582; \$485	**11.** \$891; \$594	**16.** \$500; \$642.50
7. \$250; \$275	**12.** \$595; \$833	**17.** \$248; \$266.60
8. \$495; \$594	**13.** \$480; \$600	**18.** \$660; \$767.25

THE PROFITS OF MR. BURT'S CANDY STORE

Mr. Burt has a candy shop. In March he bought candy for $500 and sold it for $860. But the difference of $360 was not all profit, because Mr. Burt paid $70 for the March rent of the store, $60 to a clerk, $15 for advertising, and $40 for heat, light, and other expenses. Hence it cost him $185 in all to run the store in March. This $185 is called his **overhead** or **operating expenses** or, briefly, his **expenses.** To find his profit you add the cost of the candy and the expenses and subtract the result from the selling price. This gives $860 − $685, or $175, **profit.**

Operating Expenses
$70
60
15
40
$185

If the expenses and the cost had amounted to *more than* the selling price, the difference would have been Mr. Burt's **loss.**

Exercises

1. Name the operating expenses of some grocery store in your neighborhood. If a grocer keeps an automobile for delivering groceries, is that part of his expenses?

2. Last year Mr. Sutton bought $5640 worth of groceries and sold them for $7896. His overhead, or operating expenses, for the year were $1579.20. What was his profit?

3. What per cent of the cost was Mr. Sutton's profit in ex. 2? What per cent of the cost were his expenses?

NOTE. The difference between the cost and the selling price is sometimes called the *gross profit;* while the *gross profit* less the *expenses* is called the *net profit.* The *gross profit* is not all profit, because it includes the operating expenses. Prominent business associations recommend that the term *gross profit* be discontinued since retail shopkeepers often misunderstand it, thinking that gross profits are real profits. The same associations recommend the use of the word *margin* in place of *gross profit.* In this book the word *profit* means *actual* or *net* profit, which is found as explained at the top of this page.

HOW MUCH SHOULD JACK ASK FOR HIS MOTOR BOAT?

Last October Jack paid \$35 for an old motor boat. It cost him \$4.50 to store it and \$10 to paint it. In the spring he wanted to sell it at a profit of 30% on the cost to pay for his effort. At what price did he have to mark it?

The cost of the boat was \$35; the expenses were \$4.50 + \$10, or \$14.50. The profit was to be 30% of \$35, or \$10.50. Hence Jack had to sell the boat for \$35 + \$14.50 + \$10.50, or \$60.

\$35.00	**Cost**
14.50	**Expenses**
10.50	**Profit**
\$60.00	**S. P.**

Cost + Expenses + Profit = Selling Price

FINDING THE SELLING PRICE

1. Paul bought an old bicycle for \$9. He paid \$1.95 for a new tire and \$.55 for repairs. What price should he ask for it to make a profit of \$2?

Find the prices at which you must sell the following if you wish to make the given profit on the cost:

	Cost	Expenses	Profit		Cost	Expenses	Profit
2.	\$20	\$5	5%	**6.**	\$12.50	\$2.20	8%
3.	\$60	\$12	6%	**7.**	\$35.75	\$10.95	10%
4.	\$100	\$35	15%	**8.**	\$49.60	\$15.70	7%
5.	\$350	\$85	18%	**9.**	\$280.40	\$112.15	15%

Find the selling price. Also find what per cent the expenses are of the cost, and what per cent the profit is of the cost:

	Cost	Expenses	Profit		Cost	Expenses	Profit
10.	\$450	\$108	\$27	**13.**	\$15.60	\$5.46	\$2.34
11.	\$890	\$267	\$89	**14.**	\$76.50	\$25.50	\$9.18
12.	\$785	\$314	\$86.35	**15.**	\$218.60	\$54.65	\$21.86

HOW MR. FIELD DETERMINES HIS PRICES

1. For 12 years Mr. Field has run the general store at Oak Center. By keeping careful accounts, he has learned that his expenses are always about 30% of the cost of his goods. He has found also that he must make a profit of 15% on the cost to earn a living. Hence he marks his goods at 30% + 15%, or 45%, more than they cost. At what price does he mark a lawn mower that cost \$9?

2. Richard finds the selling price of the lawn mower like this: Since Mr. Field marks the goods 45% above cost, Richard finds 45% of \$9, which is \$4.05. Adding \$4.05 to \$9, he gets \$13.05 as the selling price.

3. Katherine does the problem another way. To find the selling price, she takes 145% of \$9. Does this give the same answer that Richard got?

Exercises

Find the selling price in each of the following, using Richard's method. Then check the work by Katherine's method:

	Cost	Expenses	Profit
1.	\$200	30% of cost	8% of cost
2.	\$480	35% of cost	9% of cost
3.	\$1950	25% of cost	10% of cost
4.	\$7200	32% of cost	15% of cost
5.	\$25.80	40% of cost	7% of cost
6.	\$175.35	34% of cost	12% of cost
7.	\$250.50	$33\frac{1}{3}$% of cost	10% of cost
8.	\$116.24	$37\frac{1}{2}$% of cost	$12\frac{1}{2}$% of cost
9.	\$324.00	25% of cost	$12\frac{1}{2}$% of cost

10. Mr. Davis has a clothing store. He buys overcoats for \$15 and sells them for \$21. If his operating expenses are 30% of the cost, what profit does he make on each overcoat? What per cent of the cost is the profit?

COMPUTING PROFITS ON THE SELLING PRICE

So far you have found the profit and the per cent of profit on the cost. Many modern business men prefer to figure their profits on the selling price. Since commissions, discounts, and certain taxes are computed on the selling price, or total sales, it is simpler to compute the profits also on the sales. Furthermore, a merchant always knows what he receives for an article, but it is sometimes hard to separate the exact cost of the article from the operating expenses. Always state whether the per cent of profit or loss is computed on the cost or on the selling price.

Exercises

1. Joe sells eggs at 60¢ a dozen. If his profit is 15% of the selling price, how much does he make on 1 doz. eggs?

The first number is the selling price; the second, the per cent of profit on the selling price. Find the profit:

2.	\$3; 10%	**5.**	\$845; 8%	**8.**	\$42.50; 9%
3.	\$1; 12%	**6.**	\$358; 13%	**9.**	\$118.75; 7%
4.	\$50; 15%	**7.**	\$740; 11%	**10.**	\$489.60; $12\frac{1}{2}$%

Find the profit and the per cent of profit on the selling price. In ex. 11 *to* 13 *there are no expenses:*

	Cost	Selling Price		Cost	Expenses	Selling Price
11.	\$65	\$75	**14.**	\$277	\$68	\$375
12.	\$64	\$80	**15.**	\$354	\$96	\$600
13.	\$87	\$116	**16.**	\$756	\$285	\$1200

17. Morton bought 10 doz. eggs from a farmer for \$3.80 and sold them at 48¢ a dozen. He spent 35¢ for car fare in selling them and 15¢ for boxes and paper. What was his profit? the per cent of his profit on the selling price?

18. If Morton spent $1\frac{1}{2}$ hr. in getting the eggs and $2\frac{1}{2}$ hr. in selling them, how much did he make per hour?

COMPUTING EXPENSES ON THE SELLING PRICE

You have just seen that profits are often figured on the selling price. Expenses, or overhead charges, are also frequently computed as a per cent of the selling price because it is easier to compute them that way. A business man can tell his total sales for the past year from his books, and since he also knows his total expenses for the year, it is easy to find what per cent the expenses were of the total sales. Suppose that a merchant sold \$750,000 worth of goods last year with total expenses of \$225,000. Since \$225,000 is 30% of \$750,000, the merchant's expenses last year were 30% of his sales. Hence he knows about what per cent to allow for expenses this year.

By computing profits and expenses on the sales, a merchant knows how much of each dollar taken in is profit and how much of it will be used to cover expenses and cost.

A good business man tries to make his expenses as low a per cent of his sales as possible.

Exercises

These numbers show the total sales and the total expenses for a year. Find what per cent the expenses are of the sales:

	Sales	Expenses		Sales	Expenses
1.	\$8750	\$2100	**3.**	\$98,100	\$32,700
2.	\$22,500	\$6075	**4.**	\$75,450	\$22,635

5. Mr. Bates finds that his profit is 12% of the selling price and his expenses are 35% of the selling price. What per cent of the selling price is the cost of his goods?

You know that Cost + Expenses + Profit = Selling Price. Since the profit and the expenses together equal 47% of the selling price, the rest of the selling price must represent the cost.

6. Mr. Bates sells a sweater at \$6.00 (see ex. 5). How much of the \$6.00 is profit? How much of it goes for expenses? How much covers the cost of the sweater?

WHAT PRICE SHALL HE MARK THE RING?

Mr. Ladd, a jeweler, is about to mark the price of a diamond ring that cost him \$165. He has found that he must allow 35% of the selling price for expenses and 10% of the selling price for profit. At what price must he mark the ring to cover his expenses and profit as well as the original cost of the ring?

Since the expenses and profit together are 45% of the selling price, the remaining 55% of the selling price (S. P.) represents the cost of the ring, which is \$165. The selling price can now be computed as shown at the right.

55% of S. P. = \$165
1% of S. P. = \$ 3
100% of S. P. = \$300

Since 100% of the selling price is the same as the selling price, Mr. Ladd should mark the ring \$300.

To check, take 45% of \$300, which is \$135, the amount of the profit and expenses together. Subtract \$135 from \$300, and you get the cost, \$165.

Exercises

1. At what price must a storekeeper mark a tennis racket that cost him \$1.80 to allow 30% of the selling price for expenses and 10% of the selling price for profit?

Find the selling price (S. P.) *at which each of the following should be marked, and check your results:*

	Cost	Expenses	Profit
2.	\$7.14	40% of S. P.	9% of S. P.
3.	\$9.60	29% of S. P.	7% of S. P.
4.	\$3.04	27% of S. P.	9% of S. P.
5.	\$30.24	34% of S. P.	10% of S. P.
6.	\$15.39	32% of S. P.	11% of S. P.
7.	\$16.20	40% of S. P.	10% of S. P.

8. Mr. Deal bought a ring for \$90. He allowed 28% of the selling price for expenses and 12% for profit. For how much did he sell the ring?

9. On the same conditions as in ex. 8, at what price did Mr. Deal sell a ring that cost him \$150?

IMPROVEMENT TEST No. 16

Test 16 A — Addition (Time 4 min.)

1. $7\frac{1}{2}$ + $4\frac{7}{8}$ + $3\frac{3}{4}$ + $1\frac{1}{2}$

2. $2\frac{1}{3}$ + $2\frac{1}{2}$ + $5\frac{5}{6}$ + $8\frac{2}{3}$

3. $3\frac{1}{5}$ + $9\frac{2}{5}$ + $5\frac{1}{2}$ + $3\frac{9}{10}$

4. $1\frac{1}{8}$ + $6\frac{2}{3}$ + $3\frac{3}{8}$ + $1\frac{1}{8}$

5. $5\frac{1}{3}$ + $2\frac{1}{4}$ + $7\frac{1}{2}$ + $1\frac{1}{6}$

Test 16 B — Subtraction (Time 4 min.)

1. $8\frac{3}{8} - 4\frac{1}{4}$

2. $9\frac{1}{4} - 7\frac{3}{4}$

3. $7\frac{2}{3} - 4\frac{1}{2}$

4. $6 - 2\frac{3}{10}$

5. $6\frac{2}{3} - 1\frac{1}{5}$

6. $9\frac{1}{2} - 6\frac{9}{16}$

7. $3\frac{3}{5} - 2\frac{1}{2}$

8. $3 - 1\frac{5}{6}$

9. $9\frac{1}{4} - 2\frac{2}{3}$

10. $5\frac{1}{3} - \frac{5}{6}$

11. $6\frac{1}{6} - 4\frac{1}{12}$

12. $8\frac{3}{5} - 5\frac{1}{10}$

Test 16 C — Multiplication (Time 4 min. after copying)

1. $270 \times 12\frac{2}{3}$

2. $37\frac{1}{2} \times 55$

3. $288 \times 16\frac{3}{4}$

4. $5 \times \frac{5}{8}$

5. $\frac{2}{3}$ of 47

6. $6\frac{3}{4} \times \frac{5}{6}$

7. $\frac{7}{8} \times \frac{4}{5}$

8. $9\frac{1}{3} \times 7\frac{1}{2}$

9. $1\frac{9}{16} \times \frac{4}{15}$

Test 16 D — Division (Time 3 min. after copying)

1. $6 \div \frac{3}{4}$

2. $4\frac{3}{8} \div \frac{5}{16}$

3. $8\frac{2}{5} \div 12$

4. $\frac{5}{8} \div 4$

5. $5 \div 7\frac{1}{2}$

6. $1\frac{1}{2} \div 1\frac{7}{8}$

7. $\frac{7}{8} \div \frac{3}{16}$

8. $\frac{9}{10} \div \frac{12}{25}$

9. $1\frac{11}{12} \div \frac{5}{6}$

PROBLEMS IN PERCENTAGE

1. Mrs. Harris, who runs a grocery store, is selling her goods at 40% off. How much will Mrs. Arnold save on a dozen jars of jam, regularly costing $.35 each, if she buys them from Mrs. Harris?

2. Five years ago, Newtown and Redville each had 14,000 inhabitants. The population of Newtown is now 100% of what it was then, while the population of Redville has increased 100%. Find the present population of each town.

3. Mr. Leon sells furniture. If 27% of his sales usually go toward overhead expenses and if he wants 15% of his sales to be profit, at what price must he sell a living-room suite that cost him $174?

4. Arthur sells berries for Mr. Dodd at a commission of 12%. If he sells 45 boxes of strawberries @ $.38 and 25 boxes of blackberries @ $.34, how much commission does he receive in all? Find Mr. Dodd's net proceeds.

5. Our school store bought 24 tennis rackets @ $3.85, 15 cameras @ $5.95, and 72 school banners @ $.14. The discounts were 20%, 4%. Make out and receipt the invoice, supplying names, dates, and places. Compute each discount to the nearest cent.

6. Ed sold ice cream at the ball game. He purchased 8 gal. of ice cream at $2.20 a gallon. He paid $3 rent for dishes and $2.50 to Frank, who helped him. He sold 275 plates of ice cream at 10¢ each. Find the cost, expenses, total sales, and profit. Find the per cent of Ed's profit on the sales.

7. What is a "land boom"? Mr. Lyons bought lots for $8000 in a land boom and had to sell them a few weeks later for $2000. Mr. White bought $95,000 worth of property and sold it for $57,000. Which man lost the greater per cent of the amount he invested? how much greater?

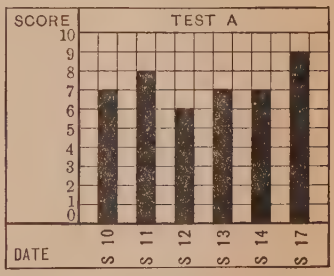
GRAPHS

Bar Graphs

Newspapers and magazines frequently use graphs to give a vivid impression of number facts.

You have learned to use a **line graph** to record your progress on an Improvement Test. Exactly the same record may be represented by a **bar graph,** as shown below.

LINE GRAPH



BAR GRAPH

How does the line graph show a score of 7 on Sept. 10? How does the bar graph show the same score?

How many squares does the bar for Sept. 11 cover? What score does this bar represent?

On Sept. 13 and 14 the score is the same. How does each graph show this? How does each show a score of 9 on Sept. 17?

Exercises

Make a line graph and a bar graph for each record:

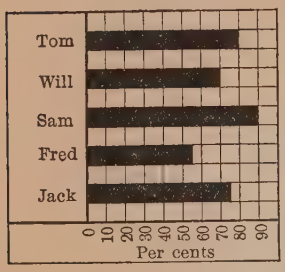
1. On Improvement Test 1 A, James had a score of 6 on Nov. 5, 8 on Nov. 6, 6 on Nov. 7, and 10 on Nov. 12.

2. Sarah made a score of 7 on an Improvement Test on Jan. 28, 9 on Jan. 29, 9 on Feb. 4, and 10 on Feb. 5.

3. Laura's mark on Test 1 A was 8 on Oct. 15, 9 on Oct. 16, 6 on Dec. 19, 9 on Dec. 20, and 10 on Dec. 21.

OTHER BAR GRAPHS

On a spelling test, Tom had 80%; Will, 70%; Sam, 90%; Fred, 55%; and Jack, 75%. To represent these facts on a bar graph, make the bars either vertical, as on the preceding page, or horizontal, as here.



In making bar graphs for numbers like 70 and 80, it is more convenient to let each square stand for 10 units. Then how many squares must Tom's bar cover to show a mark of 80? How many must Will's cover?

Since Fred's mark is 55, his bar must cover 5 whole squares and half of the sixth square. How many squares are needed to show Jack's mark of 75?

Exercises

Draw bar graphs to show the following facts:

1. Alice's marks in arithmetic for 5 months were as follows: Feb., 70%; March, 75%; April, 80%; May, 85%; and June, 90%. Let each square represent 10 units.

2. Mary weighs 60 lb.; John, 85 lb.; Sarah, 80 lb.; and Henry, 95 lb. How many units should each square represent?

3. Grade 3 has $40 in the school bank; Grade 4 has $55; Grade 5, $50; Grade 6, $85; Grade 7, $65; and Grade 8, $80.

4. There are 30 pupils in the Range School, 70 in the Cort School, 85 in the Taft School, and 15 in the Park School.

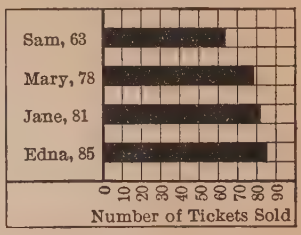
5. Ellen sold $3 worth of Christmas seals; Leo, $6.50; Frank, $9; Agnes, $1.50; and Joe, $4 worth. Let each square represent $1.

6. In a baseball series, Team A won 14 games, Team B won 4, Team C won 11, Team D won 6, and Team E won 7. Let each square represent 2 units.

USING PARTS OF A SQUARE

On this graph, which shows the number of tickets sold by four children for a school play, the bars do not each cover a whole number of squares. Sam sold 63 tickets. Since 1 square represents 10 tickets, 6 squares are covered to represent 60 tickets, and $\frac{3}{10}$ of the next square is covered to represent 3 tickets. Hence 6 whole squares and $\frac{3}{10}$ of the next square represent the 63 tickets that Sam sold.

RECORD OF TICKETS SOLD FOR A SCHOOL PLAY



To cover $\frac{3}{10}$ of a square, first think of the square as divided into 10 equal parts, as shown in A. Then cover 3 of these parts, as shown in B. When the squares are small, estimate $\frac{3}{10}$ of a square as closely as you can by the eye.

Since Mary sold 78 tickets, her bar covers 7 whole squares and $\frac{8}{10}$ of the next square. How do you show that Jane sold 81 tickets? that Edna sold 85 tickets?

Exercises

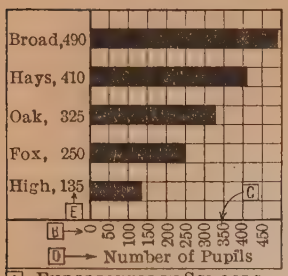
Use bar graphs to show the following facts:

1. The Fox School had a fair to raise money for a playground. On Tuesday they made \$86; on Wednesday, \$66; on Thursday, \$73; and on Friday, \$97.

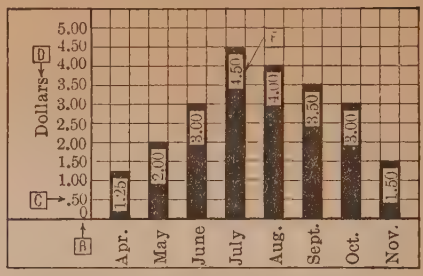
2. Out of 100 children, 32 are spending next vacation at the seashore, 24 are going to the mountains, 16 are going camping, and the rest will stay at home.

3. The average weekly earnings of workers recently were: textile mills, \$19; lumber mills, \$20; shoe factories, \$22; meat-packing industries, \$23; automobile manufacturing, \$30.

HOW TO DRAW ATTRACTIVE BAR GRAPHS



A → Enrollment in Schools of Hull



A → Family Ice Bill for 8 Months

Neat and attractive bar graphs may be made by observing the following suggestions:

1. The title of the graph should be written above or below it, as shown at A in each graph above.

2. All the bars should have the same width; the white spaces between bars should have the same width as the bars themselves.

3. The bars should be arranged in a definite order. In the first graph above, they are arranged according to length, with the longest on top. In the second graph, the arrangement follows the order of months in the year.

4. By reading the numbers, or *scale*, shown at B on each graph, you can tell what a bar represents without counting the squares. Always write the scale numbers directly opposite the lines that separate the squares, as shown at C.

5. Always write under or at the side of the scale a statement telling what the numbers represent, as shown at D.

6. In horizontal bar graphs the number represented by the bar may be written at the left of the bar, as at E. On vertical bars, the number may be written at the base of the bar or *within* the bar, as at F. This practice of printing the actual number represented by each bar is common in books and magazines.

REPRESENTING LARGER NUMBERS

Sometimes you must show numbers like 463 or 527 on a bar graph. Let each square represent 100. Then 463 will be represented by 4 squares and $\frac{63}{100}$ of the next square. Since you cannot draw exactly 0.63 of a small square, first express 0.63 correct to the nearest tenth, which gives 0.6. Then make the bar 4.6 squares long, as shown at A.

To draw 527, divide 527 by 100 and get 5.27 which, to the nearest tenth, is 5.3. Draw the bar 5.3 squares long, as at B.

To represent a larger number, like 4362, let each square stand for 1000. Dividing 4362 by 1000, you get 4.362, which is 4.4 correct to the nearest tenth. This gives 4.4 squares as the length of the bar.

To find how many squares shall represent a number, divide the number by 100 *if each square represents* 100; *divide it by* 1000 *if each square represents* 1000. *Express the result correct to the nearest tenth.*

Exercises

1. When each square represents 100, how many squares and tenths of a square are needed to represent 347? 582? 736? 258? 452? Draw a bar graph representing each number.

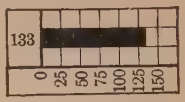
2. When each square represents 1000, how many squares and tenths of a square are needed to represent 9327? 5286? 4029? 8534? 7082? Draw a bar graph representing each number.

3. Letting 1 square = 10, draw a bar graph to show that 35 pupils were present in a class on Monday, 38 on Tuesday, 42 on Wednesday, 40 on Thursday, and 45 on Friday.

4. Letting 1 square = 1000, show by a graph the number of miles of railroad in these states: Arkansas, 4916; Indiana, 7197; Pennsylvania, 11,414; Montana, 5038.

WHEN A SQUARE REPRESENTS 25 OR 50

In all the graphs you have made so far, each square has represented 1 unit, 10 units, 100 units, or 1000 units. A square may represent any number of units. To show 133 when 1 square represents 25 units, proceed in the same way as when the scale is 100 or 1000. Divide 133, the number to be represented, by 25. This gives 5.3, correct to the nearest tenth. Hence 133 is represented by 5.3 squares, as shown in the graph.



To find the number of squares needed to represent a given number, divide the number by the value of each square and express the result correct to the nearest tenth.

Exercises

1. If the value of 1 square is 25, how many squares are needed to represent 142? 167? 218? 273? 290? 729?

2. How many squares are needed to represent 147, 324, 485, and 173 if 1 square equals 50? if 1 square equals 25?

3. Letting 1 square = 50, draw a graph showing the number of points won by these athletic teams: Team A, 336; Team B, 268; Team C, 185; Team D, 217; Team E, 78.

4. Letting each square stand for 200 ft., show by a graph the lengths of the channel spans of these famous bridges: Delaware River Bridge, Philadelphia, 1750 ft.; Hudson River Bridge, Poughkeepsie, 1500 ft.; East River Bridge, Brooklyn, 1596 ft.; Ohio River Bridge, Cincinnati, 1057 ft.

5. Letting 1 square represent \$5000, show by a bar graph that the profits of the Wearwell Shoe Company last year were \$26,100; of the Union Iron Works, \$37,000; of the Whiteman Paper Company, \$19,500; and of the Hill Bakery, \$7500.

6. Letting 1 square = \$5, show by a graph that Ella earned \$28 in June, \$36 in July, and \$14 in August.

EXERCISES

Use bar graphs to represent the following facts:

1. John has \$165 in the savings bank; May has \$220; Sam has \$356; Jane has \$190. In drawing this graph, why is it better to let each square represent 10 rather than 100?

2. The lengths in miles of some famous rivers are: Amazon, 3900; Congo, 2900; Danube, 1725; Yangtse, 3100; St. Lawrence, 2150; Missouri-Mississippi, 5431.

3. The areas of some states in square miles are: Missouri, 69,420; Kansas, 82,158; Maryland, 12,327; New York, 49,204; Colorado, 103,498; Pennsylvania, 45,146. Include your state if it is not in this list. Let 1 square = 10,000.

4. There are 763,300 telephones in California, 205,900 in Connecticut, 383,500 in New Jersey, 962,800 in Ohio, 1,780,600 in New York, and 687,700 in Pennsylvania. In drawing each bar, let 1 square = 100,000.

5. The population of Washington, D. C., was 75,000 in 1860, 132,000 in 1870, 178,000 in 1880, 230,000 in 1890, 279,000 in 1900, 331,000 in 1910, and 438,000 in 1920.

How does your graph show a steady growth in the population of Washington?

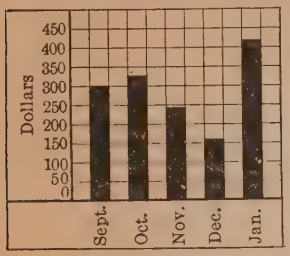
6. The areas of the continents in square miles are: North America, 8,589,257; South America, 7,570,215; Australasia, 3,312,613; Europe, 3,872,561; Asia, 17,206,000; Africa, 11,622,619. What number shall each square represent?

7. The populations of the continents are: North America, 136,000,000; South America, 64,000,000; Australasia, 9,000,000; Europe, 476,000,000; Asia, 921,000,000; Africa, 142,000,000. Let each square represent 100,000,000 people.

8. One year the grain crops of the world included corn, 4,055,238,000 bu.; wheat, 3,400,448,000 bu.; oats, 3,775,424,000 bu.; barley, 1,348,202,000 bu.; and rye, 1,424,426,000 bu.

READING BAR GRAPHS

Since you know how to make bar graphs, you should also be able to interpret such graphs when you see them in the papers.



This graph was made by the children of the Porter School to show the amounts deposited in the school bank each month.

1. How can you tell that each square represents \$50?

2. How many squares does the September bar cover? What amount does that bar represent?

You can read this bar without counting the squares by observing that the top of it is on the same level as the 300 mark on the scale at the left.

3. The October bar represents more than \$300, for the top of the bar covers about $\frac{1}{2}$ of the next square. Since 1 whole square = 50, $\frac{1}{2}$ square = 25. Adding \$25 to \$300 gives \$325 as the October savings.

4. The top of the November bar almost touches the 250 mark. Roughly, you would call the November amount "a little less than \$250." More exactly, you estimate that the top of the bar covers about $\frac{9}{10}$ of the last square. Since $\frac{9}{10}$ of 50 is 45, the bar represents \$45 more than \$200, or \$245.

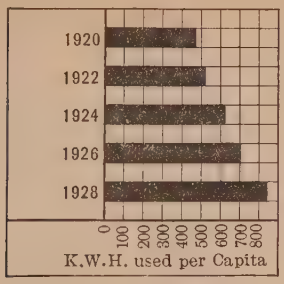
5. Determine roughly the values of the other two bars. Then read each as accurately as you can.

You see that in reading graphs the results are approximate, especially when each square represents a large number. To make it easier to read graphs, the numbers represented by each bar are often printed near it, as shown on page 226.

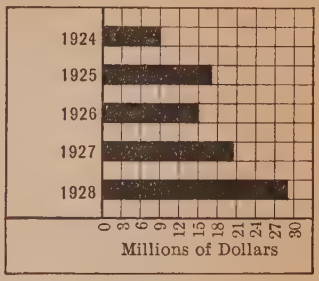
In reading graphs, find by means of the scale the value of the whole squares covered by the bar, and add to this the estimated value of the part of a square, if any, covered by the end of the bar.

CAN YOU READ THESE GRAPHS?

1. Graph 1 shows how the use of electricity in a large city increased in 8 years. In 1928 an average of 845 K.W.H. per person was used. What was the average in 1920? in each of the other years? In which year was the increase greatest?



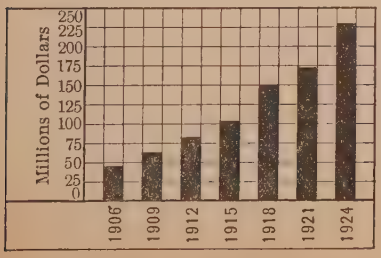
1. Use of Electricity



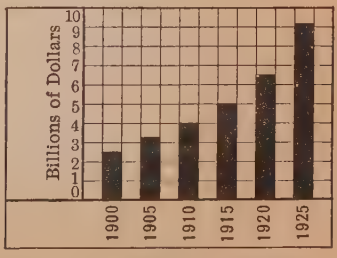
2. Record of Sales

2. Graph 2 shows the yearly sales of a business house. Estimate the total sales each year. Did they increase steadily?

3. Graph 3 shows the value of the natural gas produced in this country in certain years. Estimate the value for each year.



3. Value of Natural Gas



4. Savings Bank Deposits

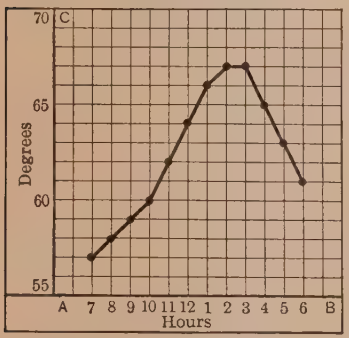
4. Graph 4 shows the increase in the amount of deposits in the savings banks of the United States. Estimate the amount represented by each bar. In which 5 years was there the largest increase in deposits? How does the graph show this?

LINE GRAPHS

George was interested to see how the temperature changes during the day. He read the thermometer each hour and wrote down the temperatures as shown in the table below.

Hour	7	8	9	10	11	12	1	2	3	4	5	6
Degrees	57	58	59	60	62	64	66	67	67	65	63	61

To get a better idea of the changes, he drew the graph below.



First he drew line AB and marked the hours on it. This was the *horizontal scale.* Next he marked the degrees on line AC, letting each space equal 1 degree (1°). This was the *vertical scale.* Since the temperature was 57° at 7 A.M., George placed a point on the graph *directly above* 7 A.M. and *directly to the right* of 57°. In the same way he placed points for each hour in the table. He connected these 12 points with a broken line.

Exercises

1. In George's graph, at what hour was it warmest?

2. What was the lowest temperature? When did it occur?

3. Can you tell from the graph what the temperature was at 11.30 A.M.? at 12.30 P.M.? at 3.30 P.M.?

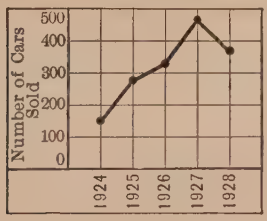
4. Draw a line graph showing these changes in temperature:

Hour	6	7	8	9	10	11	12	1	2	3	4	5	6
Degrees	58	60	62	65	68	70	72	73	74	75	72	70	70

USING A LARGER SCALE ON A LINE GRAPH

On the graph on page 232, each space on the scales represents 1 hour or 1°. Each space might also represent 10 units, 100 units, or any other number of units, just as on a bar graph. Each space on the vertical scale below represents 100 cars.

SALES RECORD OF REED BROTHERS



Year	*Cars Sold*
1924	150
1925	280
1926	327
1927	461
1928	369

To show that Reed Brothers sold 150 cars in 1924, first find 150 on the vertical scale; it is half way between 100 and 200. Place a point directly to the right of 150 and above 1924.

The sales for 1925 were 280 cars. On the vertical scale 280 is represented by 2.8 spaces, since 1 space = 100. Therefore 280 is .8 of a space above the 200 line. Place the point for the 1925 sales directly to the right of 280 and above 1925.

327 is represented by 3.27 spaces, or 3.3 spaces, to the nearest tenth. Where do you place the point for 1926?

Explain how the points for 1927 and 1928 are located.

On a line graph each point represents two numbers, one number being on the horizontal scale directly below the point, and the other on the vertical scale directly to the left of the point.

Exercise

Show on a line graph the growth of Oak Hills, as given here. How many units will each space on the vertical scale represent?

Year	*Population*
1890	3500
1900	4100
1910	5800
1920	5600

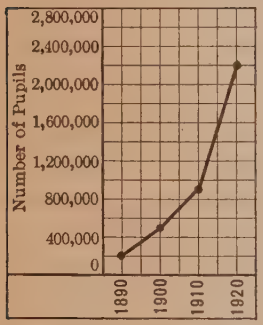
MAKING AND READING LINE GRAPHS

Show the facts of ex. 1 *to* 3 *on line graphs. Give each graph a title and indicate what the numbers on each scale represent.*

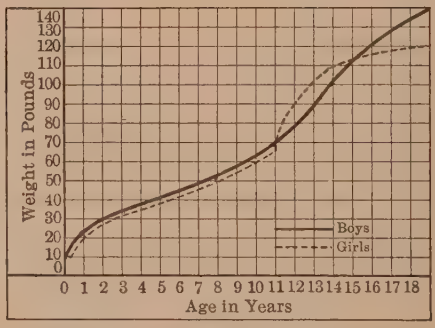
1. These average temperatures were recorded one year for the city of New York: Jan., 31°; Feb., 31°; Mar., 38°; Apr., 49°; May, 61°; June, 69°; July, 74°; Aug., 73°; Sept., 67°; Oct., 56°; Nov., 44°; Dec., 35°.

2. There were 805,000 immigrants admitted to the United States in 1921, 310,000 in 1922, 523,000 in 1923, 707,000 in 1924, 294,000 in 1925, and 304,000 in 1926.

3. What was the high-school enrollment in this country in each year shown on the first graph below?



HIGH SCHOOL ENROLLMENT



WEIGHT TABLE FOR BOYS AND GIRLS

4. In the second graph above, a comparison is made between the average weights of boys and girls. Should a boy or a girl weigh more at the age of 8? at the age of 11? of 13? of 15? of 19? From this graph tell how much you should weigh.

On this graph, instead of indicating the points by heavy dots, they are found as follows: The point indicating the weight of an 8-year-old boy is the point where the vertical line marked 8 crosses the heavy black graph line. You see that this point is located about $\frac{3}{10}$ of a space above the horizontal line marked 50; hence an 8-year-old boy weighs 53 lb.

REVIEW PROBLEMS

1. In the Drake High School, 125 pupils were graduated this year and 70 are going to college. In the Park High School, 140 pupils were graduated and 63 are going to college. Which school has the greater per cent of its graduates going to college?

2. Mr. Mohr's farm was worth \$13,400 in 1927. Since then its value has increased 135%. How much is it worth now? Draw a bar graph to show its value in 1927 and now.

3. If I sell an automobile for 30% less than it cost me, I am selling it at what per cent of the cost?

4. Mr. Dunn orders a stove listed at \$48, with discounts of 25%, 10%. He gets an extra discount of 2% if he pays within 10 days. How much will the stove cost if he pays at once?

5. The year before last the total sales of Miss Brown's Candy Shop were \$12,500, of which \$6750 covered the cost of the candy and \$4500 went for operating expenses. Last year she rented a larger store and hired extra help, thereby increasing the expenses to \$6192. The cost of the candy was \$11,008. Her sales last year totaled \$20,000. Find her profit each year and the per cent of her profit on the sales.

Show on a bar graph the per cent of Miss Brown's profit on the sales for the past 2 yr. Has it increased or decreased?

6. Using a solid line for boys and a dotted line for girls, as in ex. 4 on page 234, show on a graph their average heights in inches. Use data in the table below.

Age in Years	5	6	7	8	9	10	11	12	13	14	15	16
Boys' Heights in Inches .	42	44	46	48	50	52	53	55	57	60	62	65
Girls' Heights in Inches .	41	43	46	48	49	51	53	56	58	60	61	62

7. Anna's employer offers her either \$25 a week or \$15 a week with $6\frac{1}{2}$% commission on all her sales. If her sales average \$200 a week, which offer is better? how much better?

REVIEW OF PERCENTAGE

Express these decimals as per cents:

1.	.08	.72	.06$\frac{1}{2}$	.17$\frac{1}{2}$	1.45	2.20	.8
2.	.05	.19	.03$\frac{1}{3}$	.62$\frac{1}{2}$	3.00	10.00	.3

Express these per cents as two-place decimals:

3.	6%	4%	13%	7$\frac{1}{2}$%	12$\frac{1}{2}$%	175%	200%
4.	1%	10%	90%	4$\frac{1}{4}$%	66$\frac{2}{3}$%	100%	150%

Express these per cents as common fractions:

5.	37$\frac{1}{2}$%	75%	16$\frac{2}{3}$%	33$\frac{1}{3}$%	50%	87$\frac{1}{2}$%	70%

Find the following:

6.	15% of 750	**10.**	2$\frac{1}{4}$% of \$800	**14.**	83$\frac{1}{3}$% of \$7494
7.	25% of 368	**11.**	4$\frac{1}{2}$% of \$620	**15.**	100% of \$1000
8.	72% of 525	**12.**	5$\frac{3}{4}$% of \$964	**16.**	140% of \$9111
9.	60% of 480	**13.**	7$\frac{1}{2}$% of \$838	**17.**	250% of \$6450

Find results in the following to the nearest cent:

18.	33% of \$3573.90	**21.**	87$\frac{1}{2}$% of \$2756.21
19.	75% of \$8926.13	**22.**	130% of \$9461.07
20.	19% of \$1456.80	**23.**	325% of \$6379.99

Tell what per cent the first number is of the second:

24.	50; 250	**27.**	48; 320	**30.**	697; 820	**33.**	980; 700
25.	36; 120	**28.**	51; 300	**31.**	330; 330	**34.**	901; 530
26.	55; 220	**29.**	98; 200	**32.**	156; 650	**35.**	969; 950

Tell what numbers should be put in place of the dots:

36.	27 is 3% of . . .	**40.**	45% of . . . is 495
37.	57 is 19% of . . .	**41.**	32% of . . . is 608
38.	44 is 22% of . . .	**42.**	24% of . . . is 996
39.	85 is 17% of . . .	**43.**	69% of . . . is 1587

CHAPTER IV

HOW TO SOLVE HARD PROBLEMS

A dealer buys 28 doz. oranges at 24¢ a dozen. He sells 24 doz. of them at 35¢ a dozen. The rest of them spoil. Allowing 15% of the selling price for overhead expenses, what per cent of the selling price is his profit?

To solve a problem like this, which requires careful thinking, it is helpful to ask yourself these six questions:

1. *Do I understand every word and phrase in the problem?* What do "dealer," "overhead expenses," and "profit" mean?

2. *What am I asked to find?* You are here asked to find the per cent of profit on the selling price.

3. *What facts are given that will help me to find the answer?* There are five such facts in the above problem. What are they?

4. *By what steps can I find the answer?* For this problem, use the following steps:

(1) Find the total cost. $28 \times 24¢ = \$6.72$.
(2) Find the total selling price. $24 \times 35¢ = \$8.40$.
(3) Find the overhead expenses. 15% of \$8.40 = \$1.26.
(4) Find the profit. $\$8.40 - (\$6.72 + \$1.26) = \$.42$.
(5) Find the per cent of profit on the selling price. \$.42 is 5% of \$8.40. Hence the answer is 5%.

5. *Have I found what was asked for?* Read the problem again to make sure that you have answered the question asked.

6. *Is my work right?* Go over your work to see if your thinking has been correct and if you have used the right steps. Then check your computations. Whenever possible, estimate the answer roughly to see if your result is sensible.

HARDER PROBLEMS

1. Mrs. Lane buys $4\frac{3}{4}$ yd. of brown serge at \$1.48 a yard, $2\frac{7}{8}$ yd. of black satin at \$2.08 a yard, and a remnant of blue satin, containing $2\frac{1}{2}$ yd., for \$3.45. She gives the clerk a 10-dollar bill and two 5-dollar bills. Make change for Mrs. Lane, telling how the clerk counts it out.

2. Last week Ann worked from 3.30 P.M. to 5 P.M. on Monday, from 3.30 P.M. to 5.45 P.M. on Tuesday, from 3.45 P.M. to 5.45 P.M. on Friday, and from 1.15 P.M. to 6 P.M. on Saturday. At 32¢ an hour, find her earnings for the week.

3. A taxicab charges 15¢ for the first $\frac{1}{4}$ mi. and 5¢ for each additional $\frac{1}{4}$ mi. How much must George pay to ride $3\frac{1}{2}$ mi. with his trunk, if the trunk costs 25¢ extra? to ride $2\frac{3}{4}$ mi. without the trunk?

4. Mr. Roberts is driving from his home to Chicago, a distance of 210 mi. When he starts the trip, at 8.15 A.M., his speedometer reads 1506.5 mi. At 12.45 P.M., when he stops for lunch, it reads 1632.5 mi. If he allows 1 hr. for lunch and then continues his trip at the same average rate as in the morning, at what time will he reach Chicago?

5. During the past 5 yr. the population of Seaport has increased from 15,400 inhabitants to 21,560. If the per cent of increase over the present population during the next 5 yr. is the same as the per cent of increase during the past 5 yr., what will be the population of Seaport 5 yr. from now?

6. Mr. Green, a retail fruit dealer, bought 4 bu. of apples and Mr. Norr, another retail dealer, bought 3 bu. of apples. They both paid \$3 a bushel. Mr. Green charged 15¢ a quart and sold all his apples. Mr. Norr charged 17¢ a quart and sold 2 bu. and half of the third bushel; the rest of his apples spoiled. If their overhead expenses were equal, which man's profit was greater?

TELLING HOW TO SOLVE PROBLEMS

1. William sold subscriptions last month for a weekly magazine. He kept a record of the number of subscriptions he sold and knows the price of a subscription and his rate of commission. How can he find how much he should receive?

2. Lena recorded her time in the 100-yard dash on 6 different days last week. How can she find her average time?

3. Mother knows how much money she had on hand at the beginning of this week and how much she received during the week. She also has written down each of her expenditures. How can she find how much money she should have on hand now without counting the money? Why does she also count it?

4. Mr. Gross buys a radio on the installment plan. He pays a certain amount down and the rest in monthly installments. What three facts must he know, and then what must he do, to find how many payments he will have to make?

5. Mr. and Mrs. Horn are driving to the seashore and back. They know how many miles per hour they intend to drive and how long they will stay at the seashore. What else must they know and what must they do to determine in how many hours they will arrive home again?

6. How can you find the total cost of buying a new arithmetic, a new reader, a new history, and a new geography for every pupil in your class?

7. If you know the weight of each boy on each of two baseball teams, how can you tell how much greater the average weight of the boys on one team is than the average weight of the boys on the other team?

8. Edna earned Christmas money by making fudge. She knows how much the fudge cost, how much the boxes cost, how many boxes she sold, and how much she got per box. How can she find the amount of her profit?

UNFINISHED PROBLEMS

In each problem, one or more necessary facts have been omitted. Supply the missing facts and work the problems:

1. Marion buys 3 tickets to the Winter Festival. How much change should she receive from $5?

You must supply the cost of 1 ticket before you can solve the problem.

2. Edwin received a discount of 6% on the regular price of a football and a discount of 15% on a camera. Find the total amount he paid for both articles.

3. Mr. Brown could buy a 25-trip ticket to Newtown for $16.50. How much cheaper would that be per trip than buying a separate ticket each time?

4. At the end of a 6-hour trip, the speedometer of our car read 3437.1 mi. What was our average rate per hour?

5. A club is making pineapple lemonade for a party. The recipe calls for $\frac{1}{2}$ lb. sugar, $1\frac{1}{2}$ qt. water, 1 can grated pineapple, and 3 lemons. If 3 times the amount of the recipe is made, find the cost of the lemonade.

Finish each of the following problems by asking a question that will use all the given facts. Then answer your question:

6. Mrs. Short's gas meter read 46,400 cu. ft. on Jan. 1 and 48,100 cu. ft. on Feb. 1. She pays $1.15 per M for gas.

7. The cost of sending mail by air is 5¢ for the first ounce and 10¢ for each additional ounce or fraction thereof. Tom sends one letter which weighs $1\frac{1}{2}$ oz. and another which weighs 4 oz.

8. Paul, Henry, and Donald received $5 in all for helping Mr. Blake. Paul worked 6 hr. in all, Henry worked 9 hr., and Donald worked 5 hr.

9. The train on which Grace was riding left Belville at 1.40 P.M. It was due at Hawley at 5.25 P.M., but was half an hour late. The distance between the two towns is 102 mi.

IMPROVEMENT TEST No. 17

Test 17 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
7853	4872	1746	5639	1650
2813	7561	9641	2782	4766
2286	8182	6776	3231	5847
3494	5853	3190	6394	7164
8551	1996	7591	1726	3612
9528	4308	9728	4507	2588
9206	9499	2837	8713	4535
4868	2628	1488	7867	9607

Test 17 B — Subtraction (Time 4 min.)

1. 81540 − 47262	**2.** 71547 − 46448	**3.** 67451 − 37561	**4.** 90138 − 30178
5. 55734 − 36249	**6.** 66216 − 16880	**7.** 45052 − 20384	**8.** 86518 − 58653
9. 86200 − 42569	**10.** 85263 − 67671	**11.** 93730 − 45856	**12.** 83162 − 13945
13. 70241 − 14303	**14.** 64641 − 12774	**15.** 73219 − 57768	**16.** 98307 − 58881

Test 17 C — Multiplication (Time 4 min. after copying)

1. 35172 × 4726	**2.** 27680 × 90183	**3.** 90568 × 4387

Test 17 D — Division (Time 4 min. after copying)

If there is a remainder, carry result to the nearest tenth:

1. $67\overline{)41563}$ **2.** $52\overline{)30030}$ **3.** $49\overline{)39935}$

4. $75\overline{)49225}$ **5.** $84\overline{)16492}$

PROBLEMS FOR A GOOD THINKER

1. An elevator is licensed to carry a load of not more than 1250 lb. If the operator weighs 165 lb., can the elevator carry eight passengers whose weights are 161 lb., 113 lb., 191 lb., 170 lb., 142 lb., 187 lb., 149 lb., and 176 lb.? How many of the passengers should get off?

2. Mildred is a clerk in a department store. She gets $15 a week salary. She also gets $2\frac{1}{2}\%$ commission on the amount by which her weekly sales exceed $300. Last week her daily sales amounted to $35.27, $61.50, $49.75, $88.65, $92.47, and $168.36. How much did Mildred earn last week?

3. Mrs. Mack orders by mail articles whose shipping weights are $5\frac{1}{2}$ lb., $3\frac{3}{4}$ lb., 14 oz., $9\frac{1}{4}$ lb., and 6 oz. To have the package sent by parcel post, she must pay 8¢ for the first pound and 4¢ for each additional pound or fraction of a pound. How much must Mrs. Mack pay for postage?

4. Jane sets her watch exactly right at 5.30 P.M. Sunday. At 6 P.M. Monday, finding that it is 14 min. fast, she sets it right again. If it continues to run fast at the same rate, what time will Jane's watch indicate at 8 A.M. Tuesday?

5. Edward can walk 3 mi. in 1 hr. On his bicycle he can go 11 mi. in 1 hr. How long would it take him to walk to Dunham if he can ride there on his bicycle in 1 hr. 12 min.?

6. Mr. Briggs, who had been paying $60 a month rent, bought a 3-family house for $21,500. During the first year he occupied one of the apartments himself; another of the apartments was rented all year at $55 a month; while the third apartment was vacant during March and April and was rented the rest of the year at $58 a month. During the year Mr. Briggs paid $151.50 for coal, $67.50 for repairs, $518 for taxes, and $54 for fire insurance; he lost $1290 in interest which his $21,500 otherwise would have earned. Find how much Mr. Briggs saved or lost in a year by buying the house.

INTEREST

How Much Interest Does Fred Pay?

Fred Collins wants to open a news stand. To help him start the business, Mr. Fuller lends him \$80 *at* 6% *interest.* This means that Fred must pay 6% of \$80, or \$4.80, for the use of the money for 1 year. He pays for the use of the money just as he would pay rent for a house.

The \$80 which Fred borrows is called the **principal;** the \$4.80 which he pays for the use of it is the **interest.** The **rate of interest** is 6%; this is the per cent of the principal charged for 1 year's use of the money. If Fred kept the money for 2 yr., he would pay twice as much interest.

At the end of the year Fred pays Mr. Fuller the \$80 which he borrowed, plus \$4.80 interest, or \$84.80 in all. \$84.80 is called the **amount.** The amount at the end of any period is the sum of the principal and of the interest then due.

Exercises

Find the interest for 1 *yr. on the following:*

1. \$600 at 4%	**5.** \$500 at 6%	**9.** \$2000 at 5%
2. \$700 at 5%	**6.** \$900 at 5%	**10.** \$4000 at 4%
3. \$450 at 6%	**7.** \$660 at 4%	**11.** \$8500 at 6%
4. \$880 at 7%	**8.** \$300 at 5%	**12.** \$6000 at $4\frac{1}{2}$%

Find the interest for 1 *yr., correct to the nearest cent:*

13. \$260 at 3%	**18.** \$280 at 6%	**23.** \$1100 at 5%
14. \$395 at 6%	**19.** \$227 at 5%	**24.** \$3700 at 6%
15. \$950 at $4\frac{1}{2}$%	**20.** \$485 at $4\frac{1}{2}$%	**25.** \$4550 at 7%
16. \$475 at $5\frac{1}{2}$%	**21.** \$530 at $3\frac{1}{2}$%	**26.** \$6050 at 6%
17. \$940 at $4\frac{1}{2}$%	**22.** \$125 at $6\frac{1}{2}$%	**27.** \$2505 at $4\frac{1}{2}$%

FINDING THE INTEREST FOR SEVERAL YEARS

Walter borrowed \$300 from Mr. Fletcher for 3 yr. at 5% interest. How much interest did he pay in all for 3 years' use of the money?

Since the interest for 1 yr. is 5% of \$300, or \$15, the interest for 3 yr. is 3×\$15, or \$45 in all.

Check the work by going over it again.

\$300
.05
\$15.00 Int. for 1 yr.
3
\$45.00 Int. for 3 yr.

When money is borrowed for longer than a year, the interest is usually paid at the end of each year or of each 6 months.

Exercises

Find the total interest for 2 yr. Check the work:

1. \$450 at 6%
2. \$280 at 4%
3. \$495 at 5%
4. \$256 at 5%
5. \$785 at 6%
6. \$460 at 7%
7. \$3108 at 5%
8. \$2650 at 6%
9. \$1500 at $4\frac{1}{2}$%

Find the total interest for 3 yr. Check the work:

10. \$350 at 6%
11. \$485 at 4%
12. \$633 at 3%
13. \$475 at 6%
14. \$950 at 5%
15. \$388 at 3%
16. \$2800 at 7%
17. \$2900 at $5\frac{1}{2}$%
18. \$8508 at $4\frac{1}{2}$%

Find the total interest. Check the work:

19. \$985 for 3 yr. at 4%
20. \$850 for 4 yr. at 5%
21. \$269 for 3 yr. at 3%
22. \$767 for 2 yr. at $5\frac{1}{2}$%
23. \$2500 for 2 yr. at 6%
24. \$1275 for 2 yr. at $4\frac{1}{2}$%

25. Doris lent her brother Tom \$500, at 6% interest, to help him go to college. After 3 yr. Tom paid the money back with all the interest for the 3 yr. How much did he pay Doris?

26. Amy borrowed \$250, at 6%, so that she could take a business course. She paid back \$100 after 1 yr. and the rest 1 yr. later. How much interest did she pay in all?

FINDING THE INTEREST FOR MONTHS

1. Mrs. Bell wants to borrow \$500 for 6 mo. How much interest must she pay if the rate of interest is 6%?

First find the interest for 1 yr., which is 6% of \$500, or \$30. The interest for 6 mo., or $\frac{1}{2}$ of a year, is $\frac{1}{2}$ of \$30, which is \$15.

2. Mr. Case borrows \$210 at 5%. How much interest does he pay if he keeps the money 11 mo.?

The interest for 1 yr. is 5% of \$210, or \$10.50. Since 11 mo. = $\frac{11}{12}$ yr., the interest for 11 mo. is $\frac{11}{12}$ of \$10.50, or \$9.63.

You can indicate the entire solution for this problem in one step, as shown at the right, remembering that $5\% = \frac{5}{100}$. Cancel whenever possible. The result, to the nearest cent, is \$9.63. Check the work by going over it again.

$$\frac{\cancel{210}^{\,7}}{1} \times \frac{1}{\cancel{100}_{\,\cancel{10}_{\,2}}} \times \frac{11}{\cancel{12}_{\,4}} = \frac{77}{8} = 9.62\frac{1}{2}$$

Interest = Principal × Rate × Time.

In applying the above rule, express the time as years and parts of a year.

Exercises

Find the interest, to the nearest cent, on the following:

1. \$400 for 4 mo. at 6%
2. \$275 for 6 mo. at 5%
3. \$360 for 9 mo. at 4%
4. \$500 for 3 mo. at 7%
5. \$840 for 11 mo. at 6%
6. \$800 for 10 mo. at 5%
7. \$4575 for 6 mo. at 5%
8. \$4800 for 5 mo. at 6%
9. \$3930 for 4 mo. at 5%
10. \$6000 for 7 mo. at 4%
11. \$1500 for 2 mo. at 4%
12. \$400 for 2 yr. 6 mo. at 6%
13. \$850 for 1 yr. 3 mo. at 5%
14. \$600 for 2 yr. 4 mo. at 4%
15. \$910 for 1 yr. 6 mo. at 6%
16. \$775 for 2 yr. 6 mo. at 5%
17. \$625 for 2 yr. 2 mo. at 6%
18. \$175 for 1 yr. 8 mo. at 5%
19. \$600 for 2 yr. 5 mo. at 4%
20. \$785 for 1 yr. 9 mo. at 6%
21. \$350 for 3 yr. 6 mo. at 5%
22. \$155 for 1 yr. 6 mo. at 5%

COMPUTING INTEREST FOR MONTHS AND DAYS

1. Mr. Bates borrows \$575.75 at 6% interest. If he keeps the money only 19 days, how much interest does he pay?

In computing interest, a month is usually counted as 30 da., and a year as 12×30 da., or 360 da. Hence 19 da. $= \frac{19}{360}$ yr.

To find how much interest Mr. Bates pays, find the interest for 1 yr. and multiply the result by $\frac{19}{360}$. A shorter way is to cancel, as shown above.

$$\frac{\cancel{575.75}^{5.7575}}{1} \times \frac{\cancel{6}^{1}}{\cancel{100}_{1}} \times \frac{19}{\cancel{360}_{60}} = \frac{109.3925}{60} = 1.823$$

Mr. Bates pays \$1.82 in interest. Check the work by going over it again.

2. What is the interest on \$650 for 3 mo. 11 da. at 5%?

If 1 mo. = 30 da., then 3 mo. 11 da. = 101 da., or $\frac{101}{360}$ yr. Find the interest by cancellation, as shown above.

Exercises

Find the interest, using these principals, times, and rates:

1. \$400; 17 da.; 5%
2. \$500; 36 da.; 6%
3. \$840; 73 da.; 4%
4. \$760; 82 da.; 6%
5. \$800; 69 da.; 5%
6. \$775; 53 da.; 6%
7. \$540; 22 da.; 4%
8. \$720; 42 da.; 7%
9. \$2500; 317 da.; 6%
10. \$550; 7 mo. 9 da.; 6%
11. \$500; 4 mo. 16 da.; 6%
12. \$540; 8 mo. 25 da.; 5%
13. \$438; 3 mo. 27 da.; 5%
14. \$1250; 9 mo. 22 da.; 4%
15. \$2450; 9 mo. 23 da.; 6%
16. \$5430; 8 mo. 17 da.; 4%
17. \$120.60; 1 yr. 7 mo. 18 da.; 6%
18. \$540.60; 2 yr. 3 mo. 15 da.; 5%

19. Joseph Daniels had saved \$125. Instead of keeping the money at home, he wanted to make it earn something for him. He therefore lent it to Mr. Burt for 7 mo. 20 da. at 6%. How much did Joseph's money earn for him in that time?

FINDING THE TIME BETWEEN DATES

Mr. Mead borrowed \$750 on May 20, 1925 at 6%. Because of illness he did not pay any interest until Feb. 15, 1928, when he paid all the interest due to that date. What was the time between these two dates? How much interest did he pay?

Counting 30 da. to the month, you can find the time between May 20, 1925 and Feb. 15, 1928 in this way:

Since you cannot subtract 20 da. from 15 da., think of 2 mo. 15 da. as changed to 1 mo. 45 da. Then 45 da. − 20 da. = 25 da. Similarly, since you cannot subtract 5 mo. from 1 mo., think of 1928 yr. 1 mo. as changed to 1927 yr. 13 mo. Then 13 mo. − 5 mo. = 8 mo. and 1927 yr. − 1925 yr. = 2 yr. Hence the time between the given dates is 2 yr. 8 mo. 25 da.

yr.	*mo.*	*da.*
1928	**2**	**15**
1925	**5**	**20**
2	**8**	**25**

The interest on \$750 at 6% for 2 yr. is \$90. For 8 mo. 25 da. it is \$33.13. In all Mr. Mead paid \$90 + \$33.13, or \$123.13 interest.

Exercises

1. Mr. Sullivan borrowed \$5000 on Jan. 2, 1926 at 6%. He repaid the money and all the interest on July 16, 1928. What was the total amount he paid?

Find the interest on:

2. \$450 from May 3, 1926 to Sept. 4, 1928 at 6%.

3. \$795 from Feb. 20, 1926 to June 2, 1927 at 6%.

4. \$648 from Dec. 19, 1925 to Jan. 30, 1928 at 7%.

5. George Washington was born Feb. 22, 1732 and died Dec. 14, 1799. Find, by the method given above, how many years, months, and days old he was when he died.

NOTE. Interest seldom remains unpaid for much over 1 yr. When money is borrowed for longer than 1 yr., the interest is usually paid every 6 mo. or every year. When money is borrowed from a bank, the time is usually found by counting the exact number of days between dates, as explained on page 340, instead of subtracting the dates as shown above.

LOST INTEREST AS AN ITEM OF EXPENSE

Mr. Earl bought an automobile for \$1200. He kept it just 1 yr. and then sold it for \$900. During the year he paid a total of \$350 for garage, gasoline, etc. How much did the year's use of the car cost him?

Fred said that the car cost Mr. Earl \$650 in all because he lost \$300 when he sold the car and had expenses of \$350 besides.

Elizabeth said that if Mr. Earl had not bought the car, the \$1200 which he paid for it would have earned interest. Counting interest at 6%, she said that the car cost Mr. Earl \$72 more than \$650, or \$722 in all.

Business men look at this problem as Elizabeth did.

When money is invested in a house, a radio set, or anything else, the lost interest counts as one item of expense.

Exercises

In the following, consider that the money could earn 6%.

1. Mr. Adams bought a car for \$1800 and sold it 2 yr. later for \$1200. In the 2 yr. he paid out \$780 on the car. How much per year did his car actually cost him?

2. Mr. Green bought a lot for \$1000. He sold it 3 yr. later for \$1300, having paid \$25 taxes on it each year. Find his profit, counting lost interest as an expense.

3. Suppose that Mr. Green (ex. 2) had sold the lot for \$900. What would have been his total loss?

4. Mr. Allen bought a house for \$12,000. When he moved in, he said, "Now we don't have to pay any rent unless we count the taxes and repairs, and they will only be about \$450 a year." Was Mr. Allen right? Though he owned the house, what yearly rent did he really pay for it?

5. A miser kept his savings of \$2500 buried in a secret place. How much per year did this peculiarity cost him?

Idle money is an expense. Put it to work to earn interest.

FINDING EXACT INTEREST

In transactions between individuals, it is usual in computing interest to count 30 da. as 1 mo. and 360 da. as 1 yr. The United States government finds interest on the basis of 365 da. to the year, counting the exact number of days between dates. This is **exact interest.** Many banks in paying interest to depositors use 365 da. to the year, but in charging interest on loans, they consider 360 da. as a year.

Find the exact interest on \$3500 for 27 da. at 6%.

Counting 365 da. to the year, 27 da. $= \frac{27}{365}$ yr. The computation is made as shown at the right.

$$\frac{\cancel{3500}^{\,7}}{1} \times \frac{6}{\cancel{100}\,1} \times \frac{27}{\cancel{365}\,73} = \frac{1134}{73} = 15.53$$

The interest is less if you count 365 da. to the year than if you count 360 da. to the year, because $\frac{27}{365}$ yr. is less than $\frac{27}{360}$ yr.

Exercises

Find, to the nearest cent, the exact interest on:

1. \$498 for 73 da. at 6%
2. \$750 for 31 da. at 6%
3. \$898 for 82 da. at 4%
4. \$375 for 90 da. at 5%
5. \$508 for 30 da. at 7%
6. \$757 for 73 da. at 6%
7. \$710 for 12 da. at 5%
8. \$250 for 138 da. at 7%
9. \$475 for 290 da. at 5%
10. \$479 for 212 da. at 5%
11. \$379 for 327 da. at 7%
12. \$470 for 296 da. at 5%
13. \$560 for 193 da. at 6%
14. \$880 for 135 da. at 6%

Find the interest at 6% on the following amounts, first counting 365 da. to 1 yr. and then counting 360 da. to 1 yr. Which method gives more interest, and how much more?

15. \$2000 for 184 da.
16. \$3500 for 201 da.
17. \$4200 for 255 da.
18. \$87,000 for 85 da.
19. \$98,500 for 63 da.
20. \$125,000 for 180 da.

PAYING INTEREST SEMIANNUALLY

When people borrow money, they may agree to pay the interest *semiannually*. The interest is then paid at the end of each 6 mo. instead of at the end of the year. In computing semiannual interest, remember that the rate of interest always means the rate for a whole year.

Mr. Stone borrows \$750 at 6% and agrees to pay the interest semiannually. How much interest does he pay semiannually?

\$750
.06
2)\$45.00
\$22.50

You may first find the interest for 1 yr., which is 6% of \$750, or \$45. From this find the interest for 6 mo., which is $\frac{1}{2}$ of \$45, or \$22.50.

Or
\$750
.03
\$22.50

A shorter way of finding Mr. Stone's semiannual interest is as follows: Since at 6% *for a year, the interest for a half year* is 3%, find 3% of \$750, which is \$22.50, the same result as was obtained above.

Most savings banks now pay interest either semiannually or *quarterly* (at the end of each 3 mo.).

Exercises

1. Mr. Thomas put \$250 in the bank for his daughter Jane. The bank allows 4% interest, payable semiannually. How much interest is due Jane at the end of the first 6 mo.?

2. Mrs. Adams borrowed \$2400 at 6% to make some improvements on her house. If she pays the interest semiannually, how much does she pay every 6 mo.?

Find the semiannual interest on the following amounts:

3. \$180 at 6%	**8.** \$900 at 5%	**13.** \$75 at 6%
4. \$250 at 5%	**9.** \$470 at 6%	**14.** \$50 at 5%
5. \$640 at 7%	**10.** \$350 at 4%	**15.** \$60 at 4%
6. \$880 at 4%	**11.** \$580 at 6%	**16.** \$95 at 7%
7. \$375 at 6%	**12.** \$155 at 7%	**17.** \$25 at 6%

COMPUTING INTEREST BY THE 6% METHOD

You will now learn a short way to compute interest, known as the 6%, or 60-day, method. In this method 360 days are counted as 1 year; hence 60 days equal $\frac{1}{6}$ of a year.

Find the interest on \$500 at 6% for 60 da.; for 6 da.

Since the interest on any sum for 1 yr. at 6% is 6% of the principal, the interest for 60 da. is $\frac{1}{6}$ of 6%, or 1%, of the principal. Hence the interest on \$500 at 6% for 60 da. is 1%, or $\frac{1}{100}$, of \$500, which is \$5.

The interest for 6 da. is $\frac{1}{10}$ of the interest for 60 da.; that is, $\frac{1}{10}$ of 1%, or $\frac{1}{1000}$ of the principal. Hence the interest on \$500 at 6% for 6 da. is $\frac{1}{1000}$ of \$500, or \$.50.

To find the interest at 6% for 60 da., move the decimal point in the principal two places to the left; to find the interest for 6 da., move the decimal point three places to the left.

For other periods of time, find the interest as follows:

For 30 da., find the interest for 60 da. and take $\frac{1}{2}$ of it.
For 15 da., find the interest for 60 da. and take $\frac{1}{4}$ of it.
For 90 da., add the interest for 60 da. to that for 30 da.
For 75 da., add the interest for 60 da. to that for 15 da.
For 12 da., find the interest for 6 da. and double it.
For 1 da., find the interest for 6 da. and take $\frac{1}{6}$ of it.

Exercises

1. Find the interest on \$950 at 6% for 96 da.; for 78 da.

\$9.50 Int. for **60** da.	**\$9.50** Int. for **60** da.
4.75 Int. for **30** da.	**2.38** Int. for **15** da.
.95 Int. for **6** da.	**.48** Int. for **3** da.
\$15.20 Int. for **96** da.	**\$12.36** Int. for **78** da.

By the 6% method, find the interest at 6% on:

2. \$360 for 60 da.; for 30 da.; for 15 da.; for 6 da.

3. \$600 for 60 da.; for 30 da.; for 75 da.; for 18 da.

4. \$480 for 60 da.; for 120 da.; for 90 da.; for 12 da.

USING THE 6% METHOD

By the 6% method, find the interest at 6% on the following. Count 30 da. to 1 mo. Check the work by going over it:

1. \$550 for 36 da.
2. \$800 for 50 da.
3. \$375 for 31 da.
4. \$950 for 78 da.
5. \$200 for 123 da.
6. \$650 for 154 da.
7. \$550 for 2 mo.
8. \$2500 for 3 mo.
9. \$1150 for 5 mo.
10. \$570 for 100 da.
11. \$2650 for 3 mo. 6 da.
12. \$1100 for 2 mo. 10 da.

13. Find by the 6% method the interest on \$480 for 66 da. at 7%; at 5%.

The interest at 6% is \$5.28. Hence the interest at 1% is $\frac{1}{6}$ of \$5.28, or \$.88. Adding \$.88 to \$5.28, you get \$6.16 as the interest at 7% for 66 da.

The interest at 5% for 66 da. is found by subtracting \$.88 from \$5.28, which gives \$4.40.

\$4.80
.48
6)\$5.28 Int. at 6%
\$ .88 Int. at 1%
\$6.16 Int. at 7%

Find by the 6% method the interest on the following:

14. \$280 for 30 da. at 5%
15. \$700 for 45 da. at 7%
16. \$450 for 90 da. at 5%
17. \$360 for 72 da. at 5%
18. \$720 for 20 da. at 7%
19. \$900 for 75 da. at 5%
20. \$850 for 42 da. at 7%
21. \$2800 for 15 da. at 5%

22. Mr. Gorse had a tax bill for \$198.76 which was due Dec. 1, 1927. He paid the bill Feb. 16, 1928, and was charged 7% interest for the time it was overdue. Find by the 6% method how much interest he had to pay.

With interest at 7%, find by the 6% method the extra charge on each of these tax bills when it was finally paid:

23. \$185.76; 66 da. late
24. \$215.20; 74 da. late
25. \$184.50; 75 da. late
26. \$673.15; 96 da. late

THE COST OF OPERATING AN AUTOMOBILE

1. Mr. Scott bought an automobile for $924. He kept it for 2 yr. and then sold it for $450. The difference between $924 and $450 shows what the car lost in value because of wear and is called *depreciation*. How much did Mr. Scott lose through depreciation?

2. Mr. Scott had these expenses for his car: taxes for 2 yr., $25.71; insurance for 2 yr., $65.00; license fees each year, $10 for car and $3 for driver; fines, $15; new tire and tube, $35.45; incidentals, $37.50; garage, $8.50 monthly; alcohol for radiator, $6; repairs, washing car, etc., $96.50. How much did all these items cost for 2 yr.?

3. Mr. Scott paid $190.30 for 865 gal. of gasoline and $28.44 for 158 qt. of oil. He also paid $6 for battery service. How much did all these items cost?

4. He lost 6% interest each year on the $924 which he invested in the car. How much interest did he lose in 2 yr.?

5. The total sum of the loss through depreciation (ex. 1), the expenditures for the car (ex. 2 and 3), and the loss of interest (ex. 4) makes up the cost of operating a car. Find the total operating cost of Mr. Scott's car for 2 yr.

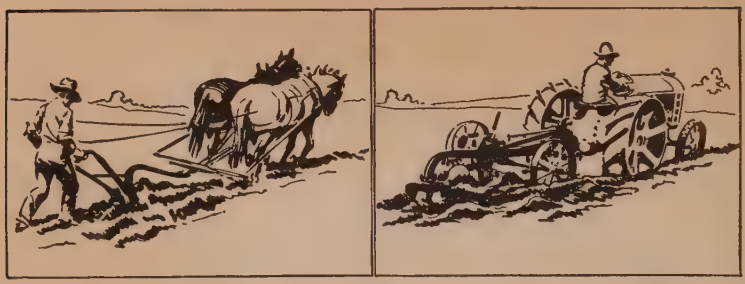
6. Mr. Scott drove his car a total of 15,743 mi. Find, to the nearest tenth of a cent, how much it cost him per mile to operate the car. Use the total operating cost found in ex. 5.

7. The railroad fare is 3.6¢ per mile. Compare this with Mr. Scott's expense per mile for operating the car.

8. When Mr. Scott and 5 friends travel in his car, how much is their total saving per mile on railroad fare? How much is their total saving on a 65-mile trip?

9. How much does each of the 6 men in ex. 8 save on a 65-mile trip if they travel in Mr. Scott's car instead of using the railroad and share the operating expense equally?

FOR PLOWING — HORSES OR A TRACTOR?



Mr. Pearson has a 50-acre farm in New Jersey. Formerly he used horses for plowing and sowing. Now he uses a tractor.

1. His horses were valued at \$185 each. By investing this money in horses he lost 5% interest yearly on the value of each. He also lost 8% yearly of the value of each through the depreciation of the horses. How much per horse did he lose in interest and depreciation every year?

2. The yearly expenses for feeding and keeping each horse were: (1) $3\frac{1}{4}$ tons of hay at \$16 a ton; (2) 56 bu. of oats at \$.48 a bushel; (3) 52 bu. of corn at \$.68 a bushel; and (4) shelter, care, shoeing, etc., \$54 a year. Find the total yearly expenses of feeding and keeping each horse.

Add to these expenses the loss found in ex. 1, to find the total yearly cost per horse.

3. Each of the horses averaged 960 hr. of work yearly. Find the hourly cost for the labor of a horse.

Use total yearly cost per horse found in ex. 2.

4. A man whom Mr. Pearson hired plowed $1\frac{4}{5}$ acres with a 2-horse team in a day of 10 hr. Using the cost per hour found in ex. 3, find how much the labor of the horses cost for the day.

5. The man whom Mr. Pearson hired (see ex. 4) was paid \$2.50 a day. What was the total cost of the day's plowing? What was the cost per acre?

6. Last year Mr. Pearson sold his horses and bought an all-purpose tractor outfit for \$1750. He allows 12% yearly on the cost of the tractor for depreciation, 5% yearly for lost interest, and \$32 a year for shelter. These three items make up what is called the *machine cost* of the tractor. How much is the total machine cost of the tractor for 1 yr.?

7. The tractor uses in a year 924 gal. of fuel costing 18¢ a gallon and 74 gal. of lubricating oil at 68¢ a gallon. The repairs each year average 6% of the original price. The total of these items gives the *operating cost* of the tractor. Find the operating cost for 1 yr.

8. Find from ex. 6 and 7 the total yearly cost of maintaining and operating the tractor.

9. The farmer uses his tractor 696 hr. during the year for all kinds of work. Using the total cost obtained in ex. 8, find the cost of the tractor for an hour's work.

10. Mr. Pearson hires a man who plows $7\frac{1}{2}$ acres with the tractor in a 10-hour working day. What is the machine and operating cost of the tractor for the day?

11. The man who runs the tractor receives \$3.75 as his day's wages. What is the total cost of plowing the $7\frac{1}{2}$ acres? What is the cost per acre of plowing with the tractor?

12. Compare the cost of plowing an acre with horses (ex. 5) with the cost of plowing an acre with a tractor (ex. 11). How much does Mr. Pearson save per acre by plowing with a tractor instead of with horses?

13. For what other work besides plowing can a tractor be used? Which is less trouble to keep, horses or a tractor? Which can be used for longer hours at a time? Which can finish the work faster if a storm is approaching?

14. Name the advantages of having a tractor on a large farm. Does it pay to keep a tractor on a very small farm?

COMPOUND INTEREST

Joe Monroe won $500 for an essay. He put it in a savings bank at 4% interest. The bank paid interest semiannually, on Jan. 1 and July 1.

Joe put his $500 in the bank on Jan. 1, 1926. On July 1, $10 interest for the first 6 mo. was due him. How was this interest computed? Joe was told that if he left the $10 interest in the bank, it would be added to his principal, so that he would have $510 earning interest for the next 6 mo. Joe did this and on Jan. 1, 1927, he got 6 mo. interest on $510, which was $10.20.

Joe also left this $10.20 in the bank, making his principal $520.20. The next interest, due July 1, 1927, was figured on $520, since *savings banks do not pay interest on parts of a dollar.*

$500.00	**First Principal**
10.00	**Interest for 6 mo.**
510.00	**Prin., July 1, 1926**
10.20	**Int. for 6 mo.**
520.20	**Prin., Jan. 1, 1927**
10.40	**Int. for 6 mo.**
530.60	**Prin., July 1, 1927**
10.60	**Int. for 6 mo.**
541.20	**Prin., Jan. 1, 1928**
10.82	**Int. for 6 mo.**
552.02	**Prin., July 1, 1928**
11.04	**Int. for 6 mo.**
$563.06	**Prin., Jan. 1, 1929**

From the work above, tell how much Joe would have in all in the bank by Jan. 1, 1929. How much in all will his $500 earn during these three years?

By leaving his interest in the bank, Joe received *interest on interest* as well as on the original principal. This is called **compound interest.** If Joe had taken his interest out of the bank each 6 mo., he would have received **simple interest.** In compound interest, the principal always increases; in simple interest, it is always the same.

Joe's interest was compounded semiannually. If compound interest is credited every 3 mo., it is said to be compounded quarterly. Many savings banks now credit interest quarterly.

DOES MONEY GROW FASTER AT COMPOUND OR AT SIMPLE INTEREST?

1. If Joe received only simple interest at 4% on his $500, the bank would pay him $10 every 6 mo. How much simple interest would the $500 earn at 4% in 2 yr.? in 3 yr.?

2. Does $500 earn more in 3 yr. at 4% simple interest or at 4% compounded semiannually (see page 256)? how much more?

3. Continue the work on page 256 up to Jan. 1, 1931, and show that the new principal at that time is $609.44. At 4% compound interest, how much does the $500 earn in 5 yr.?

4. How much does $500 earn in 5 yr. at 4% simple interest? How much more is the compound interest for 5 yr. (see ex. 3)?

5. If the work on page 256 were continued up to 10 yr., 15 yr., etc., it would show that $500 at 4% interest compounded semiannually would grow to the amounts given in Column *A* below. Column *B* shows the amounts to which $500 would grow at 4% simple interest. Column *C* shows the advantage of compound interest over simple interest.

	A *At Compound Interest*	*B* *At Simple Interest*	*C* *Advantage of Compound Int.*
In 3 yr. $500 grows to	$ 563.06	$560	$3.06
In 5 yr. $500 grows to	609.44	600	9.44
In 10 yr. $500 grows to	742.78		
In 15 yr. $500 grows to	905.34		
In $17\frac{1}{2}$ yr. $500 grows to	999.50		
In 20 yr. $500 grows to	1103.46		
In 25 yr. $500 grows to	1345.00		
In 35 yr. $500 grows to	1999.78		
In 50 yr. $500 grows to	3622.32		

Complete Columns *B* and *C*, and study the results carefully.

Money earns much more at compound interest than it does at simple interest at the same rate. After a few years the advantage of compound interest is very striking.

ROY OPENS A SAVINGS ACCOUNT

When Roy Howland began depositing money in the Franklin Savings Bank, he received a *pass book*, in which a record was kept of the amounts of money he deposited and withdrew.

The first page of Roy's pass book gave this information:

1. Our present rate of interest is 4% per annum.
2. Interest is credited to depositors quarterly on Jan. 1, Apr. 1, July 1, and Oct 1. If left in the bank, it draws interest from the day on which it is credited.
3. Deposits made on the first, second, or third business day of any month draw interest from the first day of that month, if left until the end of that quarter of the year. Deposits made after the third day of the month draw interest from the first day of the next month.
4. No interest is allowed on sums drawn during a quarter.
5. Interest is not paid on fractional parts of a dollar.

This bank compounds interest quarterly; it *credits*, or adds, the interest to the depositor's total amount every 3 mo. Many savings banks now compound interest quarterly.

Exercises

Read the above rules and answer these questions:

1. On Oct. 15 Roy put $100 in the bank. He withdrew it the next Jan. 1. How much interest did he get on it?

Since the money was not deposited during the first three days of October, it drew no interest for this month. The $100 was in the bank only two complete months before the end of the quarter on Jan. 1. Hence it drew interest for 2 mo. at 4%.

2. On Apr. 1 Roy deposited $200. He withdrew it on June 28 and received no interest at all. Why not?

3. Roy deposited $100 on August 10 and withdrew it on Oct. 3. How much interest did he receive on it?

4. Is it better to draw money on Apr. 1 or on Mar. 31? Why?

5. Why is it better to deposit money on the first three days of the month than on the fourth or the fifth day?

INTEREST ON SAVINGS BANK ACCOUNTS

Here is a page from the pass book of Anna Lee. The interest regulations are like those on page 258. Interest is at 4%.

Date	Deposits	Withdrawals	Interest	Balance
Jan. 2	100			100
Jan. 19	10			110
Feb. 3	20			130
Feb. 25		25		105
Mar. 2	17			122
Mar. 15		10		112
Apr. 1			1 05	113 05

The first deposit of \$100 was made before Jan. 3. This drew interest for 3 mo. because the *balance* column, which gives the total amount in the bank on any date, shows that Anna always had at least \$100 in the bank during this quarter. The interest on \$100 for 3 mo. = \$1.

The \$10 deposited on Jan. 19 and the \$20 deposited on Feb. 3 were equivalent to a deposit of \$30 on Feb. 1. This \$30 would have drawn interest for 2 mo. if left till the end of the quarter (March 31); but \$25 of it was withdrawn on Feb. 25. Hence only \$5 drew interest for 2 mo. This interest amounted to \$.03.

Amount	*Time*	*Interest*
\$100	3 mo.	\$1.00
5	2 mo.	.03
7	1 mo.	.02
	Total	\$1.05

Show that only \$7 of the \$17 deposited on Mar. 2 drew interest for 1 mo. Compute the interest.

The total interest for the quarter = \$1 + \$.03 + \$.02, or \$1.05.

Exercise

In this account, is \$1.16 the correct interest at 4%?

Date	Deposits	Withdrawals	Interest	Balance
Jan. 2	100			100
Jan. 19	30			130
Feb. 25		10		120
Mar. 2	15			135
Mar. 15		5		130
Apr. 1			1 16	131 16

FINDING THE INTEREST ON SAVINGS ACCOUNTS

Show how interest is computed on these accounts if compounded quarterly at 4% according to the regulations on page 258:

1.

Date	Deposits	Withdrawals	Interest	Balance
July 3	15			15
July 24	7			22
Aug. 2	12			34
Aug. 14		5		29
Sept. 1	18			47
Oct. 1			0 30	47 30

2.

Date	Deposits	Withdrawals	Interest	Balance
Oct. 1	Bal. 47 30			47 30
Oct. 3	9			56 30
Oct. 20	15			71 30
Nov. 3	15			86 30
Nov. 30	28			114 30
Dec. 11		10		104 30
Jan. 1			0 82	105 12

Ex. 2 continues ex. 1. The balance of $47.30 in ex. 1 was carried forward to ex. 2; hence the interest in ex. 2 includes interest on $47 for 3 mo.

3. Here is the next quarter of the above account. Find the interest due Apr. 1 and the balance on that date:

Date	Deposits	Withdrawals	Interest	Balance
Jan. 1	Bal. 105 12			105 12
Jan. 2	20			125 12
Feb. 1	18			143 12
Feb. 14	5			148 12
Feb. 28	13			161 12
Mar. 3	15			176 12

IMPROVEMENT TEST No. 18

Test 18 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$4\frac{5}{8}$	$3\frac{2}{3}$	$8\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{5}{6}$
$2\frac{1}{4}$	$5\frac{2}{3}$	$9\frac{1}{5}$	6	$2\frac{1}{2}$
$6\frac{1}{2}$	4	$7\frac{3}{5}$	$5\frac{1}{5}$	$8\frac{1}{4}$
$3\frac{1}{8}$	$1\frac{1}{4}$	$8\frac{7}{10}$	$8\frac{2}{3}$	$2\frac{5}{12}$

Test 18 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$9\frac{7}{8}$	$6\frac{1}{4}$	8	$8\frac{1}{2}$
$8\frac{1}{2}$	$4\frac{5}{16}$	$7\frac{1}{5}$	$3\frac{1}{3}$
5.	**6.**	**7.**	**8.**
$7\frac{7}{10}$	$9\frac{1}{16}$	4	$2\frac{5}{8}$
$3\frac{1}{2}$	$5\frac{3}{16}$	$1\frac{5}{6}$	$2\frac{1}{8}$
9.	**10.**	**11.**	**12.**
$9\frac{1}{3}$	$4\frac{5}{6}$	7	$8\frac{3}{4}$
$2\frac{5}{6}$	$1\frac{3}{4}$	$5\frac{9}{10}$	$2\frac{3}{5}$

Test 18 C — Multiplication (Time 4 min. after copying)

1.	2.	3.
\$2.50	\$1.12$\frac{1}{2}$	\$1.68
$2\frac{3}{4}$	16	$21\frac{5}{8}$

4. $\frac{2}{5}$ of $\frac{5}{6}$	**5.** $11\times\frac{3}{4}$	**6.** $4\frac{1}{6}\times4\frac{2}{5}$
7. $\frac{5}{6}$ of 50	**8.** $3\frac{1}{2}\times50$	**9.** $5\frac{1}{3}\times1\frac{7}{8}$

Test 18 D — Division (Time 3 min. after copying)

1. $\frac{5}{6}\div10$	**2.** $\frac{7}{8}\div\frac{1}{16}$	**3.** $54\div3\frac{3}{5}$
4. $15\div\frac{5}{8}$	**5.** $4\frac{5}{8}\div\frac{3}{8}$	**6.** $5\frac{5}{8}\div3\frac{1}{3}$
7. $72\div\frac{2}{3}$	**8.** $6\frac{1}{8}\div7$	**9.** $8\frac{4}{5}\div3\frac{1}{7}$

UNITED STATES POSTAL SAVINGS SYSTEM

Through the Postal Savings System the United States coöperates with other savings institutions in encouraging systematic saving. If you are 10 years of age or over and have one dollar or more to invest, you may go to a post office and buy postal-savings certificates. These certificates will show how much you have deposited. You should present them at the same post office when you want your money back.

Money invested in postal savings earns simple interest at 2% for each *full* year that it remains on deposit. It earns interest for a partial year at the rate of $\frac{1}{2}$ of 1% for each *full* period of 3 months. Money withdrawn in less than 3 months receives no interest. Interest begins on the first day of the month following the month in which the money is deposited. For example, a deposit made on Mar. 16, 1931, begins to earn interest on April 1, 1931.

Postal-savings certificates come only in denominations of \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$200, and \$500. Amounts less than \$1 may be saved by purchasing postal-savings stamps at 10¢ each. A postal-savings card with 10 savings stamps affixed will be accepted as a deposit of \$1.

Exercises

1. What denomination of certificates will you get if you invest \$8 in postal savings? if you invest \$43? \$28? \$75? \$87?

2. On Dec. 27, 1930, Mary bought \$25 worth of postal-savings certificates. She took the money out on Feb. 14, 1931. How much interest did she receive?

3. Herman invested \$50 in postal savings on Dec. 31, 1929. How much interest had his \$50 earned by Apr. 1, 1931?

4. Ada invested \$20 on Dec. 4, 1929, and \$30 on Dec. 16, 1930. She has not taken out any interest. How much interest is due her on the day you do this problem?

COMMERCIAL BANKS

Keeping Money in a Commercial Bank

A business man usually has a *checking account* in a *commercial bank.* No interest is paid on small checking accounts, but on large accounts a low rate of interest is sometimes allowed. The depositor can draw his money from a commercial bank by a written and signed order called a **check.**

Besides allowing its depositors to have checking accounts, a commercial bank also renders them these services:

1. It offers them a safe place to keep their money.
2. It provides them with blank check books without charge.
3. It cashes checks on other banks for them.

Mr. Miller has a checking account at the Dayton National Bank. When he deposits money there, he indicates on a **deposit slip** how much of the money is in bills, how much in coin, and how much in checks, and where each check comes from. For a check from a local bank, he gives the name of the bank, as "Dayton Trust Company"; for a check from another city, he writes only the city.

Deposited by
Henry P. Miller
in the
DAYTON NATIONAL BANK
Dayton, Ohio
March 20, 1928

Bills	37	00
Coin	1	65
Checks		
Dayton Trust Co.	12	40
Chicago	90	00
	141	05

Exercises

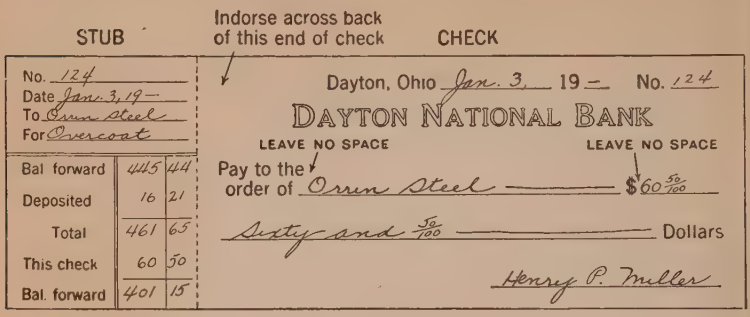
Hugh Moore made these deposits in the Stanton Northern Bank. Write a deposit slip like the one above for each date.

1. Feb. 23, 1928: bills, $185; coin, $44.75; check on National Bank, $252.65; check on Erie, $51.13; check on Denver, $523.

2. Feb. 27, 1928: bills, $44; coin, $23.25; check on Farmers Bank, $400; check on Akron, $310.08; check on Troy, $21.90.

BANK CHECKS

Mr. Miller, who owed $60.50 to Mr. Steel for an overcoat, paid the bill by writing this check for him:



STUB

No. 124
Date Jan. 3, 19—
To Orin Steel
For Overcoat

Bal forward	445	44
Deposited	16	21
Total	461	65
This check	60	50
Bal. forward	401	15

Indorse across back of this end of check

CHECK

Dayton, Ohio Jan. 3, 19— No. 124

DAYTON NATIONAL BANK

LEAVE NO SPACE LEAVE NO SPACE

Pay to the order of Orin Steel $60 50/100

Sixty and 50/100 Dollars

Henry P. Miller

To the left of the dotted line is the *stub* of the check, which stays in the check book. To the right of the dotted line is the *check*, which Mr. Miller sends to Mr. Steel.

The Correct Form for Checks

Examine the above check and answer these questions:

1. Why does Mr. Miller send a check rather than cash?

2. In what bank is Mr. Miller's money deposited?

3. To whom are the words "Pay to" addressed?

4. Why is the amount written both in words and in figures?

5. Why is a waved line put after the name of Mr. Steel and also after the amount written in words?

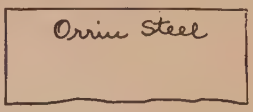
6. Why should no space be left between the sign $ and $60\frac{50}{100}$ or before the amount written in words?

7. Why is the amount written $60\frac{50}{100}$ instead of $60.50? If the amount is exactly $60, it is written $60\frac{00}{100}$ or $60\frac{no}{100}$.

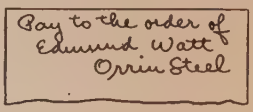
8. Make out ten checks on the Farmers Bank of Saratoga, using a different amount, name, and date on each. Sign them.

INDORSING A CHECK

When Mr. Steel gets the check for $60.50 from Mr. Miller, he can present it at the Dayton National Bank or at the bank where he keeps his account and get $60.50 for it. But he must first *indorse* the check by writing his name across the back of the left end of it. This is called an *indorsement in blank*. By indorsing the check, Mr. Steel acknowledges the receipt of the money.



Instead of depositing the check in the bank, Mr. Steel could use it to pay a bill from Mr. Watt. He would indorse it *in full*, making it payable only to Mr. Watt or his order. Such an indorsement is safer than a blank indorsement in case the check is lost.



No matter where the check is cashed, it eventually goes back to the Dayton National Bank, where $60.50 is deducted from Mr. Miller's balance. At the end of the month, the canceled check is returned to Mr. Miller, together with the other checks he made out during the month. These checks serve as receipts for the bills he has paid. The bank also sends Mr. Miller a *statement* each month, showing his bank balance.

Exercises

1. J. G. Snow wrote Tom Pell a check for $52.60 on the Grant Bank. Write the check and indorse it in blank.

2. What does indorsing a check signify? Tom Pell indorsed the check 9 hr. before he went to the bank. Was this wise?

3. On June 3 Tom deposited the check in the Concord Trust Company, together with $29 in bills, $6.73 in coin, and a check on Boston for $403.13. Write the deposit slip.

4. Tom wrote a check on the Concord Bank for $23.80 for Mary Lee. Mary sent it to Mrs. Rose Breen in payment of a bill. Make out the check and indorse it in full for Mary.

KEEPING A RECORD OF BANK BALANCES

Every time Mr. Miller writes a check, he fills out the stub of the check. After he sent Mr. Steel's check for $60.50 (page 264), the stub, shown at the right, remained in his check book. The last item on this stub, which is labeled "Balance forward," shows that Mr. Miller had $401.15 left in the bank after paying that check.

No. 124		
Date Jan. 3, 1928		
To O. Steel		
For Overcoat		
Bal. forward	445	44
Deposited	16	21
Total	461	65
This check	60	50
Bal. forward	401	15

The first item under the double line, also called "Balance forward," shows that Mr. Miller had $445.44 in the bank after writing check No. 123. He had also deposited $16.21 since drawing that check, making the total balance $461.65 before he wrote check No. 124.

Why is it important for a business man to keep a record of his bank balance on the stubs of the checks? If Mr. Miller gives Mr. Judd a check for $500 when his balance in the bank is only $401.15, he *overdraws his account* and the bank may refuse to pay anything to Mr. Judd when he presents the check.

Exercises

On Jan. 4 Mr. Miller wrote a check for $50. He filled out the stub of the check as shown here.

No. 125		
Date Jan. 4, 1928		
To R. Brown		
For Repairs		
Bal. forward	401	15
Deposited		
Total	401	15
This check	50	00
Bal. forward	351	15

1. What does the first item under the double line show? How does this balance compare with the last balance on the stub of Jan. 3, shown above?

2. Has any deposit been made since the last check?

3. How much did Mr. Miller have in the bank after paying this check?

4. What is the safest time to fill in the stub of a check?

MR. MILLER BALANCES HIS CHECK BOOK

These are the stubs in Mr. Miller's check book immediately after the two stubs on page 266. Compute the totals and balances that should be put in place of the stars:

1.

No. 126
Date Jan. 5, 1928
To M. Short
For Books

Bal. Forward	351	15
Deposited	15	13
Total	*	
This check	12	50
Bal. Forward	*	

2.

No. 127
Date Jan. 6, 1928
To R Cohn
For Furniture

Bal. Forward	*	
Deposited	20	00
Total	*	
This check	245	75
Bal. Forward	*	

3.

No. 128
Date Jan. 6, 1928
To H. Mason
For Groceries

Bal. Forward	*	
Deposited	00	00
Total	*	
This check	65	00
Bal. Forward	*	

4. Make out on the Dayton National Bank the three checks recorded on the above stubs. Indorse the checks for the men to whom they are made out.

How a Storekeeper Uses a Bank Account

George Hale, a grocer, had $739.50 in the Franklin Bank on July 5. On July 6 he deposited $12.20 in coin, $68 in bills, a check for $44.50 on Mt. Vernon, a check for $36.20 on the Hamilton Bank, and a check for $19.68 on the Union Bank.

Later in the same day he sent a check for $61.10 to the Star Dairy for milk and cream. He also made out a check for $22.40 to Robert Curtis for store fixtures.

1. Make out Mr. Hale's deposit slip of July 6.

2. What was his bank balance after he made this deposit?

3. Show how Mr. Hale wrote out the first check to the Star Dairy; also show how he filled out the stub.

4. Find his balance after the payment of the first check.

5. Write the check for Robert Curtis and its stub.

6. What was Mr. Hale's bank balance at the end of the day?

BORROWING MONEY FROM A BANK

It is a considerable expense to a bank to take care of the checking accounts of its depositors and to collect for them checks drawn on other banks. To repay itself, the bank lends out at interest a large part of the money deposited in it. The rates of interest for such loans are limited by law. The loans are often made for 3 mo. or less.

Mr. Hart, a shopkeeper, needs $500 to buy toys which he will sell for Christmas. He borrows this amount for 30 da. from a bank. Mr. Hart has to give the bank a written promise that he will pay back the $500 in 30 da. Such a promise is called a **note** or a **promissory note**. Here is Mr. Hart's note:

$ 500.00 Oakland, Nov. 1 1928

Thirty days after date I promise to pay

to the order of The Merchants Bank

Five Hundred 00/100 Dollars

Value received

No. 2310 Due Dec. 1 1928 Henry C. Hart

Mr. Hart is the **maker** of the note. The amount he borrows is the **face** of the note. The face of this note is $500.

The bank might ask Mr. Hart to deposit stocks, bonds, or other valuables as security. Such security is called **collateral.** Or Mr. Hart might have to get another man in good financial standing to indorse his note as security.

In the above note, interest is not mentioned because the bank collects the interest *in advance.* On Nov. 1, when Mr. Hart borrows the money, he pays interest on $500 at 6% for 30 da., which is $2.50. Instead of getting the full $500 from the bank, he gets only $500 − $2.50, or $497.50. At the end of the 30 da. he pays back $500 to the bank.

BANK DISCOUNT

The $2.50 interest which the bank collects in advance from Mr. Hart (page 268) is called **bank discount.** When the bank deducts $2.50 interest from the $500, it **discounts** the note. The $497.50 which Mr. Hart receives is the **proceeds.** The proceeds equal the face of the note less the bank discount.

On pages 244 and 250 you found that on loans between two persons and on savings deposits the interest is generally collected *at the end* of each year or of each half year or quarter year. Commercial banks usually collect the interest *in advance*, bank discount being merely a technical term which means interest in advance. On certain other types of notes, such as demand notes, banks do not collect the interest in advance.

Bank discount, which is profit to the bank, must not be confused with the discount allowed to a purchaser on the list price of goods.

If Mr. Hart cannot pay the note when it is due, the bank may extend the loan and require him to make out a new note.

Exercises

Compute interest by the 6% method.

1. Mr. Hart later borrowed $700 from a bank for 60 da. Write the note. Who keeps the note until it is due? Why?

2. In ex. 1, what was the bank discount at 6%? How much money did the bank actually give Mr. Hart? How much did he have to pay back after 60 da.?

3. George Bell borrows $1000 for 90 da. from the Pacific Bank. Make out the promissory note. What is the face of the note? At 7%, find the bank discount and the proceeds.

4. John Martin borrows $400 from the Liberty Bank for 66 da. Write the note. Who is the maker of the note? What is its face? At 6%, find the bank discount and the proceeds.

5. With interest at 6%, how much money would Mr. Stahl receive from the bank if he borrowed $450 for 30 da.? if he borrowed $200 for 60 da.? $725 for 90 da.?

FINDING BANK DISCOUNT AND PROCEEDS

Write the promissory notes which the men named below make out to the banks from which they borrow:

	Maker	Bank	Face	Time
1.	Paul Thomas	Mechanics Bank	$350	36 da.
2.	Roger Doyle	First National	$700	90 da.
3.	Henry Ferris	Public National	$480	10 da.
4.	John Curtis	Corn Exchange	$115	60 da.
5.	Mark Wilson	Genesee County	$800	20 da.
6.	Frank Warden	Farmers Exchange	$625	72 da.
7.	James Nolan	Second National	$280	15 da.
8.	Adam Morley	National City	$1250	90 da.
9.	Edgar White	Union Trust	$2200	116 da.

10. Using the 6% method, compute the bank discount at 6% and the proceeds of each note in ex. 1 to 9.

11. On Oct. 1, Mrs. Owen put $400 in the savings bank at 4% interest, compounded quarterly. On Dec. 2, she needed $400 to purchase a supply of Christmas candies for her store. She wanted to take the money from the savings bank. A friend advised her rather to borrow the $400 for 30 da. at 6%. Which plan was more economical, and how much more?

Mrs. Owen's bank has the interest regulations given on page 258.

Find the proceeds on notes for the following amounts discounted at 5%. *Use the* 6% *method, as given on page* 252:

12. $150 due in 90 da.
13. $325 due in 60 da.
14. $450 due in 66 da.
15. $750 due in 30 da.
16. $900 due in 96 da.
17. $475 due in 120 da.
18. $550 due in 102 da.
19. $290 due in 45 da.
20. $370 due in 15 da.
21. $240 due in 135 da.
22. $180 due in 165 da.
23. $850 due in 2 mo. 6 da.
24. $400 due in 3 mo. 15 da.
25. $520 due in 4 mo. 18 da.

IMPROVEMENT TEST No. 19

Test 19 A — Reduction (Time 4 min. after copying)

Change to decimals, correct to the nearest hundredth:

1. $\frac{4}{9}$ **2.** $\frac{8}{11}$ **3.** $\frac{29}{36}$ **4.** $\frac{11}{12}$

5. $\frac{1}{7}$ **6.** $\frac{5}{16}$ **7.** $\frac{41}{64}$ **8.** $\frac{21}{32}$

Test 19 B — Multiplication (Time 4 min. after copying)

1. 432 × .06	**2.** 7.64 × .37	**3.** 57.2 × .42	**4.** .027 × 9200
5. 907 × 8.1	**6.** 8.09 × .034	**7.** 6.57 × .05	**8.** .018 × .3900
9. 400 × .85	**10.** .005 × .08	**11.** 2.92 × .149	**12.** 0.81 × 9000

Test 19 C — Division (Time 3 min. after copying)

1. $6.2\overline{).36022}$ **2.** $.094\overline{)862.92}$ **3.** $7.9\overline{)56.80337}$

Test 19 D — Division (Time 3 min. after copying)

Find each quotient correct to the nearest tenth:

1. $53\overline{)246.4}$ **2.** $.75\overline{)748}$ **3.** $.92\overline{)55.9}$

Test 19 E — Division (Time 3 min. after copying)

Divide, annexing enough zeros to each dividend to make the quotient come out exact:

1. $64\overline{)464}$ **2.** $96\overline{)324}$ **3.** $7800\overline{)2.73}$

GEOMETRIC CONSTRUCTIONS

Squares and Rectangles

Many common objects, like doors, windows, and picture frames, are shaped like rectangles. Every rectangle has four square corners, or **right angles,** which look like this: ⌊ ⌈ ⌉ ⌋

In the rectangle shown above, two of the sides are longer than the other two. When a rectangle has all four sides equal, it is called a *square*.

Look around the room and find fifteen different rectangles. How many things can you find with the shapes of squares?

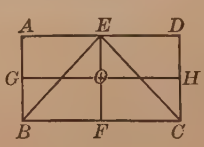
Right Angles and Perpendiculars

1. Tell which of the following are right angles:

2. Tell how many right angles you find in each figure:

3. When one line meets another so as to make a right angle, the lines are said to be *perpendicular* to each other. Line AC is perpendicular to line AB because angle A is a right angle. Is line AB also perpendicular to line AC?

4. In this figure point out all the perpendicular lines you can. Give each of your answers like this: EF is perpendicular to GH at O.



Is AE perpendicular to EF? Is AE perpendicular to EB? Is AE perpendicular to EC? Give a reason for each of your answers.

HOW TO DRAW PERPENDICULAR LINES

To draw a rectangle, you should know how to make an exact right angle by drawing one line perpendicular to another. This can be done by using a ruler and a draftsman's triangle, as shown at the right. One of the corners of the triangle is a right angle. In Fig. 2 below, the angle x of the triangle is the right angle. A draftsman's triangle is usually made of wood or celluloid; but such a triangle may easily be made of heavy cardboard. If you have no triangle, make one about 6 in. tall and 4 in. wide at the bottom.

To draw a line perpendicular to line AB at point C, place the edge of your ruler along line AB, as shown below in Fig. 1. Then place your triangle on the ruler, as in Fig. 2, so that its

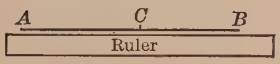


FIG. 1

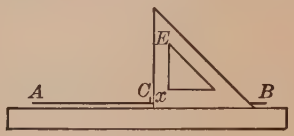


FIG. 2

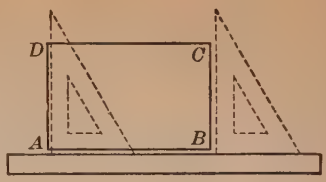
right angle x has one edge close to the ruler and the other edge, E, just touching point C. Then draw a line from C along edge E of the triangle. This line will be perpendicular to AB at C.

Before drawing the perpendicular line, press the ruler down firmly with the thumb and the little finger of the left hand and hold the triangle close to the ruler with the other three fingers, as shown in the picture at the top of this page. Always use a well-sharpened pencil.

Mark other points, such as R, S, and T, on line AB and draw lines perpendicular to AB at these points. Must you change the position of your ruler in drawing these perpendiculars?

DRAWING SQUARES AND RECTANGLES

1. Draw a rectangle 6 in. long and 4 in. high.



First draw the base AB, making it 6 in. long. Then draw a line perpendicular to AB at B, using the ruler and the triangle. Next slide the triangle to the left along the ruler until its left-hand edge touches point A, and draw line AD perpendicular to AB. Make AD and BC each 4 in. long; then draw a line connecting points D and C.

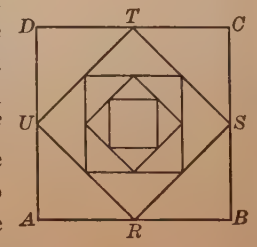
Draw these rectangles, using a ruler and a triangle:

2. 5 in. by 3 in.
3. 4 in. by 5 in.
4. 6 in. by $\frac{1}{2}$ in.
5. 2 in. by 3 in.
6. 3 in. by 1 in.
7. 1 in. by 4 in.
8. 4 in. by $1\frac{1}{2}$ in.
9. $2\frac{1}{2}$ in. by 5 in.
10. 5 in. by 5 in.
11. 5 in. by 2 in.
12. 6 in. by 6 in.
13. 8 in. by 8 in.

14. The pattern shown at the right often appears on oilcloth. Make an enlarged copy of it, making each small square 1 in. on a side.

Rule the bottom line and draw perpendiculars to it at A, B, C, D, E, and F. Mark off points on the end lines (A and F) 1 in. and 2 in. from the bottom, and draw the horizontal lines.

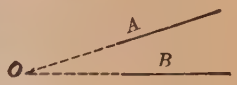
15. Jenny saw this design in a magazine. She decided to embroider it on the corners of a linen table scarf, making the largest square 5 in. on a side. The directions say that the largest square, $ABCD$, is to be drawn first. Then the middle point of each of its sides is found with a ruler, and these four middle points are joined in order, to form the square $RSTU$. Each of the smaller squares is made by joining the middle points of the sides of the next larger one. Show Jenny how to draw this design on paper.



PARALLEL LINES

1. The two lines at the right are *parallel* because, no matter how far they are extended in either direction, they never meet.

2. Parallel lines are always the same distance apart. Lines A and B are not parallel, because they are closer together on the left. If lines A and B are extended as shown by the dotted lines, they meet at O.

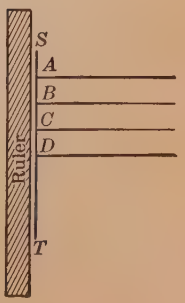


3. Are the lines at C parallel? at D? at E? at F? at G?

4. Are the top and the bottom of the blackboard parallel? the opposite edges of your desk? the opposite edges of your ruler? Find other parallel lines in your classroom.

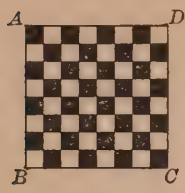
How to Draw Parallel Lines

Suppose you wish to draw several parallel lines 1 in. apart. Draw a line ST and locate points A, B, C, and D on it 1 in. apart. Next lay your ruler along line ST. Place your triangle close to the ruler, and draw a line from A perpendicular to ST, as on page 273. Now, holding the ruler firmly in place, slide your triangle along it to a point B and draw another line perpendicular to ST. Slide the triangle again to C and draw another perpendicular; then slide it to D and draw another perpendicular. Then the four lines A, B, C, and D are all parallel. Are A, B, C, and D each perpendicular to ST?



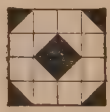
HOW TO DRAW A CHECKERBOARD

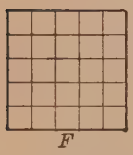
1. Joan wants to draw a checkerboard with each small square 1 in. on a side. She first draws square *ABCD* 8 in. on a side, using a ruler and a triangle. She then draws the small squares by making seven horizontal parallel lines 1 in. apart and seven vertical parallel lines 1 in. apart (see page 275).

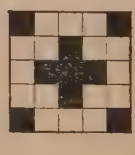


2. Fred says that he can draw the checkerboard with a ruler without using a triangle. He places points 1 in. apart on each of the sides of square *ABCD* and connects the opposite points. See which way is quicker, Fred's or Joan's. Does Fred need a triangle to draw the large square *ABCD* accurately?

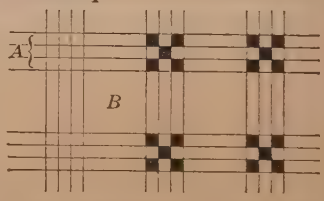
3. Rule squares like *E* and *F* and copy these designs:





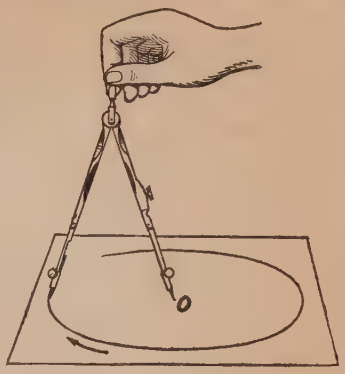


4. Mary found this design on the marquisette curtains in her room. Copy it with your ruler and triangle, letting the horizontal and the vertical parallel lines in each group, like the four lines at *A*, be $\frac{1}{8}$ in. apart. Make each large white square, like *B*, $\frac{5}{8}$ in. on each side. Make your design 6 in. square.

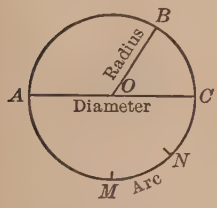


DRAWING CIRCLES

This picture shows how to draw a circle with a pair of compasses. Suppose that the points of the compasses are set 2 in. apart and O is to be the center of the circle. The steel point of the compasses is set on the paper at O and the pencil end is moved around to draw the circle. Every point on this circle is 2 in. from the center O, since it was drawn with the points of the compasses 2 in. apart.



In the picture below, the line OB, connecting the center with a point on the circle, is called a **radius.** The plural is **radii.** All radii of the same circle have the same length. The line AC, formed by two radii in a straight line, is a **diameter.** A diameter passes through the center of the circle and is twice as long as a radius. A part of a circle, like the curved line MN, is called an **arc.**



Exercises

1. Wheels, coins, and the full moon are circles. Name five other things that are circles.

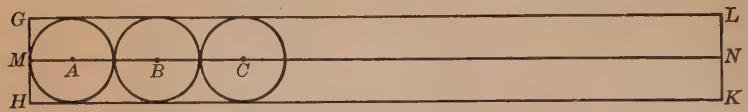
2. Set the points of your compasses 1 in. apart and draw a circle. On the circle draw three radii and three diameters. How long is each diameter?

3. Draw circles with radii of $1\frac{1}{2}$ in., $\frac{3}{4}$ in., and $2\frac{1}{4}$ in. How long is the diameter of each of these circles?

4. Draw a circle on the blackboard, using as radius a string with a piece of chalk tied to the end of it.

THE CIRCLE IN DESIGNS

1. The above border was used on the cover of a book. To make it, use your ruler, triangle, and compasses, thus:



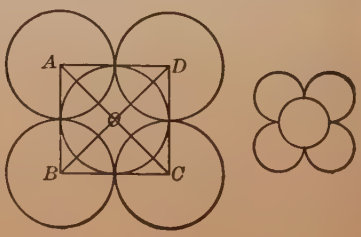
Draw rectangle *GHKL* 1 in. wide and 8 in. long. Then draw *MN* half way between *GL* and *HK*. Mark point *A* $\frac{1}{2}$ in. from *M*, point *B* 1 in. from *A*, point *C* 1 in. from *B*, etc. With a radius of $\frac{1}{2}$ in., draw circles with *A*, *B*, *C*, etc. as centers.

Color the circles or draw pictures in them, as shown above.

2. Determine how this design is made and copy it, making the circles with a radius of $\frac{3}{4}$ in. Which lines will you draw first?

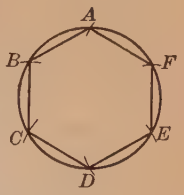
3. This is a drawing of Mrs. Hall's garden. Each circle has the same point as center. Copy the drawing, making the circles, in order, $\frac{1}{2}''$, $1\frac{1}{2}''$, $2''$, and $3''$ in radius. The straight section of the walk, leading outside the garden, should be $\frac{1}{2}''$ wide.

4. Draw a square *ABCD* 2 in. on a side. Draw lines *AC* and *BD* meeting at *O*. With a 1-inch radius, draw five circles, using *O*, *A*, *B*, *C*, and *D* as centers. Erasing all the lines within the central circle and all the straight lines, you get a design which looks like the second design at the right.



DRAWING A HEXAGON IN A CIRCLE

Draw a circle with a radius of 1 in. Keeping the points of the compasses 1 in. apart, and taking any point on the circle, such as A, as center, draw an arc cutting the circle at B. With the same radius and with B as center, draw another arc cutting the circle at C. In the same way draw arcs cutting the circle at D, E, and F. If your work has been carefully done, another arc, drawn with F as center, will cut the circle at point A, at which you started. The circle is now divided into six equal arcs. By joining points A, B, C, D, E, and F with straight lines, you get a **regular hexagon,** which has six equal sides and six equal angles.

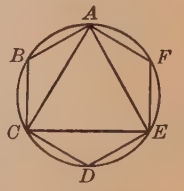


Exercises

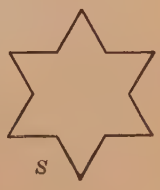
1. In the hexagon above, how does the length of each side compare with the length of the radius of the circle?

2. Draw regular hexagons in circles whose radii are 2 in., $1\frac{1}{2}$ in., $1\frac{1}{4}$ in., and $2\frac{1}{2}$ in.

3. Draw a regular hexagon in a circle. Then draw lines AC, CE, and EA to form a triangle within the hexagon. Measure the sides of the triangle. How do their lengths compare? Triangle ACE is called an **equilateral triangle**, because each of its sides has the same length.



4. In the figure in ex. 3, draw a second triangle whose corners are B, F, and D. This will give figure R at the right. Erase certain lines in R and you will get a six-pointed star, as shown in S.



OTHER FIGURES IN CIRCLES

1. In Fig. 1 below, divide a circle into six equal arcs. With the same radius and with A as center, draw inside the circle an arc from F to B. Draw similar arcs with B, C, D, E, and F as centers. This gives the design shown here. Color it.

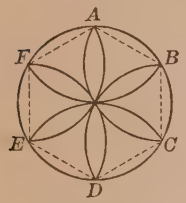


FIG. 1

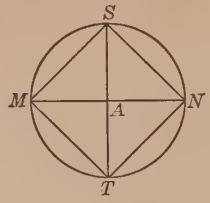


FIG. 2

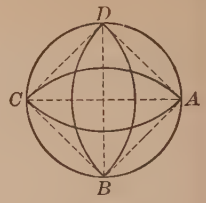
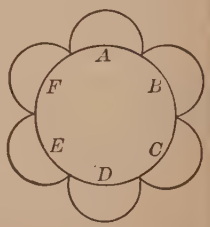


FIG. 3

2. In Fig. 2 above, draw a circle with center A and diameter MN. With your ruler and triangle, draw diameter ST, perpendicular to MN at A. Connect S, M, T, and N in order. This gives a square *inscribed* in a circle.

3. In Fig. 3 above, draw a circle and inscribe a square $ABCD$ in it. With a radius equal to AB and with A as center, draw an arc from B to D. Draw similar arcs with the other corners of the square as centers.



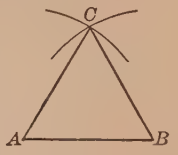
4. Divide a circle into six equal arcs, as shown at the right. Then, with a radius half as long, draw six arcs outside the circle, using points A, B, C, etc. as centers.

5. Copy these designs. Use a radius of 2″ for the large

circles in the first two designs and one of 3″ in the third design.

EQUILATERAL TRIANGLES

In ex. 3 on page 279 you drew in a circle a triangle, called an *equilateral triangle*, whose sides were all equal in length. An equilateral triangle can also be drawn as described below:



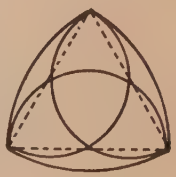
If you want an equilateral triangle 2 in. on a side, draw a line AB, 2 in. long. With a radius of 2 in., and with A as center, draw an arc above the line. With the same radius and with B as center, draw another arc crossing the first one at C. Draw AC and BC. Are the three sides of the triangle equal in length?

Exercises

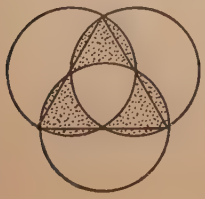
1. Draw equilateral triangles whose sides are each 1 in.; 2 in.; $1\frac{1}{2}$ in.; $1\frac{3}{4}$ in.; $2\frac{1}{2}$ in.; 3 in.

2. This linoleum pattern is made of equilateral triangles. Make it three times as large as it is here.

3. For this design, draw a dotted equilateral triangle 3 in. on a side. Draw the outside arcs with the corners of the triangle as centers and a side of the triangle as radius. For the inside arcs, use a radius of $1\frac{1}{2}$ in., with the middle points of the sides of the triangle as the centers of the circles.

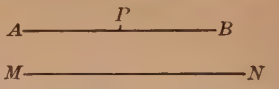


4. Copy each of the following designs on a larger scale:

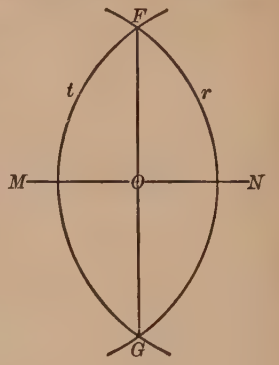


HOW TO DIVIDE A LINE INTO HALVES

Line AB is 1 in. long. You can easily divide it into halves with a ruler by finding point P $\frac{1}{2}$ in. from A. But as line MN is a little over $1\frac{1}{8}$ in. long, you cannot find its exact middle with a ruler whose smallest divisions are eighths of an inch.



To divide line MN into halves, use your compasses. With M as center and a radius somewhat longer than half of MN, draw arc r, as shown here. With the same radius and with N as center, draw arc t. These arcs meet at F and at G. Draw FG, which will cut MN at O, the middle point of MN. A line like FG, which cuts another into two equal parts, *bisects* the other line. FG is called the *bisector* of line MN.



Exercises

1. Use your compasses to bisect lines whose lengths are 2 in., 4 in., 5 in., and $1\frac{1}{2}$ in. Check your drawings with a ruler.

2. Draw bisectors of lines $1\frac{7}{8}$ in., $2\frac{3}{8}$ in., and $3\frac{1}{4}$ in. long.

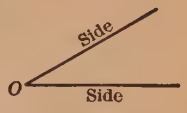
3. Draw a line a little over $3\frac{5}{8}$ in. long. Bisect it with your compasses. Then bisect each half of the line. Into how many equal parts is the line now divided?

4. Divide a line $7\frac{1}{4}$ in. long into fourths; into eighths.

5. Mary wants to use this design as a border for a picture. The border is to measure $6\frac{3}{8}$ in. across the top and to contain 8 equal rectangles, each $\frac{1}{2}$ in. high. With compasses divide the length of the border into eight equal parts; then make the border.

THE SIZE OF AN ANGLE

You know that a square corner is called a right angle. Any two straight lines meeting at a point form an **angle.** The lines are the **sides** of the angle and the point, *O*, is the **vertex.**



The commonest kinds of angles are: (1) the **right angle;** (2) the **acute angle,** which is less than a right angle, and (3) the **obtuse angle,** which is greater than a right angle.

ACUTE ANGLE

RIGHT ANGLE

OBTUSE ANGLE

The size of an angle depends on how far the sides are turned away from each other. On the first clock below, at 8.55, the hands form an acute angle. As the minute hand turns away

from the hour hand, the angle increases, so that at 9 o'clock the hands form a right angle. As the minute hand turns away further, the angle increases further; at 9.05 it is evidently an obtuse angle. Hence an angle increases in size as one side turns away from the other.

The length of the sides of an angle has no effect on the size of the angle. The angles at the right represent the same amount of turning, as shown by the arrows, and are therefore of the same size.

The size of an angle depends on how far one side is turned away from the other and not on the length of the sides.

LETTERING AND READING ANGLES

Angles are named by the use of letters. In Fig. 1, angle M is designated by one capital letter at the vertex. The same

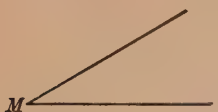


FIG. 1

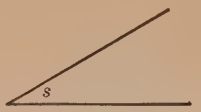


FIG. 2

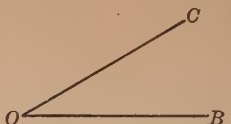
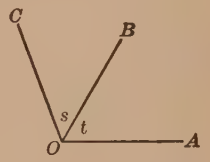


FIG. 3

angle may be named angle s by placing a small letter s within the angle near the vertex, as in Fig. 2. In Fig. 3, angle BOC is named by capital letters at the ends of the sides and at the vertex. Angle BOC may also be read "angle O".

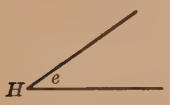
In the figure at the right, "angle O" may mean angle AOC, angle AOB, or angle BOC. Hence in this case you must name each angle with three letters or call the smaller angles "angle s" and "angle t."

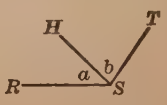


When an angle is named by three letters, the letter representing the vertex is written between the other two letters.

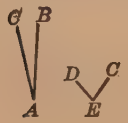
Exercises

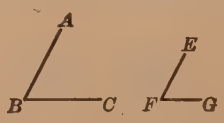
1. Read every angle in these figures, give all its possible readings, and tell if it is acute, right, or obtuse.





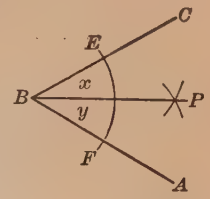
2. Read each angle below and tell what kind of angle it is. Tell which angle, if either, in each pair of angles is larger.





HOW TO BISECT AN ANGLE

To bisect a line means to divide it into two equal lines. *To bisect an angle* means to divide it into two equal angles.

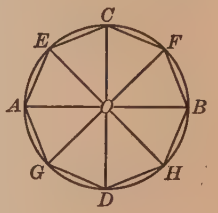


To bisect angle ABC, place the steel point of the compasses at B as center. With any convenient radius, draw an arc cutting the sides of the angle at E and F. Then with EF as radius draw an arc having E as center and another arc having F as center. The arcs meet at P. Draw line BP, which bisects the angle ABC, making angle x equal to angle y. BP is called the **bisector** of angle ABC.

Bisecting Angles

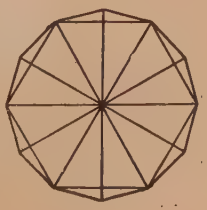
1. Draw five angles of different sizes and bisect each of them.

2. In a circle draw two diameters, AB and CD, perpendicular to each other at O. With the compasses bisect angles BOC and COA and extend the bisectors as diameters to meet the circle at F, G, E, and H. Connecting the eight points in order, you get an eight-sided figure, or **octagon.** The barrel of a gun often has this shape.



3. If you bisect four of the angles at the center of the octagon in ex. 2 and extend these bisectors as diameters until they meet the circle, you can draw a figure with 16 equal sides. Try it, using a circle 3″ in radius.

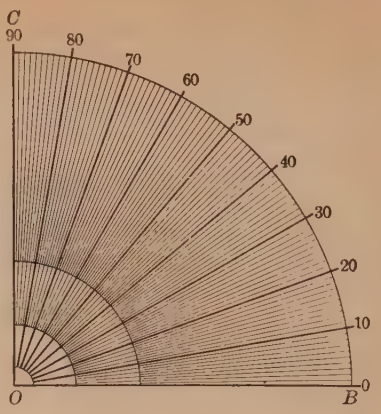
4. Inscribe a hexagon in a circle (as on page 279) and connect its corners with the center. Bisect three of the six angles thus formed at the center and extend the bisectors as diameters. Then draw a regular twelve-sided figure and erase the circle.



HOW ANGLES ARE MEASURED

Just as a line is measured by unit lengths like the inch, so an angle is measured by a unit angle called a **degree.**

Draw a quarter circle with O as center. Draw CO perpendicular to OB, making a right angle BOC. If this right angle is divided into 90 very small equal angles, each of the small angles is a *degree.* Hence a right angle contains 90 degrees, or 90°. The sign ° means *degree.*



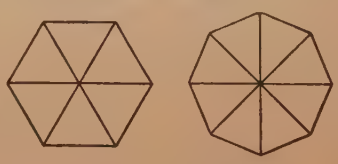
In the figure at the right, the 90 small angles or degrees in the angle BOC also divide the arc BC into 90 small equal arcs. Each of these small arcs is a *degree of arc.* Hence the degree is a unit for measuring arcs as well as angles.

This figure shows that four right angles containing 90° each may be drawn at the center of a circle. Hence a total of $4 \times 90°$, or 360°, may be drawn around a point. An entire circle also contains 360° of arc.

Exercises

1. How many degrees are there in $\frac{1}{2}$ of a right angle?

2. In these figures, how many degrees are there in each of the equal angles at the center? Remember that all the angles at the center equal 360°.



3. How many degrees of arc are there in $\frac{1}{2}$ of a circle?

IMPROVEMENT TEST No. 20

Test 20 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
7558	7294	5836	2498	3848
4576	4345	1233	8868	2877
5993	7076	9469	6876	6175
7575	8220	9549	6223	3959
7806	4985	7403	9691	5731
9362	4649	9622	9867	3943
8785	7883	1916	1521	4154
2434	6499	3818	5711	7489

Test 20 B — Subtraction (Time 3 min.)

1. 2919509	2. 8588024	3. 7534062
1650832	1196435	6390686
4. 9783264	5. 4820154	6. 9692149
1246858	3385274	4346179
7. 6107517	8. 9748309	9. 7024453
1805269	3291410	3151582

Test 20 C — Multiplication (Time 4 min. after copying)

1. 94817	2. 32605	3. 10748
5319	6254	8276

Test 20 D — Division (Time 4 min. after copying)

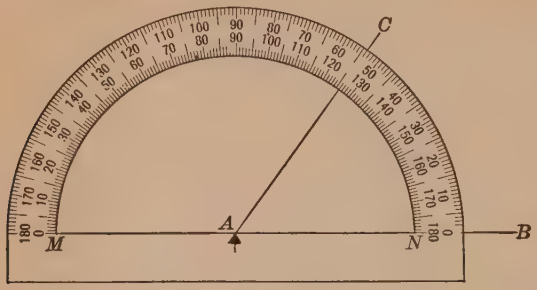
If there is a remainder, carry result to the nearest tenth:

1. $19\overline{)17163}$ 2. $78\overline{)55624}$ 3. $43\overline{)254173}$

4. $68\overline{)41352}$ 5. $36\overline{)29358}$

AN INSTRUMENT FOR MEASURING ANGLES

Angles are measured with an instrument called a **protractor.** It has a semicircular edge marked off into 180°. The upper edge, MN, of its base is the diameter of the semicircle, and the arrow at A in the middle of MN is the center.



To measure angle BAC with a protractor, place the protractor over angle BAC so that the arrow of the protractor is at the vertex A, and the edge MN of the protractor lies along side AB of the angle, as shown above. Side AB of the angle should pass through the "zero" mark on the scale of the protractor. The side AC now points to 54°; hence angle BAC contains 54°.

If either side of the angle is not long enough to meet the protractor, extend the side as much as necessary.

Exercises

1. Make angles like these and measure each with a protractor:

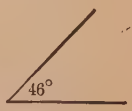
2. Draw a circle with a 2-inch radius. Draw two diameters perpendicular to each other. Bisect each of the four angles thus formed and find with your protractor how many degrees each of the final eight angles contains.

MAKING AN ANGLE OF ANY SIZE WITH A PROTRACTOR

1. Suppose you want to draw an angle of 54°. Draw line AB, which will be one side of the angle; let A be the vertex. Place your protractor over the line AB with the arrow at vertex A, and with the "zero" mark on the scale lying on line AB, as shown in the picture at the top of page 288. Next find the 54° mark on the protractor and put a dot by it on the paper. Call this dot D. Then take the protractor off the paper and draw line AD. You now have angle BAD, which contains 54°.

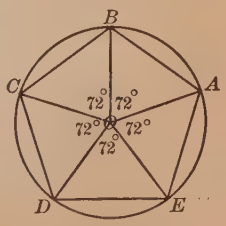
2. Draw an angle of 76°, as explained above. Then check by measuring the angle again to see if it contains 76°.

3. Make angles of 60°, 30°, 45°, 70°, 22°, 15°, 100°, 120°, and 135°. Write the number of degrees in each angle, as shown here. Then measure each angle again to check your work.



How to Draw a Regular Pentagon

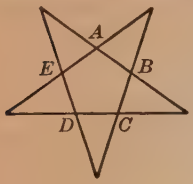
The figure $ABCDE$ has five equal sides; it is a **regular pentagon.** The five angles at the center are all equal. Since all the angles drawn around a point total 360°, each of these five angles must contain $\frac{1}{5}$ of 360°, or 72°.



To draw a regular pentagon, first draw a circle and its radius OA. Using OA as one side and O as vertex, draw an angle of 72° with a protractor. This gives angle AOB equal to 72°. With OB as one side, draw another angle of 72° and call it angle BOC. Continue, using the second side of each angle as the first side of the next angle. Finally, join points A, B, C, D, and E in order. You now have a regular pentagon, $ABCDE$.

MAKING DESIGNS AND PATTERNS

1. Using the method at the bottom of page 289, draw a regular pentagon in a circle whose radius is 1 in. Draw another pentagon with a $1\frac{1}{2}$-inch radius; a 2-inch radius.

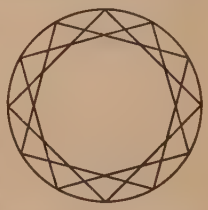


2. Draw a regular pentagon $ABCDE$ in a circle whose radius is 1 in. Extend all the sides of the pentagon as shown here, and you will have a five-pointed star.

3. Draw a regular pentagon in a circle having a 2-inch radius. Bisect the angles at the center and extend the bisectors to cut the circle. Then join in order all the points on the circle and you will have a figure of ten equal sides, called a **decagon.**

How many degrees should there now be in each angle at the center of your decagon? Measure each angle with a protractor to see if it is exactly right.

4. Divide a circle into twelve equal arcs. Then join every fourth point to form the design shown at the right.



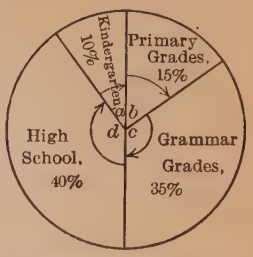
5. How many degrees will each angle contain if fifteen equal angles are drawn at the center of a circle? Draw fifteen such equal angles at the center and extend their sides until they meet the circle. Then draw a figure of fifteen equal sides.

6. If you can make this pattern accurately, you are proficient in geometric designing. First study the design carefully to see how each part of it is made. Then make it, letting the large rectangle be $7\frac{1}{2}$ in. long. When the design is finished, color it.

GRAPHS

Circle Graphs

You have learned to make bar graphs and line graphs. To show the relation of all the parts of anything to the whole of it, you often use a **circle graph.** A circle graph may be used, as at the right, to show that the kindergarten sold 10% of all the tickets for a school play; the primary grades sold 15%; the grammar grades, 35%; and the high school, 40%.



TICKETS SOLD FOR PLAY

To show the kindergarten's sales, you must use 10% of the circle. There are 360° in a circle; 10% of 360° = 36°. Hence at the center of the circle you draw with a protractor angle a equal to 36°.

Alongside of angle a draw angle b to show that the primary grades sold 15% of the tickets. 15% of 360° is 54°. Hence angle b should equal 54°.

How many degrees has angle c, which represents the grammar grades? After c is drawn, the rest of the circle will represent the high school sales. Do you have to draw angle d?

As a check, find 40% of 360°, which gives 144° as the size angle d should be. Then measure angle d with a protractor to see if it equals 144°.

To draw a circle graph when the parts of a whole are given as per cents, first multiply 360° *by each per cent; then mark off at the center of the circle angles containing the number of degrees required to represent each per cent.*

Exercises

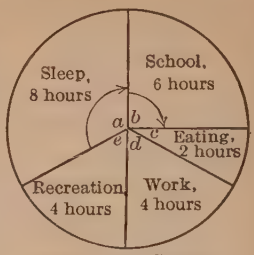
1. Draw a circle graph to show the candy sales according to seasons: autumn, 26%; winter, 39%; spring, 21%; summer, 14%. Compute the angles correct to the nearest whole degree.

2. Draw a circle graph to show that a family used 20% of its income for food, 25% for rent, $16\frac{2}{3}$% for clothing, $28\frac{1}{3}$% for other expenses, and 10% for savings.

WHEN THE PARTS OF THE WHOLE ARE EXPRESSED AS NUMBERS

Each day John spends 8 hr. in sleep, 6 hr. in school, 4 hr. in recreation, 4 hr. in work, and 2 hr. in eating. Draw a circle graph showing how John spends his day.

The parts of John's day are expressed as numbers instead of per cents. To draw a circle graph in this case, proceed as follows:



How John Spends the Day

1. Add all the hours, getting a total of 24 hr. in the day.

2. Find what part of the day John spends in sleep. Since he sleeps 8 hr., he devotes $\frac{8}{24}$, or $\frac{1}{3}$, of the day to sleep.

3. Since the whole day is represented by a circle of 360°, $\frac{1}{3}$ of the day is represented by $\frac{1}{3}$ of 360°, or 120°. Draw central angle a, of 120°, to represent the time John spends in sleep.

4. Similarly, John spends $\frac{6}{24}$, or $\frac{1}{4}$, of the day in school. Since $\frac{1}{4}$ of 360° is 90°, draw angle b, of 90°, to show the time John spends in school.

5. What part of the day is spent in eating? in work? Find how many degrees will represent each of these in the circle and draw angles c and d.

6. What does the remaining part of the circle represent? How many degrees should there be in this part? Check your work by measuring angle e.

Exercise

A sum of $240 was donated to the Thanksgiving fund of the Burchard School. The Literary Society gave $70; the Art Club, $50; the Parents' League, $64; the Tennis Club, $16; and the Athletic Association, $40. What part of the total sum did each of these organizations give? Show these facts on a circle graph.

DRAWING CIRCLE GRAPHS

Draw circle graphs for ex. 1 *to* 7. *Check the work.*

1. A boy has a collection of 180 stamps, of which 80 are from countries in North America, 50 from Europe, 30 from South America, and 20 from Asia.

2. In a class of 40 pupils, 9 were graded A on a test, 16 got B, 10 got C, and the rest got D.

3. In our school, 35% of the pupils gave English as their favorite study, 30% gave arithmetic, 20% gave history, and 15% gave geography.

4. Of the 250 graduates of a girls' high school, 150 will go to college, 75 will go to work, and 25 will stay at home.

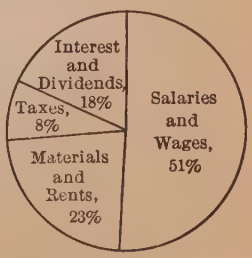
5. Of 960 children in the Grace School, 240 are in the third grade, 200 in the fourth, 184 in the fifth, 128 in the sixth, 112 in the seventh, and 96 in the eighth grade.

6. Of the world's telephones, 61% are in the United States, 29% are in Europe, and 10% are in all other countries.

7. Japan produces 48% of the world's raw silk; China, 21%; Italy, 9%; and the rest of the world, 22%.

8. Find in your geography book facts expressed as per cents of a whole. Show them on a circle graph.

9. A telephone company published this graph to show its expenditures one year. Tell what this graph means. What should you do to find out if this graph is constructed correctly?

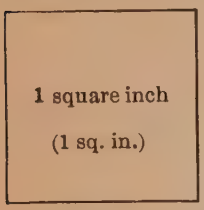


10. Dick figures that he spends 40% of his earnings for board, 25% for clothing, 15% for incidentals, and 10% for recreation, and that he saves 25%. Why can't you show these facts on a circle graph? Correct the per cent that Dick saves and make the graph.

MEASUREMENTS

Squares and Rectangles

You have seen that a four-sided figure containing four right angles is a **rectangle.** If its four sides are all of the same length, it is a **square.**



A square whose sides are each 1 in. long is called a *square inch.* What is a square foot? a square yard?

When you find how many square inches, square feet, or square yards there are in a surface, you find the **area** of the surface.

The figure at the right represents a rectangle 5 in. long and 3 in. wide. By dividing it into square inches, you find that it contains 3×5, or 15, square inches. Hence its area is 15 sq. in.

Area of rectangle = length × width

Letting A stand for area, l for length, and w for width, you can write the above rule in a kind of shorthand, thus:

$$A = l \times w$$

This is called a **formula.** You may also write it thus:

$$A = lw$$

When two letters are written side by side in a formula, you multiply them. Thus, lw means l times w.

Exercises

Find the areas of these rectangles, using the formula:

1. 7 in. × 2 in.
2. 8 ft. × 3 ft.
3. 6 ft. × 4 ft.
4. 3 yd. × $7\frac{1}{2}$ yd.
5. $4\frac{1}{2}$ in. × $3\frac{1}{2}$ in.
6. $5\frac{1}{4}$ ft. × $2\frac{3}{4}$ ft.
7. 2.35 in. × 1.3 in.
8. 9.41 ft. × 8.5 ft.
9. 8.5 ft. × 4.25 ft.

10. Draw a rectangle 6 in. long and $4\frac{1}{2}$ in. wide. Divide it into square inches and show that its area is 27 sq. in.

THE AREA OF A SQUARE

Since a square is a rectangle having four equal sides, you find the area of a square by multiplying one side by itself. A square whose side is 4 in. long has an area of 4×4 sq. in., or 16 sq. in.

When you multiply a number by itself, you *square the number*. Thus, you square 4 when you multiply 4 by 4 and get 16. The *square* of 4 is 16.

A brief way to write 4×4 is 4^2, which is read "4 square." The ^{2} should be written a little above the number to be squared.

If you represent each side of a square by s, the formula for its area is written as follows:

$$A = s \times s$$

or, in shorter form, $A = s^2$

EXPRESSING DIMENSIONS IN THE SAME UNIT

1. Find the area of a room 5 yd. long and 11 ft. wide.

In finding the area of a rectangle, you must express the length and the width in the same unit before multiplying. Both must be in inches or both in feet or both in yards, etc. In this problem, change 5 yd. to 15 ft. Then the area is 15×11 sq. ft., or 165 sq. ft.

2. At 40¢ a square foot, how much will the tile cost for a bathroom floor 8 ft. 6 in. by 5 ft. 6 in.?

First express 8 ft. 6 in. as $8\frac{1}{2}$ ft. and 5 ft. 6 in. as $5\frac{1}{2}$ ft. Then, since $A = lw$, the area is $8\frac{1}{2} \times 5\frac{1}{2}$ sq. ft., or $46\frac{3}{4}$ sq. ft. Tiling is sold by the whole square foot, hence 47 sq. ft. must be bought.

Find the areas of these rectangles in square feet:

3. 1 ft. 6 in. $\times$ 9 in.

4. 10 ft. $\times$ 6 ft. 3 in.

5. 10 ft. 2 in. $\times$ 18 ft.

6. 13 ft. 6 in. $\times$ 12 ft.

7. 7 ft. 8 in. $\times$ 7 ft. 6 in.

8. 4 ft. 9 in. $\times$ 2 ft. 8 in.

9. 5 ft. 4 in. $\times$ 5 ft. 3 in.

10. 5 ft. 6 in. $\times$ 4 ft. 3 in.

FINDING AREAS

1. How much will it cost Mrs. Daniel to cover a floor 18 ft. by 14 ft. with linoleum at $2.10 per square yard?

2. The linoleum that Mrs. Daniel buys (ex. 1) comes in strips 6 ft. wide. Draw a plan of the room and show how the strips are laid to avoid waste. How long is each 6-foot strip when it is ready to be laid?

3. Linoleum may also be purchased 9 ft. wide or 12 ft. wide. Could Mrs. Daniel (ex. 1) use the kind which is 9 ft. wide? If so, show how the strips would be laid to avoid waste. How many running feet of the 9-foot width would she need? How could she use the 12-foot width without waste?

4. At 28¢ per square foot, find the cost of cementing the floor of Mr. Hay's cellar, which is 18 ft. by 20 ft. 4 in.

5. Jane is making a patchwork quilt for a doll's bed. It is to be 18 in. wide and 27 in. long. She wants each patch to be a 3-inch square. How many patches does she need?

6. At 36¢ a square foot, find the cost of tiling a bathroom floor that measures 7 ft. 6 in. by 6 ft.

Oriental rugs larger than 9 ft. by 6 ft. are often sold to stores by the square foot. At $2.25 a square foot, find how much a store pays for one rug of each of these sizes:

7. 8 ft. 8 in. by 9 ft.

8. 6 ft. by 10 ft. 8 in.

9. 8 ft. by 10 ft. 3 in.

10. 6 ft. 8 in. by 8 ft. 9 in.

11. 6 ft. 3 in. by 9 ft. 6 in.

12. 8 ft. 9 in. by 11 ft. 10 in.

Find the areas of these rectangles in square yards:

13. 18 yd. × 10 yd.

14. $7\frac{1}{3}$ yd. × $7\frac{1}{3}$ yd.

15. 12 yd. × 10 yd. 2 ft.

16. $18\frac{3}{4}$ yd. × 13 yd. 1 ft.

17. 6 yd. $1\frac{1}{2}$ ft. × 6 yd. $1\frac{1}{2}$ ft.

18. 11 yd. 1 ft. × 7 yd. $1\frac{1}{2}$ ft.

SQUARE MEASURE

Review this table of *square measure:*

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
43,560 sq. ft. or 160 sq. rd. = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Exercises

1. By drawing a square foot on the board and dividing it into square inches, show that 144 sq. in. = 1 sq. ft. Also show that 9 sq. ft. = 1 sq. yd.

2. May's garden covers $\frac{1}{4}$ A.; Rob's is 125 ft. by 96 ft. Which garden is larger? how much larger?

3. Mr. Allen's garden is 40 rd. by 30 rd. How much fertilizer must he buy for it if 900 lb. are required per acre?

4. Mrs. Jenkins sold a plot of her land 20 rd. by 28 rd. for $75 an acre. How much did she receive for it?

5. Mr. Grant is building a concrete sidewalk $2\frac{1}{2}$ ft. wide and 108 ft. long. Find the cost at $1.95 a square yard.

6. Mr. Marvin owns some land 66 ft. by 250 ft. Find, to the nearest hundredth, what decimal part of an acre he has.

Tell what numbers should be put in place of the dots:

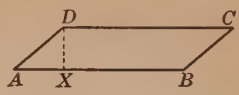
7. 16 sq. yd. = . . . sq. ft.
8. 320 acres = . . . sq. mi.
9. 450 sq. ft. = . . . sq. yd.
10. 180 sq. in. = . . . sq. ft.
11. $\frac{1}{2}$ A. = . . . sq. rd.
12. $\frac{3}{4}$ A. = . . . sq. ft.
13. $\frac{1}{4}$ sq. mi. = . . . A.
14. 4356 sq. ft. = . . . A.

15. Mrs. Thompson's dining room is 12 ft. by 14 ft.; her living room is 15 ft. by 18 ft. For one coat of varnish 1 gal. will cover 600 sq. ft. Find, to the nearest half gallon, how much varnish she must buy to give the floors of both rooms two coats.

PARALLELOGRAMS

You have learned on page 275 that lines like those at the right are called **parallel lines** because they are always the same distance apart.

You are now going to study a figure, $ABCD$, which is called a **parallelogram** because its opposite sides are parallel. Sides AB and DC are parallel and sides AD and BC are parallel. The side AB, on which the parallelogram stands, is its **base** and the dotted line DX, perpendicular to the base at X, is its **height.**

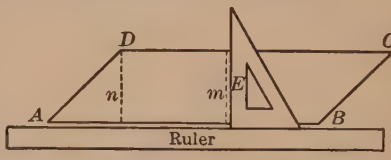


The slanting side AD is *not* the height of the parallelogram because A is not a right angle. Is AD longer or shorter than DX?

How to Draw the Height of a Parallelogram

Since the height of a parallelogram is perpendicular to its base, you can draw the height with a ruler and a triangle.

To draw the height of $ABCD$, lay a ruler along the base AB and place a triangle on it as shown here. Then draw a line m along edge E of the triangle. This line is the *height* because it makes a right angle with the base. The height may be drawn at any point on the base. Hence n is also the height because it is perpendicular to the base and has the same length as m.



Exercises

1. With your ruler and triangle draw parallelograms about the same shape as those below. Draw the height of each.

2. Measure the base and the height of each of the parallelograms that you drew in ex. 1.

THE AREA OF A PARALLELOGRAM

Draw a parallelogram $ABCD$ and draw its height CX, as in Fig. 1. Cut off the triangle CXB with a pair of scissors and

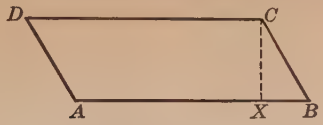


FIG. 1

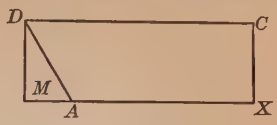


FIG. 2

place it in the position of M in Fig. 2. You have then built, from the parallelogram $ABCD$, a rectangle with the same base, the same height, and the same area as the parallelogram. The area of the rectangle equals its base times its height. Hence

$$\textit{Area of parallelogram} = \textit{base} \times \textit{height}$$

or, briefly,

$$A = bh$$

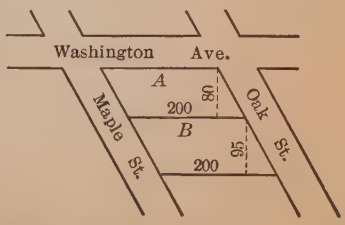
Here b stands for the *base* and h stands for the *height.*

Exercises

1. Alice's flower bed has the shape of a parallelogram with a base of 20 ft. and a height of 8 ft. Find its area.

Use the formula $A = bh$. If $b = 20$ and $h = 8$, what does A equal?

2. Oak Street and Maple Street are parallel streets which cut Washington Avenue diagonally. Lot A has a base of 200 ft. and a height of 80 ft. Lot B has a base of 200 ft. and a height of 95 ft. Find the area of each lot.

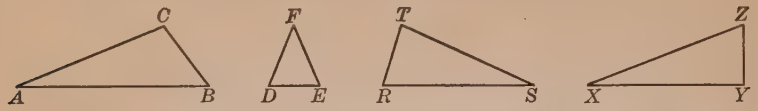


Find the areas of the following parallelograms:

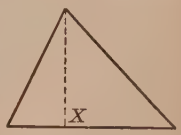
	BASE	HEIGHT		BASE	HEIGHT
3.	11 ft.	9 ft.	**6.**	14 ft. 4 in.	$12\frac{1}{4}$ ft.
4.	14 rd.	$9\frac{1}{4}$ rd.	**7.**	4 yd. 2 ft.	3 yd. 1 ft.
5.	$12\frac{1}{2}$ ft.	$8\frac{1}{2}$ ft.	**8.**	15 ft. 6 in.	9 ft. 4 in.

TRIANGLES

1. A **triangle** is a figure with three sides. Its *base* is the side on which it stands. Thus, the base of triangle ABC is AB. Name the bases of the other triangles below.



2. The *height* of a triangle is the distance *from its highest point to its base.* In the triangle at the right, the dotted line, which makes a right angle with the base at X, is the height. Hence the height of a triangle is *perpendicular* to its base.



You can draw the height of a triangle with a ruler and a triangle just as you draw the height of a parallelogram. (See page 298.)

3. Draw triangles shaped like those at the top of the page. Draw the height of each, using a ruler and a triangle.

Finding the Area of a Triangle

From a piece of cardboard cut two triangles exactly like A and place them together to form B. You see that B is a

parallelogram made from 2 equal triangles; hence each triangle is $\frac{1}{2}$ the parallelogram. Since the area of the parallelogram equals its base times its height, the area of each triangle is $\frac{1}{2}$ as large.

$$\textit{Area of triangle} = \tfrac{1}{2} \times \textit{base} \times \textit{height}$$

That is, $A = \frac{1}{2}bh$ or $A = \dfrac{bh}{2}$

FINDING THE AREAS OF TRIANGLES

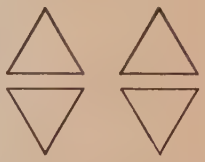
Find the area of a triangular flower bed whose base is 30 ft. and whose height is 15 ft.

Here $b = 30$ and $h = 15$. Using the formula $A = \frac{bh}{2}$, you multiply 30 by 15 and divide by 2, as shown at the right. The area of the bed is 225 sq. ft.

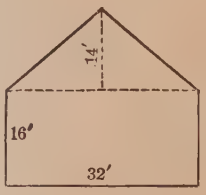
$$\frac{\overset{15}{\cancel{30}} \times 15}{\cancel{2}_{1}} = 225$$

Exercises

1. In the garden at Seaside Park they are laying out 4 triangular flower beds, as shown at the right, each having a base of 20 ft. and a height of $17\frac{1}{2}$ ft. What is the total area of the 4 beds?



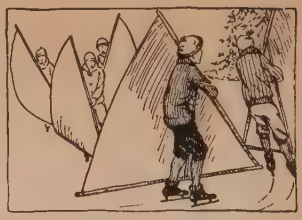
2. The picture shows one end of Mr. Brown's barn. It is made of what two kinds of figures? Find its total area. The barn has two sides with a total area of 2560 sq. ft., and also another end just like that in the picture. How many gallons of paint must Mr. Brown buy to paint the barn if 1 gal. of paint covers 300 sq. ft.? The smallest size can holds 1 gal.



Find the areas of triangles with these dimensions:

	Base	Height		Base	Height
3.	$9\frac{1}{2}$ rd.	7 rd.	**9.**	18 yd. 1 ft.	6 yd.
4.	12 in.	7 in.	**10.**	10 ft. 3 in.	8 ft.
5.	15 yd.	8 yd.	**11.**	15 yd. 1 ft.	9 yd.
6.	30 ft.	16 ft.	**12.**	20 ft. 6 in.	10 ft.
7.	$15\frac{1}{2}$ ft.	13 ft.	**13.**	17 ft. 6 in.	10 ft.
8.	18 yd.	25 ft.	**14.**	12 ft. 8 in.	$18\frac{1}{2}$ ft.

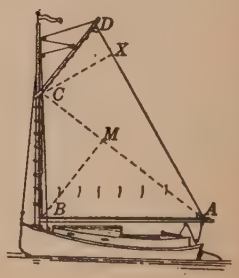
THE WINGED SKATERS HAVE A RACE



1. In a sail race on ice skates where the boys used triangular sails, the rules said that no sail should contain more than 20 sq. ft. Find the areas of the following sails. Which boys were not allowed to compete? Which boys had little chance to win because of small sails?

	Base of Sail	Height of Sail		Base of Sail	Height of Sail
Dick,	5 ft.	5 ft.	Fred,	5 ft.	6 ft.
John,	6 ft.	7 ft.	Paul,	6 ft.	$6\frac{2}{3}$ ft.
Ralph,	6 ft.	$6\frac{1}{2}$ ft.	Will,	$5\frac{1}{2}$ ft.	$6\frac{1}{2}$ ft.
Harry,	$5\frac{3}{4}$ ft.	$7\frac{1}{4}$ ft.	Earl,	$5\frac{3}{4}$ ft.	$7\frac{1}{2}$ ft.

2. This picture shows George's catboat. To get the area of its sail, draw line AC to divide it into two triangles. First find the area of triangle ACD, using AD as the base and CX as the height. Then find the area of triangle ABC, using AC as the base and BM as the height. By measuring, George found that $AD = 23$ ft., $CX = 8$ ft., $AC = 21$ ft., and $BM = 10$ ft. What is the total area of the sail?

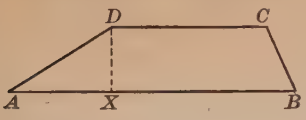


Find the areas of these triangular sails, first expressing both the base and the height in feet:

	Base	Height		Base	Height
3.	9 ft. 6 in.	8 ft.	**6.**	12 ft.	8 ft. 9 in.
4.	13 ft. 3 in.	11 ft.	**7.**	9 ft. 9 in.	7 ft. 6 in.
5.	15 ft. 2 in.	12 ft.	**8.**	9 ft. 4 in.	7 ft. 6 in.

TRAPEZOIDS

This figure is a trapezoid. In a trapezoid two opposite sides are parallel, while the other two are not parallel. Thus, sides AB and DC are parallel, but sides AD and BC are not parallel.

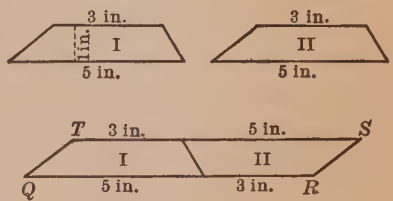


The parallel sides AB and DC are the *bases*, and DX is the *height*. The height of a trapezoid is perpendicular to its base.

Which figures below are trapezoids? Draw their heights.

Finding the Area of a Trapezoid

Cut from paper two equal trapezoids like I and II. Make the upper base 3 in. long, the lower base 5 in., and the height 1 in. Turning II upside down and placing it at the side of I, you will get a long parallelogram $QRST$, as shown above.



The height of parallelogram $QRST$ is 1 in.; its base is 5 in. + 3 in., or 8 in., which is the sum of the parallel sides of one of the trapezoids. The area of the parallelogram is 8×1 sq. in., or 8 sq. in. Since each trapezoid is half as large as the parallelogram, the area of one trapezoid is $\frac{1}{2}$ of 8 sq. in. Hence you have this rule:

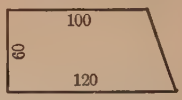
$$\textit{Area of trapezoid} = \frac{\textit{height} \times \textit{sum of parallel sides}}{2}$$

Let a = lower base, b = upper base, and h = height.

Then $$A = \frac{h(a+b)}{2} \quad \text{or} \quad A = \tfrac{1}{2}\,h(a+b)$$

PROBLEMS ABOUT TRAPEZOIDS

1. Mr. Carr has a lot shaped like a trapezoid. The parallel sides are 100 ft. and 120 ft. long, and the height is 60 ft. Find the area of the lot.

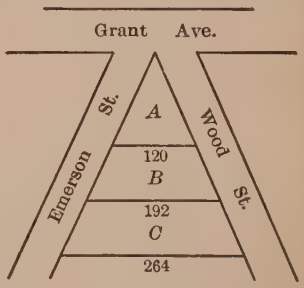


Use the formula $A = \frac{1}{2}\,h(a + b)$. Since $a = 120$, $b = 100$, and $h = 60$, the area $= \frac{1}{2} \times 60 \times 220$. How do you get the number 220? What is the area of Mr. Carr's lot in square feet?

2. Walter made a sail in the shape of a trapezoid, with the upper base 3 ft., the lower base 6 ft., and the height 4 ft. George made a triangular sail with a base of 7 ft. and a height of $5\frac{1}{2}$ ft. Which sail has the larger area? how much larger?

3. The stage of an auditorium is shaped like a trapezoid with bases of 30 ft. and 22 ft. and a depth of 14 ft. Find the area of the stage floor.

4. On this map Lot A has a height of 125 ft., while Lots B and C each have a height of 75 ft. Their other dimensions in feet are given on the map. Find the area of each lot.



Find the areas of these trapezoids:

	Bases	Height		Bases	Height
5.	8 in., 6 in.	4 in.	**12.**	8′ 6″, 9′ 3″	5′ 4″
6.	18 ft., 7 ft.	8 ft.	**13.**	15′ 9″, 9′ 3″	8′ 6″
7.	12 ft., 9 ft.	7 ft.	**14.**	80 ft., 60 ft.	65 ft.
8.	$7\frac{1}{2}$ ft., $6\frac{1}{2}$ ft.	10 ft.	**15.**	38 ft., 27 ft.	$29\frac{1}{2}$ ft.
9.	30 ft., 20 ft.	14 ft.	**16.**	$4\frac{1}{3}$ yd., $5\frac{1}{3}$ yd.	6 yd.
10.	15 rd., 12 rd.	8 rd.	**17.**	$10\frac{1}{2}$ rd., 12 rd.	8 rd.
11.	20 yd., 22 yd.	19 yd.	**18.**	$8\frac{1}{3}$ yd., $7\frac{2}{3}$ yd.	$6\frac{1}{2}$ yd.

IMPROVEMENT TEST No. 21

Test 21 A — Reduction (Time 3 min.)

Change to decimals:

1. 47%	**2.** 6%	**3.** 375%	**4.** $28\frac{1}{2}\%$	**5.** $7\frac{1}{2}\%$
6. 85%	**7.** 1%	**8.** 400%	**9.** 1000%	**10.** $4\frac{1}{4}\%$

Change to common fractions in lowest terms:

11. 80%	**12.** 28%	**13.** $87\frac{1}{2}\%$	**14.** 64%	**15.** 65%

Change to per cents:

16. $\frac{2}{3}$	**17.** $\frac{3}{4}$	**18.** 7	**19.** .45	**20.** $.08\frac{1}{2}$
21. $\frac{3}{8}$	**22.** $\frac{1}{20}$	**23.** 6	**24.** 3.50	**25.** $.16\frac{3}{4}$

Test 21 B — Comparing Numbers (Time 3 min. after copying)

What numbers should be put in place of the dots?

1. 46 is . . .% of 92 **2.** 91 is . . .% of 140

3. 51 is . . .% of 600 **4.** 161 is . . .% of 2300

5. 207 is . . .% of 450 **6.** 145 is . . .% of 2000

Test 21 C — Per Cent of Increase (Time 4 min. after copying)

These numbers show the total sales of stores for two successive months. Find the per cent of increase:

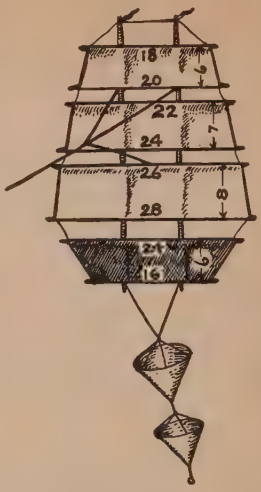
1. \$2475; \$3168 **2.** \$7456; \$9320 **3.** \$3430; \$9604

NOTE. In scoring a test like Test 21 A, first find what per cent of the examples you have right. Then find your score from this per cent, counting 88% as 9, 84% as 8, etc.

SOME INTERESTING KITES

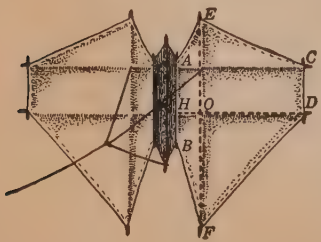
In ex. 2, 4, *and* 5, *find results to nearest tenth.*

1. The kite shown here is called a Chinese junk kite. The three upper trapezoids represent sails and the dark trapezoid at the bottom represents the hull of the junk. The pulling or lifting power of a kite depends upon its total surface or area. Find the total area of the sails and the hull of this kite. The dimensions in inches are given on each trapezoid.



2. In a gentle breeze the wind pushes against a kite with a force of 2 lb. for each square foot of the kite's area, while in a strong breeze the pressure averages $3\frac{1}{2}$ lb. per square foot. How many pounds of pressure would there be against the Chinese junk kite in ex. 1 in a gentle breeze? in a strong breeze?

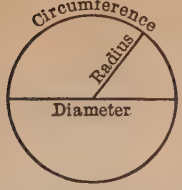
3. In this butterfly kite, each wing is the same size. To find the area of the right wing, divide it into two trapezoids, *ABFE* and *EFDC*. What lines form the bases and the height of each trapezoid? Find the area of the right wing if $AB = 14$ in., $EF = 3$ ft., $CD = 8$ in., $OD = 18$ in., and $HO = 4$ in.



4. Find the total area of the butterfly kite in square feet if the area of the body (the shaded part) is 72 sq. in. For each wing use the area in ex. 3.

5. How many pounds of pressure would there be against the butterfly kite in a gentle breeze? (See ex. 2 and 4.)

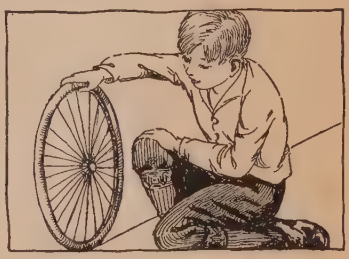
THE CIRCLE



On page 277 you learned how to draw a circle. A line drawn from the center to a point on the circle is a **radius.** A line drawn through the center, with its ends on the circle, is a **diameter.** You see that a diameter is twice as long as a radius. All the radii of a given circle are of the same length. Are all the diameters of a given circle also equal in length?

The distance around a circle is called its **circumference.**

Comparing the Circumference with the Diameter



Bert Gray was told that the length of the circumference of any circle is always a little over 3 times the length of its diameter. To find out if this is true, Bert measured the circumference and the diameter of his bicycle wheel as follows:

To find the circumference of the wheel, Bert chalked a mark on the tire and placed the wheel so that this mark touched a certain point on a straight line on the floor. He then rolled the wheel along the straight line until the mark on the tire again touched the floor. The wheel had then gone around once. Bert next measured the distance on the straight line between the starting and stopping points, and found that it was 88 in. Hence the circumference of the wheel was also 88 in.

Bert then measured the diameter of his wheel and found that it was 28 in. long. He divided 88 by 28 and obtained $3\frac{1}{7}$, or 3.14. Hence the circumference of the wheel was 3.14 times as long as its diameter.

MEASURING CIRCLES

1. On page 307, why didn't Bert measure the circumference of the wheel with a ruler?

2. Anna rolled a plate along a straight line and found that its circumference was about $25\frac{1}{8}$ in. long. She then measured the diameter, which was about $7\frac{15}{16}$ in. By dividing, she found that the circumference was about 3.16 times as long as the diameter.

$$\frac{201}{\cancel{8}_{1}} \times \frac{\cancel{16}^{2}}{127} = \frac{402}{127} = 3.16$$

3. Measure, to the nearest $\frac{1}{16}$ in., the circumference and the diameter of a bicycle wheel, a plate, and other circular objects, using Bert's method. Divide the length of the circumference of each by its diameter, and enter the result, to the nearest hundredth, in a table like this:

Object	Circumference	Diameter	Circum. ÷ Diam.
1. Bert's bicycle wheel	88 in.	28 in.	3.14
2. Plate Anna measured	$25\frac{1}{8}$ in.	$7\frac{15}{16}$ in.	3.16
3. Wheel I measured	_____	_____	_____
4. Plate I measured	_____	_____	_____
5. Phonograph record	_____	_____	_____
6. Automobile tire	_____	_____	_____
7. Hoop	_____	_____	_____
8. Saucer	_____	_____	_____
9. Tomato can	_____	_____	_____

The circumference of a tin can is found by making a mark on one of the circular ends and rolling the can on its side so that the circular end follows a straight line. The diameter is the greatest width across the end of the can.

4. Compare the results in the last column of the table in ex. 3. Is the circumference of each circle a little over 3 times as long as its diameter?

If you find any result in the last column which is smaller than 3 or much larger than 3, you have made a mistake, which is probably due to some inaccuracy in your measurements. In such a case, carefully repeat your measurements and divide again to see if you can get a result closer to 3.14.

A QUICK WAY TO FIND THE CIRCUMFERENCE

If the measurements of circumferences and diameters on page 308 were made very carefully, you would find that the circumference is always 3.14 times as long as the diameter. Therefore if you know the diameter of a circle, you can multiply it by 3.14, or by $3\frac{1}{7}$, to find the circumference.

$$Circumference = 3.14 \times Diameter$$

Let C stand for Circumference, D for Diameter, and the Greek letter π (pronounced "pie") for 3.14. Then

$$C = \pi \times D \qquad \text{or} \qquad C = \pi D$$

Since a diameter equals twice the radius (r), you may write

$$D = 2r$$

Hence $C = \pi 2r$

which is the same as $C = 2\pi r$

The letter π represents 3.1416, to the nearest ten-thousandth; but it is usually more convenient to let $\pi = 3.14$ or $3\frac{1}{7}$.

Finding the Circumference by the Formula

1. Find the circumference of a wheel 14 in. in diameter.

Using the formula $C = \pi D$ and letting $\pi = 3\frac{1}{7}$, you have $C = \frac{22}{7} \times 14$.

2. If the radius of a wheel is 14 in., find the circumference, using the formula $C = 2\pi r$ and letting $\pi = 3\frac{1}{7}$.

Using $\pi = 3\frac{1}{7}$, find the circumferences of these circles:

3. $r = 21$ ft. **5.** $D = 50$ ft. **7.** $r = 7$ yd. **9.** $D = \frac{3}{4}$ mi.

4. $r = 35$ in. **6.** $D = 38$ in. **8.** $r = 6$ ft. **10.** $r = 140$ yd.

Tell what numbers should be put in place of the dots, first letting $\pi = 3.14$ and then letting $\pi = 3\frac{1}{7}$. Compare the answers:

11. If $D = 7$, $C = \ldots$ **15.** If $D = 21$, $C = \ldots$

12. If $r = 7$, $C = \ldots$ **16.** If $r = 42$, $C = \ldots$

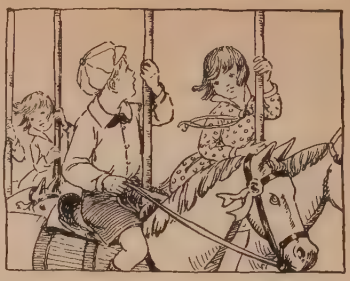
13. If $r = 49$, $C = \ldots$ **17.** If $r = 56$, $C = \ldots$

14. If $D = 42$, $C = \ldots$ **18.** If $D = 9.8$, $C = \ldots$

WHICH CHILD HAD THE LONGER RIDE?

In the following problems, let $\pi = 3\frac{1}{7}$.

Joan and Bob are on the merry-go-round. Bob's horse is 15 ft. from the center and Joan's is 12 ft. from the center.



1. Which child rides farther in 1 revolution of the merry-go-round? how much farther?

The distance of each horse from the center is the radius; the distance each child rides in 1 revolution is the circumference of the circle.

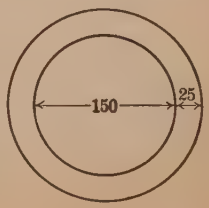
2. How much farther does one child ride than the other in 7 revolutions of the merry-go-round? in 30 revolutions?

3. If Joan's horse were 14 ft. from the center and Bob's 11 ft., how much farther would Joan go in a ride of 35 revolutions than Bob?

Other Problems

1. A bicyclist rides around a circular track 150 ft. in diameter. How far does he ride in 1 trip? in 5 trips? How many times must he go around the track to go 1 mi.?

2. If the track in ex. 1 is 25 ft. wide and the inner diameter is 150 ft., find the diameter of the outside edge of the track. In one trip around the track, how many feet farther must a bicyclist riding on the outside edge go than one on the inside edge?



Find the distances around circular tracks with these dimensions:

3. A radius of 91 ft.

4. A radius of 42 ft.

5. A radius of 48 ft.

6. A radius of 54 ft.

7. A diameter of 196 ft.

8. A diameter of 168 ft.

9. A diameter of 200 ft.

10. A diameter of 420 ft.

A FORMULA FOR THE AREA OF A CIRCLE

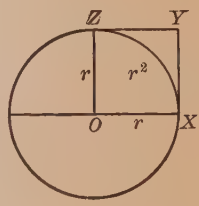
Mr. Bruce is cementing the bottom of a circular skating rink whose radius is 10 yd. He must find its area in square yards in order to find out how much cement to order.

Mr. Bruce has learned that the formula for the area of a circle is $A = \pi r^2$. This formula means that the area (A) equals $3\frac{1}{7}$ times the square of the radius (r^2). r^2 means $r \times r$.

Mr. Bruce uses the formula thus:

The formula is:	$A = \pi r^2$
Since $r = 10$,	$A = 3\frac{1}{7} \times 10 \times 10$
Or,	$A = 314\frac{2}{7}$

Hence the area of the rink is $314\frac{2}{7}$ sq. yd.



You can better understand the meaning of r^2 by studying the figure at the right. The square $OXYZ$ has each side equal to a radius (r) of the circle; therefore its area is $r \times r$, or r^2. The area of the circle is $3\frac{1}{7}$ times as large as the area of this square, or πr^2.

Exercises

1. Use the formula to find how many square feet of cement are needed for a circular rink 35 ft. in radius.

2. Measure, to the nearest sixteenth of an inch, the diameters of at least four of the following: a round table, a large coin, a circular centerpiece, a round cake, the top of a tin can, a phonograph record, the bottom of a pail. Find the radius of each and then compute the area.

3. Amy has a circular flower bed 16 ft. in diameter and another bed $14\frac{1}{2}$ ft. square. Which is larger? how much larger?

Find the areas of circles with these radii. Use $\pi = 3\frac{1}{7}$:

4. 28 in. **5.** 42 in. **6.** 7 mi. **7.** 30 ft. **8.** 18 yd. **9.** 77 yd. **10.** 63 in. **11.** 35 ft. **12.** 49 yd. **13.** 44 ft.

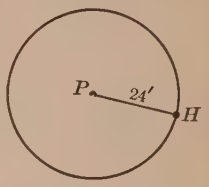
MEASURING CIRCLES

In the following exercises, let $\pi = 3\frac{1}{7}$.

1. A circular swimming pool is 56 ft. in diameter. Find the area of the bottom of the pool in square feet. How much will it cost to cement the bottom at $2.35 per square yard?

2. If Tom swims around the pool in ex. 1, keeping close to the edge, about how many feet does he swim?

3. In a field a horse (H) is tied to a post (P) by a rope 24 ft. long, as shown at the right. How many square feet has he to graze in?

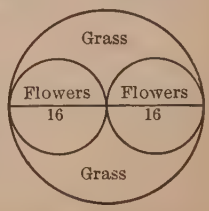


4. If the length of the rope in ex. 3 were doubled, how many times as great an area would the horse have to graze in?

5. Jim stained the top of a circular table $3\frac{1}{2}$ ft. in diameter. How many square feet did he stain?

6. Ruth's birthday cake was baked in a round pan 14 in. across; Ann's was baked in a rectangular pan 14 in. long and 11 in. wide. Both cakes were the same height. Which girl had the larger cake?

7. A circular garden plot 32 ft. in diameter contains 2 flower beds, each 16 ft. in diameter, as shown at the right. The rest of the plot is to be sown with grass. How many square feet are left for grass?



Find the circumferences and the areas of circles having the following dimensions:

8. A radius of 14 ft.
9. A radius of 12 yd.
10. A radius of 10 mi.
11. A radius of $10\frac{1}{2}$ ft.
12. A radius of 112 ft.
13. A diameter of 154 in.
14. A diameter of 210 ft.
15. A diameter of 350 ft.
16. A diameter of 100 yd.
17. A diameter of 520 mi.

REVIEW PROBLEMS

1. Draw a bar graph showing that the numbers of miles of railways in certain countries are as follows: Brazil, 17,907; China, 6818; Germany, 35,209; Norway, 2141; Russia, 42,504; United Kingdom, 23,734. Let 1 square = 10,000 miles.

2. Draw a line graph to show the daily attendance record of your class last month. Did the attendance improve?

3. Last year, \$8104.32 of the total sales of Mr. Hadley's general store covered the cost of the goods, \$2782.08 covered overhead expenses, and \$1209.60 covered profits. Find what per cent of the total sales each of these items was. Then show these per cents on a circle graph.

4. James Horton, a hardware dealer, ordered from Paul Brooks 12 kettles @ \$1.70, 8 carpet sweepers @ \$3.75, 4 wheelbarrows @ \$2.85, and 24 tool grinders @ \$3.20. The discounts were $33\frac{1}{3}\%$, 10% and the terms were 30 da. net, 2% 10 da. Mr. Horton paid in 5 da. Make out and receipt the invoice, indicating the discounts and the net amount. Write the check which Mr. Horton sent to Mr. Brooks.

5. On Jan. 1 Mrs. Peel put \$600 in a savings bank at 4%, compounded quarterly. On Mar. 24 she needed \$600 to pay a bill. Instead of withdrawing the money from the bank, she borrowed \$600 for 30 da. at 6%. How much did she save or lose by not withdrawing the money?

6. Using the 6% method, find the bank discount and the proceeds if you borrow \$850 for 72 da. at 7%.

7. Ann keeps her money in a bank at $4\frac{1}{2}\%$, compounded quarterly. On Apr. 1 she had \$48 in the bank. She deposited \$12 on Apr. 2, withdrew \$4 on Apr. 26, and deposited \$24 on May 15 and \$8.70 on June 8. Make out the page in her pass book for this period. How much had she in the bank on July 1, including interest?

REVIEW

Find the interest on the following by the 6% method:

1. \$800; 60 da. at 6%
2. \$450; 90 da. at 6%
3. \$100; 36 da. at 6%
4. \$400; 30 da. at 5%
5. \$900; 66 da. at 5%
6. \$1000; 6 da. at 5%
7. \$2500; 10 da. at 6%
8. \$3600; 126 da. at 6%
9. \$1200; 100 da. at 6%
10. \$5000; 130 da. at 5%

Find the exact interest, to the nearest cent, on:

11. \$660 for 50 da. at 6%
12. \$820 for 60 da. at 7%
13. \$438 for 67 da. at 6%
14. \$700 for 55 da. at 5%
15. \$350 for 74 da. at 4%
16. \$956 for 146 da. at 7%
17. \$200 for 157 da. at 6%
18. \$610 for 101 da. at 4%
19. \$207 for 219 da. at 5%
20. \$600 for 307 da. at 7%

Find the proceeds on notes for these amounts, discounted at 6%:

21. \$500 due in 96 da.
22. \$620 due in 72 da.
23. \$240 due in 102 da.
24. \$426 due in 120 da.
25. \$960 due in 2 mo. 6 da.
26. \$640 due in 2 mo. 18 da.
27. \$360 due in 3 mo. 12 da.
28. \$280 due in 4 mo. 15 da.

Find the areas of rectangles of these dimensions:

29. $6\frac{1}{2}$ in. $\times$ 6 in.
30. $8\frac{1}{4}$ ft. $\times$ 5 ft.
31. $3\frac{1}{3}$ yd. $\times$ 2 yd.
32. $1\frac{1}{2}$ rd. $\times 1\frac{1}{2}$ rd.
33. $6\frac{1}{3}$ yd. $\times 2\frac{2}{3}$ yd.
34. $9\frac{1}{2}$ rd. $\times 3\frac{1}{2}$ rd.
35. $6.1' \times 2.2'$
36. $4.3' \times 3.8'$
37. $9.5'' \times 7.6''$

Find the areas of these parallelograms:

38. Base $10\frac{1}{2}'$; height $6'$
39. Base $12\frac{1}{3}'$; height $18'$
40. Base $15''$; height $6\frac{1}{4}''$
41. Base $11''$; height $4\frac{5}{8}''$

42. Find areas of triangles with dimensions as in ex. 38–41.

Find the circumferences and the areas of circles with radii of:

43. 7 in. **44.** 14 ft. **45.** 4 mi. **46.** 35 rd. **47.** 12 yd.

CHAPTER V

THREE TYPES OF PROBLEMS IN PERCENTAGE

Last year you used per cents in three types of problems:

1. A bronze statue weighing 50 lb. is 88% copper. How many pounds of copper are there in the statue?

Here you *find a given per cent of a number.*

To find 88% of 50 lb., first change 88% to .88. Then multiply 50 by .88, which gives 44.00, or 44.

There are 44 lb. of copper in the statue.

Check the work by going over it again.

2. Mr. Orr earned \$40 last week. He saved \$3. What per cent of his earnings did he save?

Here you *find what per cent one number is of another.*

First find what fractional part \$3 is of \$40. \$3 is $\frac{3}{40}$ of \$40. Changing $\frac{3}{40}$ to a two-place decimal, you get $.07\frac{1}{2}$. Since per cent means hundredths, $.07\frac{1}{2} = 7\frac{1}{2}\%$.

$$40\overline{)3.00} \qquad .07\tfrac{1}{2} = 7\tfrac{1}{2}\%$$
$$\underline{2\ 80}$$
$$20$$

Mr. Orr saved $7\frac{1}{2}\%$ of his earnings.

Check by taking $7\frac{1}{2}\%$ of \$40 to see if it equals \$3.

3. Jack receives \$9 for selling a radio. If this is 15% of the price of the radio, find the price of the radio.

Here you *find a number when a per cent of it is given.*

15% of the price = \$9.00
Then 1% of the price = \$9.00 ÷ 15, or \$.60
and 100% of the price = 100×\$.60, or \$60.00

Since 100% of the price is the entire price, the radio costs \$60.

To check, see if 15% of \$60 equals \$9.

WHAT A PROFIT OF 9.7% MEANS

Last year the total sales of the Ives Car Company were $1,955,000, and 9.7% of these sales was profit. How much was the company's profit last year?

First change 9.7% to a decimal. To do this, write 9.7% as $\frac{9.7}{100}$. Then divide 9.7 by 100 by moving the decimal point two places to the left; this gives .097. Hence 9.7% = .097. To find the company's profit, multiply $1,955,000 by .097. What is the company's profit?

Check the multiplication by going over it again or by casting out 9's.

To change a per cent to a decimal, drop the sign % and move the decimal point two places to the left.

Thus, 27.3% = .273; 6.45% = .0645; 0.3% = .003.

Finding Profits

1. Mr. Rose averages a profit of 13.5% on his sales. Express 13.5% as a decimal. What is Mr. Rose's profit when he sells $460 worth of goods? $380 worth? $500 worth?

Express these per cents as decimals:

2. 14.6%	**4.** 17.4%	**6.** 83.25%	**8.** 8.2%	**10.** 0.3%
3. 36.1%	**5.** 99.3%	**7.** 62.33%	**9.** 3.5%	**11.** 0.18%

Find these profits correct to the nearest cent:

12. 15% of $850	**16.** 3.4% of $730	**20.** 0.6% of $1450
13. 1.5% of $850	**17.** 2.5% of $775	**21.** 0.8% of $2050
14. 15.5% of $850	**18.** 6.3% of $862	**22.** 50.8% of $1800
15. 1.55% of $850	**19.** 5.7% of $104	**23.** 4.25% of $6600

24. A candy factory sold $31,000,000 worth of candy in a year at 4.1% profit. How much profit was that?

25. The United States produces 580,000,000 tons of coal yearly, of which West Virginia produces 15.7%. How many tons of coal does West Virginia produce in a year?

FINDING PER CENTS TO THE NEAREST TENTH

Arthur borrowed \$120 for 1 yr. from Mr. Gordon and paid him \$3 interest. What rate of interest did he pay?

The rate was $\frac{3}{120}$ of \$120. Changing $\frac{3}{120}$ to a decimal, you get .025. To change .025 to a per cent, move the decimal point two places to the right and annex the sign %, just as you do in changing .46 to 46%. You find that .025 = 2.5%.

Hence Arthur paid interest at the rate of 2.5%.

Check the work by finding 2.5% of \$120 to see if you get \$3.

$$\begin{array}{r} .025 = 2.5\% \\ 120\overline{)3.000} \\ 2\,40 \\ \hline 600 \\ 600 \\ \hline \end{array}$$

To change a decimal to a per cent, move the decimal point two places to the right and annex the sign %.

Thus, .153 = 15.3%; .1825 = 18.25%; and .007 = 0.7%.

Exercises

Change these decimals to per cents:

1. .157	**4.** .755	**7.** .067	**10.** .9675	**13.** .0875
2. .432	**5.** .125	**8.** .095	**11.** .4933	**14.** .0909
3. .625	**6.** .048	**9.** .003	**12.** .6667	**15.** .0625

16. Ed sold an automobile for \$480 and received \$12 commission. What per cent of the selling price was that?

17. The Red Lion team won 18 out of 26 games. Find, to the nearest tenth of 1%, what per cent of the games they won.

Changing $\frac{18}{26}$ to a four-place decimal, you get .6923, or 69.23%. Correct to the nearest tenth of 1%, 69.23% is 69.2%.

Hence the team won 69.2% of the games.

Find per cents in ex. 18 *to* 22 *to the nearest tenth of* 1%.

18. Louise spends \$18 of her salary of \$28 for room and board. What per cent of it does she spend this way?

Find what per cent the first number is of the second:

19. 12, 160 **20.** 43, 75 **21.** 85, 400 **22.** 87, 350

COMPUTING THE STANDING OF BASEBALL CLUBS

Here are the actual records of the number of games won and lost up to a certain date by the teams of the National League. The standing of the teams is found by computing, to the nearest tenth of 1%, what per cent each team has won of all the games it has played.

Team	*Won*	*Lost*	*P. C.*	*P. C.*
Pittsburgh	54	39	.581	58.1
Cincinnati	56	43		
St. Louis	52	44		
Chicago	50	46	↑	↑
Brooklyn	49	49		
New York	46	48	*A*	*B*
Boston	38	57		
Philadelphia	37	56		

1. The Pittsburghs have won 54 games and lost 39 games; hence they have played 93 games in all. To learn their standing, find what per cent 54 is of 93. Carry the result to four decimal places. This gives .5806, which is .581 correct to thousandths, or 58.1% to the nearest tenth of 1%.

```
.5806
93)54.0000
   46 5
   ----
    7 50
    7 44
    ----
     600
     558
     ---
      42
```

The newspapers usually report the standing of the clubs as shown in Column *A* at the top of the page. This method is used only in baseball records and is never found in other statistical reports. The letters *P. C.* above a column mean *per cent;* hence the numbers under *P. C.* in Column *A* should represent per cents. But the rating .581 does not represent .581%, since the Pittsburghs have won 58.1%, and not .581%, of their games. Hence, to be technically correct, the ratings should be written as shown in Column *B*. In Column *A*, where the ratings are reported as three-place decimals like .581, it would be correct if the heading were "Rating" or "Standing" instead of *P. C.*

2. Find, to the nearest tenth of 1%, what per cent of all its games each of the other teams above has won.

3. What would the ratings of the Bostons and of the Philadelphias be if they were found only to the nearest whole per cent? Could you then tell which of these teams is ahead? Then why are the ratings computed to the nearest tenth of 1%?

WHICH TEAM WAS AHEAD?

When the Chicagos had won 50 games and lost 48, the Brooklyns had won 51 games and lost 49. Compute the standing of each team to find which team was ahead.

If the standings of both teams are computed to the nearest tenth of 1%, you find that each team won 51.0% of its games. Hence the teams seem to be tied. But if the computations are carried to another decimal place, the Chicagos have 51.02% and the Brooklyns 51.00%; hence the Chicagos are ahead. When two teams stand the same to the nearest tenth of 1%, the results are found to the nearest hundredth of 1% to determine which team is ahead.

Exercises

Find ratings to the nearest tenth of 1%. If two ratings then seem equal, find them to the nearest hundredth of 1%:

1. AMERICAN LEAGUE

Team	*Won*	*Lost*	*P. C.*
New York	64	34	
Cleveland	56	44	
Philadelphia	51	47	
Detroit	52	48	
Washington	48	46	
Chicago	50	49	
St. Louis	41	57	
Boston	30	67	

2. AMERICAN LEAGUE

Team	*Won*	*Lost*	*P. C.*
New York	65	34	
Cleveland	57	44	
Philadelphia	51	48	
Detroit	52	49	
Chicago	51	49	
Washington	48	47	
St. Louis	41	58	
Boston	31	67	

3. NATIONAL LEAGUE

Team	*Won*	*Lost*	*P. C.*
Pittsburgh	62	46	
St. Louis	65	50	
Cincinnati	65	51	
New York	58	54	
Chicago	59	55	
Brooklyn	55	62	
Boston	47	67	
Philadelphia	42	68	

4. NATIONAL LEAGUE

Team	*Won*	*Lost*	*P. C.*
Pittsburgh	65	47	
St. Louis	69	50	
Cincinnati	68	51	
Chicago	61	57	
New York	58	58	
Brooklyn	57	64	
Boston	47	70	
Philadelphia	43	71	

FINDING BATTING AVERAGES

1. When Bell of St. Louis had been at bat (*A. B.*) 386 times, he had made 136 hits (*H.*). What was his batting average?

You must find what per cent his number of hits was of the number of times he had been at bat. Since 136 is 35.2% of 386, Bell's average was 35.2%.

2. Find the batting averages of these players, to the nearest tenth of 1%. If this gives two players a tie, find their averages to the nearest hundredth of 1%:

Player	*A. B.*	*H.*	*P. C.*
Brown	389	133	
Donohue	71	24	
Cuyler	379	126	
Roush	349	116	
Williams	196	65	
Walker	374	122	
Gibson	46	15	

Player	*A. B.*	*H.*	*P. C.*
Goslin	345	121	
McNulty	37	13	
Collins	330	115	
Welch	89	30	
Bennett	78	26	
Simmons	382	123	
Miller	255	82	

3. When the members of the Cincinnati National Team had been at bat 3910 times, they had made a total of 1023 hits. What was the team's batting average?

4. Find the batting average of each team:

Team	*A. B.*	*H.*	*P. C.*
Cincinnati	3510	1023	
Pittsburgh	3481	1005	
Philadelphia	3490	1000	
Chicago	3625	1034	

Team	*A. B.*	*H.*	*P. C.*
St. Louis	3599	1021	
New York	3520	1000	
Boston	3548	1006	
Brooklyn	3579	968	

5. The leading National League pitcher recently was Meadows of Pittsburgh. He had pitched 18 games and won 14. What per cent of the games pitched had he won?

6. A pitcher won 11 games out of all he had pitched, and lost 4. What per cent of his games had he won?

IMPROVEMENT TEST No. 22

Test 22 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
1183	8469	7774	7235	7832
327	7712	2886	558	625
6917	7876	432	3635	8488
9865	8635	3902	2671	6237
2752	631	7489	9783	816
928	7698	4164	2455	2998
1414	5402	410	9596	925
2849	4710	6967	3479	5190
4792	9854	959	932	3875

Test 22 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
904260	467225	846643	778803
128436	123764	269454	289650
5.	**6.**	**7.**	**8.**
731091	750931	451833	570000
586134	164066	282355	348762
9.	**10.**	**11.**	**12.**
351616	828319	825921	976742
218287	471730	586225	678045

Test 22 C — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.	5.
489	648	901	947	735
764	158	437	908	265

Test 22 D — Division (Time 3 min. after copying)

If there is a remainder, find result to the nearest tenth:

1. $23\overline{)14375}$ **2.** $92\overline{)65236}$

3. $78\overline{)32692}$ **4.** $49\overline{)28861}$

THE EXPENSES OF A GREAT CITY

One year New York spent $21,792,122 for fire protection. What per cent, to the nearest tenth of 1%, was this expense of the city's total expenditure of $399,618,885?

To work this problem, you may find what per cent 21,792,122 is of 399,618,885, as in *A*. The answer is 5.5%.

A.

$$399618885\overline{)21792122.0000}\quad .0545 = 5.5\%$$

19980944 25
1811177 750
1598475 540
212702 2100
199809 4425
12892 7675

B.

$$39962\overline{)2179.0000}\quad .0545 = 5.5\%$$

1998 10
180 900
159 848
21 0520
19 9810
1 0710

A quicker way is first to express 399,618,885 and 21,792,122 in *round numbers* as 399,620,000 and 21,790,000. Then the expenses for fire protection are $\frac{21,790,000}{399,620,000}$, or $\frac{2179}{39962}$, of the total expenditures. Changing this fraction to a per cent, as in *B*, you again get 5.5% as answer.

In round numbers 399,618,885 becomes 399,620,000 because the thousands' digit is more than 5, and 21,792,122 becomes 21,790,000 because the thousands' digit is less than 5.

Exercises

Express figures in round numbers to the nearest thousand or ten thousand. Find answers to the nearest tenth of 1%.

1. One year the city of New York spent $94,758,925 for education out of a total expenditure of $399,618,885. What per cent of the total were the expenses for education?

2. In the same year the city of New York spent $34,421,939 for public health. What per cent of the total was that?

3. Find what per cent of the total expenses each of these expenditures was: $16,236,971 for state taxes; $24,392,784 for hospitals and homes.

FACTS ABOUT POPULATION

Express figures in round numbers to the nearest thousand or ten thousand. Compute per cents to the nearest tenth of 1%.

1. At the 1920 census, the United States had a total population of 105,710,620. The number of people living in cities was 54,304,603. What per cent of the entire population was this?

2. The population of Mississippi was 1,790,618, of which 240,121 people lived in cities and 1,550,497 in the country. What per cent of the inhabitants of Mississippi lived in cities? What per cent lived in the country?

3. In the state of Texas, which had a population of 4,663,228, there were 1,512,689 persons living in centers of 2500 or more inhabitants. What per cent of the population of the state lived in these centers? If the rest of the population lived in the country, what per cent lived in the country?

4. New Jersey had 3,155,900 inhabitants, of whom 2,474,936 lived in cities and 680,964 lived in rural districts. What per cent of the total population lived in cities? What per cent lived in rural districts?

5. New York State had 10,385,227 inhabitants, of whom 1,795,387 lived in country districts. What per cent lived in the country? What per cent lived in cities?

6. Rhode Island had a total population of 604,397, with 589,180 people living in cities. What per cent of the population of this state lived in cities?

7. Which of the states in ex. 2 to 6 had a large per cent of their inhabitants living in cities? Which state had the largest per cent living in country districts?

8. In 1910 the United States had a rural population of 49,806,146 and an urban population of 42,166,120. Was a greater per cent of the total population living in cities in 1910 or in 1920 (see ex. 1)?

PER CENTS SMALLER THAN 1%

1. For collecting a bill for \$320, Mr. Nash charges $\frac{1}{2}$ of 1% of the amount of the bill. How much does he charge?

To find $\frac{1}{2}$ of 1% of \$320, first find 1% of \$320, which is \$3.20. Then find $\frac{1}{2}$ of \$3.20. The charge is \$1.60.

$\frac{1}{2}$ of 1% is sometimes written $\frac{1}{2}$%; but the form $\frac{1}{2}$ of 1% is much easier to interpret.

1% of \$320 = \$3.20

$\frac{1}{2}$ of 1% of \$320 = \$1.60

2. A bank charges Mr. Blake 0.1% for collecting \$600 for him. How much does the bank get?

Change 0.1% to a decimal by moving the decimal point two places to the left. This gives .001. Multiplying \$600 by .001 gives \$.60. Hence the bank gets \$.60.

In business 0.1% is usually written as $\frac{1}{10}$ of 1%. In science, however, the form 0.1% is more often used.

\$ 600
.001
\$.600

Exercises

1. Each workman in the Fulton Factory contributes $\frac{1}{5}$ of 1% of his wages to a sick fund. If Mr. Case receives \$180 a month in wages, how much does he give to the fund?

Find the following:

2. $\frac{1}{2}$ of 1% of \$800

3. $\frac{1}{4}$ of 1% of \$400

4. $\frac{3}{4}$ of 1% of \$840

5. $\frac{1}{10}$ of 1% of \$90

6. 0.5% of \$840

7. 0.6% of \$750

8. 0.2% of \$255

9. 0.1% of \$700

10. 0.1% of \$720

11. 0.2% of \$445

12. 0.4% of \$245

13. 0.3% of \$110

14. When the United States had 105,710,620 inhabitants, the District of Columbia had 437,571. What per cent of the country's population had the District of Columbia?

Express the figures in round numbers and find the answer to the nearest tenth of 1%.

15. Which is greater, $\frac{1}{2}$ of 1% of \$200 or 50% of \$200? $\frac{1}{2}$ of 1% of \$1000 or 0.5% of \$1000?

PER CENTS LARGER THAN 100%

1. The whole of a number is 100% of it because 100% = 1.00, or 1. Thus, 100% of the pupils in your class is the same as 1 × the number of pupils, or all the pupils.

2. Similarly, 200% = 2.00, or 2; hence 200% of a number is 2 × the number. 300% of a number is 3 × the number.

3. To find 135% of \$1900, change 135% to 1.35. Then multiply \$1900 by 1.35, which gives \$2565. Why should the answer be more than \$1900?

Exercises

Express the following as decimals:

1. 400%	**3.** 500%	**5.** 209%	**7.** 780%	**9.** 201%
2. 354%	**4.** 115%	**6.** 162%	**8.** 468%	**10.** 1200%

Express these as per cents, using the rule on page 317:

11. 2.84	**13.** 1.75	**15.** 18.08	**17.** 4.12	**19.** 3.
12. 3.05	**14.** 4.62	**16.** 10.05	**18.** 1.93	**20.** 15.

21. Last summer there were 380 boys at Camp Sloan. Now the enrollment is 130% as great. Find the present enrollment.

22. Mr. Mack's income this year was 3 times as great as last year. Express this in percentage language.

Find the following and check the work:

23. 125% of \$868	**27.** 128% of \$495	**31.** 1200% of \$890
24. 300% of \$550	**28.** 302% of \$166	**32.** 2000% of \$330
25. 812% of \$127	**29.** 115% of \$980	**33.** 1525% of \$180
26. 401% of \$336	**30.** 785% of \$288	**34.** 1850% of \$425

35. Mr. Foster earns \$50 a week. Mr. Burns earns 150% as much. How much does Mr. Burns earn?

36. Five years ago Mr. Amson had \$4500 in the savings bank. He now has 350% of that amount. How much has he now?

INCREASE AND DECREASE

1. Five years ago Mr. Gay's land was worth $2400. Its value has since increased 150%. What per cent of the former value is the present value? How much is the land worth now?

The former value is 100% of itself. The land is now worth 150% more, or 250% of its former value. What is 250% of $2400?

2. During the World War an imported dye increased 300% in price. What per cent was the later price of the original price? If the dye cost $2 a pound before the war, what was its cost during the war?

3. In May eggs cost 25% less than in January. What per cent was the May price of the January price?

Tell what numbers should be put in place of the dots:

4. 150% more than $500 = 250% of $500, or $1250.

5. 100% more than $250 = . . .% of $250, or . . . .

6. 40% less than $2000 = . . .% of $2000, or . . . .

7. $30 increased by 20% of itself = 120% of $30, or . . . .

8. $20 increased by 80% of itself = . . .% of $20, or . . . .

9. In 5 yr. a boy's weekly salary was raised from $8 to $28. The increase was what per cent of the original salary?

The amount of increase was $28 − $8, or $20. You must find what per cent $20 is of $8. $20 ÷ $8 = 2.50, or 250%. Hence the boy's salary was increased 250%.

$$8\overline{)20.00}\quad 2.50 = 250\%$$

10. In one year a factory increased the value of its daily output from $2600 to $3120. Find the per cent of increase.

Below are the profits of stores for two successive years. Find, to the nearest tenth of 1%, the per cent of increase or decrease:

11. $5600; $7000	**14.** $7800; $7098	**17.** $3640; $3093
12. $3680; $3450	**15.** $1200; $2580	**18.** $5600; $6652
13. $8350; $9698	**16.** $6336; $7128	**19.** $4500; $3987

FEEDING PLANTS

1. This picture shows a field of corn. To the part on the right a fertilizer containing the substances needed for plant growth had been applied. The part on the left received no fertilizer. The fertilized part produced 58.6 bu. per acre, while the unfertilized part yielded 28.5 bu. per acre. Find, to the nearest tenth of a per cent, the per cent of gain in production due to fertilizer.

2. An unfertilized farm in a certain state produced 42.8 bu. of sweet potatoes per acre. A neighboring farm that was well fertilized produced 203.5 bu. per acre. Find, to the nearest tenth of a per cent, the per cent of increase in production due to fertilizer.

3. An unfertilized field yielded 127 bu. of onions per acre, while a fertilized field produced 756 bu. per acre. Find the per cent of increase due to fertilizer.

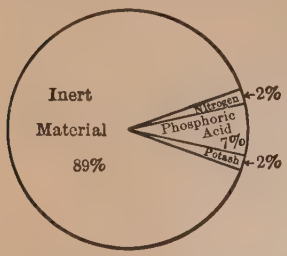
4. An unfertilized acre of land produced 112 boxes of oranges, while a fertilized acre of the same kind of land produced 420 boxes. Find the per cent of increase due to fertilizer.

5. A sugar plantation of 1450 acres was fertilized at a cost of \$16 per acre. The fertilization increased the production by 1.6 tons of sugar per acre. How many extra tons of sugar were produced? If sugar sold for \$92.75 a ton, find the total net profit from fertilizing the farm.

Net profit = value of extra sugar − cost of fertilizer.

WHAT SHALL WE FEED TO PLANTS?

A complete fertilizer contains *nitrogen*, *phosphoric acid*, and *potash*. Most fertilizers also contain much *inert material*, which is dry soil or other substances that do not contain any plant food. The graph at the left shows the comparative amounts of each kind of plant food in a ton of a complete fertilizer.



CONTENTS OF A COMPLETE FERTILIZER

The table below names ten common fertilizers and tells what per cent of each is nitrogen, phosphoric acid, and potash, and what each costs per ton.

TRADE NAME	NITROGEN	PHOSPHORIC ACID	POTASH	COST PER TON OF 2000 LB.
Nitrate of Soda	15.0%			$59.50
Sulfate of Ammonia	20.4%			72.50
Dried Blood	14.3%			58.00
Muriate of Potash			52.0%	39.50
Sulfate of Potash			48.0%	42.00
Kainit			13.0%	13.80
Acid phosphate		16.0%		15.00
Ground Bone	3.0%	24.0%		46.00
Tankage	6.2%	11.0%		48.50
Cottonseed Meal	7.0%	2.5%	1.5%	51.50

1. Find the number of pounds of nitrogen, of phosphoric acid, and of potash in a ton of each of the above fertilizers.

2. Find, to the nearest tenth of a cent, the cost per pound of the nitrogen in nitrate of soda; in sulfate of ammonia; in dried blood. In which is nitrogen cheapest per pound?

In ex. 2–3 assume that the inert material costs nothing.

3. Is potash cheapest per pound in muriate of potash, in sulfate of potash, or in kainit?

COMMERCIAL FORMS

Depositing Money in a Commercial Bank

You learned last year (pages 263 to 270) that a commercial bank renders many services to its depositors in connection with their checking accounts and that it lends out at interest much of the money that it receives in deposits. A checking account is a great convenience for a business man. The opportunity to borrow money from a bank when necessary is also an advantage.

When John Lee deposits money in the bank where he has a checking account, he fills out a **deposit slip** (see page 263). This deposit slip tells how much money he is putting in the bank and how much of it is in coin, in bills, and in checks.

Mr. Lee can draw any of his money from the bank by means of a **check.** If he owes $220.30 to Roy Page of Detroit, he writes the check shown below. This check is an order on the Empire Bank of New York, where Mr. Lee keeps his money, to pay $220.30 to Mr. Page. Mr. Lee mails this check to Detroit to Mr. Page. What happens after that to the check is shown on the next page.

New York, Nov. 2, 19— No. 4267

THE EMPIRE BANK

Pay to the order of Roy Page ——— $220 30/100

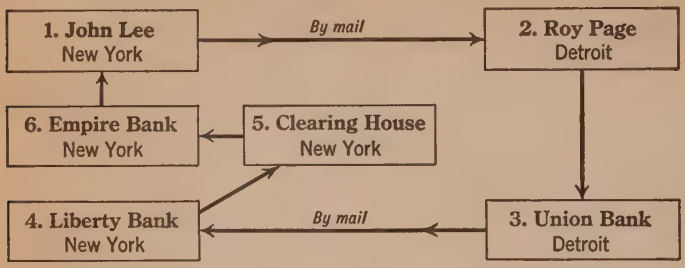
Two hundred twenty and 30/100 ——— Dollars

John Lee

Before Mr. Page can deposit this check in his bank or receive cash for it, he must *indorse* it by writing his name on the back of it across the left-hand end. Show two different ways of indorsing a check (see page 265).

THE TRAVELS OF A CHECK

The check at the bottom of page 329 is written because John Lee of New York owes $220.30 to Roy Page of Detroit. Its whole history is one of travel, as is shown below.



Step 1. John Lee has a checking account in the Empire Bank of New York. He writes a check for $220.30 on this bank and mails it to Roy Page in Detroit.

Step 2. The check arrives at Mr. Page's office in Detroit.

Step 3. Mr. Page indorses the check and deposits it in the Union Bank of Detroit, where $220.30 is added to his account.

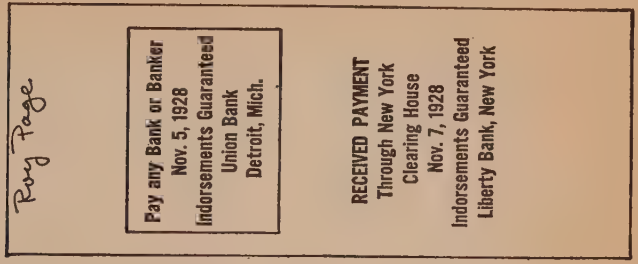
Step 4. Most banks in large cities like New York, Chicago, San Francisco, Detroit, and Philadelphia do business with one bank in each of the other large cities. The Union Bank indorses Mr. Lee's check and mails it to the Liberty Bank in New York, with which it does business.

Step 5. The check arrives at the Liberty Bank in New York, and this bank sends it by messenger the next morning to the New York Clearing House, where it is handed over to a messenger from the Empire Bank. Thus Mr. Lee's check gets back to the Empire Bank, on which it was drawn.

Step 6. The Empire Bank deducts $220.30 from Mr. Lee's account. At the end of the month the bank returns the canceled check to him. If he is wise, he will keep this check as a receipt.

TRACING THE TRAVELS OF A CHECK

When Mr. Lee's check (see page 329) is returned to him, he finds across the back of it the three indorsements shown below. These record the check's travels since the day he first wrote it.



1. Mr. Page indorsed the check at the Union Bank just before depositing it. Why did he wait until then to indorse it?

2. Which indorsement shows that the Union Bank of Detroit sent the check to another bank? Does this indorsement show where they sent it? What statement on the back of the check shows that the Union Bank sent the check to the Liberty Bank?

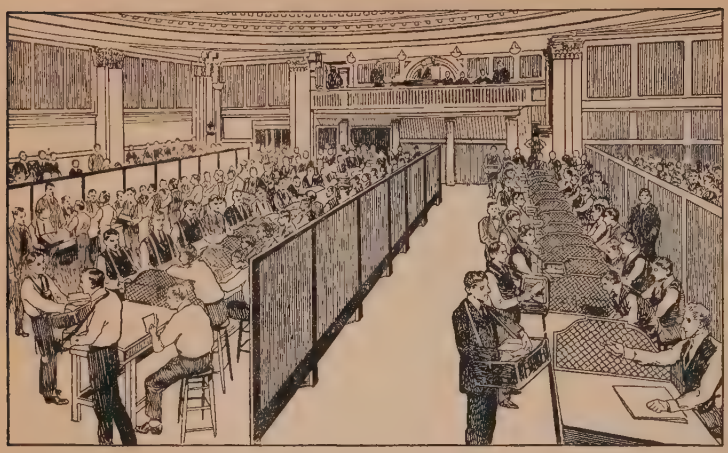
3. On what day and where does the Liberty Bank turn the check over to the Empire Bank?

4. Ask some one who has a checking account to show you some checks that have come back from the bank. Study these checks and their indorsements and try to diagram their travels.

5. Rose Mason sends a check for $54.20 on the Copley Bank of Boston to Ada Jones of Denver. Mrs. Jones deposits it in the Commercial Bank of Denver, which sends it to the National Bank of Boston. This bank takes it to the Boston Clearing House. Where does it go then? Diagram its travels.

6. Guy Adams writes Fred Towns of Chicago a check for $3.50 on the First National Bank of St. Louis. Mr. Towns has his account in the Central Bank of Chicago, which does business with the Columbia Bank of St. Louis. Diagram the check's travels.

THE WORK OF THE CLEARING HOUSE



Every large city in the United States has a clearing house organized and maintained by the banks of that city. At each clearing house messengers from the banks of the city meet daily to exchange the checks which those banks have against one another and to balance the difference.

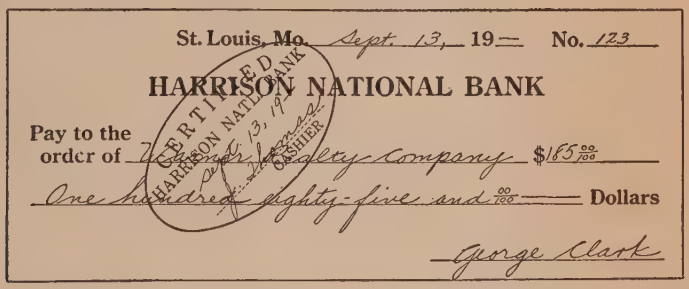
The above picture shows the interior of the New York Clearing House while the banks are exchanging checks. At each desk is seated a man representing some New York bank. In front of each desk stands a messenger from another bank, delivering checks which his bank holds against the bank represented at that desk. The messengers who are standing move from desk to desk until they have delivered to the proper banks all the checks in the leather cases which they are carrying.

The total amount of the checks that change hands in a clearing house is called the **clearings.** In New York the clearings are often over $1,000,000,000 a day. The actual exchange of all the checks representing this huge amount is made in the Clearing House within 15 minutes.

CERTIFIED CHECKS

The Warner Realty Company of Chicago sends George Clark of St. Louis a bill for $185 and asks for a **certified check** in payment. This means that the company wants Mr. Clark's bank to guarantee that he has enough money to pay the check.

Mr. Clark makes out a check for $185. The cashier at his bank stamps it " Certified," as shown below.


St. Louis, Mo. Sept. 13, 19— No. 123
HARRISON NATIONAL BANK
Pay to the order of Warner Realty Company $185.00
One hundred eighty-five and 00/100 Dollars
George Clark
CERTIFIED
HARRISON NATIONAL BANK
Sept. 13, 19
CASHIER

The bank deducts $185 at once from Mr. Clark's account and holds it until the check comes in for payment.

Exercises

1. The Warner Realty Company, to which Mr. Clark sends the above certified check, has an account with the Capitol Bank of Chicago, which does business with the Traders Bank of St. Louis. Make a diagram showing the travels of the check. St. Louis has a clearing house.

2. Mr. Clark has $389.50 in the bank before the above check is certified. What is his balance after it is certified?

3. After the check is certified, Mr. Clark deposits $31 in bills, $42.20 in coin, and a check for $45. Make out the deposit slip. What is his bank balance then?

4. If Mr. Clark then presents a check for $400 to his bank to be certified, will the cashier certify it? Why not? If the check were for $69, would the bank certify it?

PAYING A BILL BY A BANK DRAFT

Alan Curt, who runs a small shop in Seattle, must pay in advance for $112.44 worth of goods which he orders from James Robb of Chicago. He therefore buys a **bank draft** for this amount from The Mechanics Bank of Seattle.

Seattle, Wash., Sept. 2, 19— No. 1275

THE MECHANICS BANK

Pay to the order of Alan Curt ——— $112.44

One hundred twelve and 44/100 ——— Dollars

To THE FIRST NATIONAL BANK
Chicago, Ill.

Henry Macy
Cashier

In this draft the Seattle bank orders The First National Bank of Chicago to pay $112.44. Mr. Curt has the draft made out payable to himself, rather than to Mr. Robb, so that his name will appear on it, thus showing that he sent the draft.

Mr. Curt now indorses the draft as follows: "Pay to the order of James Robb. Alan Curt," and mails it to Mr. Robb.

When Mr. Robb receives the draft, he deposits it in his bank in Chicago, which sends it to the Chicago Clearing House. Here it is turned over to The First National Bank of Chicago. The draft is finally returned to The Mechanics Bank of Seattle, which keeps it.

A bank draft is really a check of one bank on another bank.

Exercises

1. In the bank draft above, which bank is ordered to pay Mr. Curt? Where is its name printed on the draft? On a check, where is the name of the bank that is ordered to pay?

2. In the above draft, which bank orders the payment of the money? Who orders payment on a check?

3. Why does Mr. Curt send a bank draft instead of a check?

COLLECTING A BILL BY A SIGHT DRAFT

You have learned that banks provide a convenient means of paying bills. Banks also help in the *collection* of bills.

If the Chelsea Coal Company of Pittsburgh sends Charles Moss of Baltimore a bill for $450 and he fails to pay it, the company may make out this **sight draft:**

$450 00/100 Pittsburgh, Pa., Jan. 31, 1928

At sight pay to the order of

The Hamilton National Bank, Pittsburgh

Four hundred fifty and 00/100 Dollars

Value received and charge to the account of

To Charles Moss
Baltimore, Md.

Chelsea Coal Company
per B. P. S.

The Chelsea Coal Company leaves this draft with The Hamilton Bank, where it has an account. This bank indorses the draft and sends it to a bank in Baltimore. A representative of the Baltimore bank presents the draft to Mr. Moss. If he owes the money and wants to keep his business reputation, Mr. Moss will pay the $450 at once. The Baltimore bank will then send the money to The Hamilton Bank of Pittsburgh, which will credit it to the Chelsea Coal Company's account after deducting about $\frac{1}{10}$ of 1% of the $450 as a collection fee.

Exercises

1. In a sight draft, the name of the one who is ordered to pay is written in the lower left-hand corner. Is this true of a bank draft? of a check?

2. Is Mr. Moss more likely to pay his debt after receiving a sight draft from a bank in his own city than after receiving a letter from Pittsburgh from the Chelsea Coal Company? Why?

3. How does The Hamilton Bank indorse the draft before sending it to Baltimore?

MAKING OUT BANK DRAFTS AND SIGHT DRAFTS

Use the drafts on pages 334 *and* 335 *as models.*

1. To pay George Wood of Chicago $274.50, Morris Wells bought a bank draft from the Pacific Bank of Los Angeles, which does business with the American Bank of Chicago. Write the draft in favor of Mr. Wells and indorse it in full for him.

2. The Pacific Bank charged a fee of $\frac{1}{10}$ of 1% for the draft. What fee did Mr. Wells have to pay? How much did he pay in all for the draft?

3. Mr. Wood deposited the draft in the La Salle Bank of Chicago. Where would this bank send it? Trace the travels of the draft, using a diagram.

4. Write a bank draft for $160.30 of a bank in St. Louis on a bank in Philadelphia. Let it be made out to the order of Ruth Cook and indorsed to the order of Jane Downes.

5. Philip Galt of St. Paul owed Max Thorp of New Orleans $685. As Mr. Galt did not pay the money when it was due, Mr. Thorp made out a sight draft payable to the order of the Colonial Bank of New Orleans. Write the draft.

6. In ex. 5, describe what happened from the time the Colonial Bank got the draft until Mr. Thorp got the money.

7. If the Colonial Bank charged Mr. Thorp $\frac{1}{10}$ of 1% for collecting the $685, how much of the $685 did he receive?

8. What is the purpose of a sight draft? of a bank draft?

9. You owe Albert Meek of Portland, Maine, $75.30 and you have no checking account. What kind of draft will you send him? Write the draft and indorse it.

10. Thomas Stone of Little Rock, Ark., has failed to pay $813.50 which he owes you. What kind of draft will you use to collect the money? Write the draft, using the name of some bank in your city. If your bank's fee is $\frac{1}{10}$ of 1%, how much will you receive in all?

IMPROVEMENT TEST No. 23

Test 23 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$7\frac{5}{16}$	$6\frac{2}{3}$	$2\frac{3}{5}$	$4\frac{1}{4}$	$8\frac{5}{6}$
$9\frac{1}{2}$	$4\frac{1}{6}$	$1\frac{1}{4}$	$\frac{1}{8}$	$3\frac{1}{3}$
$\frac{3}{4}$	$3\frac{5}{6}$	$1\frac{1}{5}$	$7\frac{1}{3}$	$2\frac{3}{4}$
$1\frac{7}{16}$	$8\frac{1}{2}$	$1\frac{3}{10}$	$5\frac{1}{4}$	$9\frac{5}{12}$

Test 23 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$5\frac{1}{8}$	$4\frac{3}{4}$	$9\frac{3}{16}$	$6\frac{3}{4}$
$2\frac{5}{8}$	$3\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{1}{3}$
5.	**6.**	**7.**	**8.**
4	$7\frac{2}{3}$	$9\frac{2}{5}$	$8\frac{5}{8}$
$1\frac{5}{6}$	$3\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{4}$
9.	**10.**	**11.**	**12.**
6	$6\frac{2}{5}$	$8\frac{5}{6}$	$3\frac{5}{16}$
$2\frac{7}{10}$	$3\frac{1}{3}$	$6\frac{7}{8}$	$2\frac{1}{8}$

Test 23 C — Multiplication (Time 4 min. after copying)

1. 48	**2.** \$1.75	**3.** $\$1.37\frac{1}{2}$
$7\frac{5}{8}$	$6\frac{3}{4}$	23
4. $\frac{5}{8}$ of 35	**5.** $\frac{15}{16}\times\frac{4}{5}$	**6.** $8\frac{1}{8}\times\frac{4}{15}$
7. $\frac{5}{6}$ of 64	**8.** $\frac{8}{9}$ of $4\frac{5}{16}$	**9.** $2\frac{4}{5}\times5\frac{5}{6}$

Test 23 D — Division (Time 4 min. after copying)

1. $\frac{7}{8}\div 2$	**2.** $18\div\frac{3}{8}$	**3.** $3\frac{1}{2}\div5\frac{1}{4}$
4. $\frac{9}{16}\div\frac{5}{12}$	**5.** $6\frac{3}{8}\div 3$	**6.** $1\frac{2}{3}\div8\frac{1}{3}$
7. $2\frac{1}{2}\div 4$	**8.** $6\div1\frac{4}{5}$	**9.** $12\frac{3}{8}\div\frac{3}{4}$
10. $9\frac{3}{8}\div 6$	**11.** $\frac{15}{16}\div2\frac{1}{12}$	**12.** $13\frac{1}{5}\div3\frac{1}{7}$

SENDING MONEY BY A POSTAL MONEY ORDER

Another safe way of sending money, besides the check and the bank draft, is the **postal money order.** Mr. Moore buys a money order for $50 from a post office in his home town and sends it to Mr. Lent. Mr. Lent presents it to a post office in his town and receives $50.

At the right are the fees for postal money orders. The fee for Mr. Moore's order for $50 is 18¢. Hence the order costs $50.18 in all.

Amount	*Fee*
$0.01 to $2.50	5¢
2.51 to 5.00	7¢
5.01 to 10.00	10¢
10.01 to 20.00	12¢
20.01 to 40.00	15¢
40.01 to 60.00	18¢
60.01 to 80.00	20¢
80.01 to 100.00	22¢

Exercises

How much, including the amount of the order and the fee, would you pay for each of these postal money orders?

1. $2.00 **2.** $18.00 **3.** $25.00 **4.** $62 00 **5.** $80.00 **6.** $80.20 **7.** $75.00 **8.** $11.50 **9.** $1.20 **10.** $2.80 **11.** $4.75 **12.** $5.12 **13.** $20.02 **14.** $33.80 **15.** $99.50 **16.** $100.00

17. No money order is issued for more than $100. To send $150 by postal money order, you must buy two orders, one for $100 and one for $50. How much would these orders cost you?

18. What postal money orders must you buy, and what is the total fee, to send $120.12? to send $135.50? $240.35? $76.60?

19. Mr. Ford buys postal money orders for $100, $74.30, $65.48, $170, $4.02, and $160.21. How much could he save on them by having a checking account?

20. Money can also be sent by express money order and by telegraph. Find out what you can about these methods.

SIMPLE INTEREST

Computing Interest by the 6% Method

Before learning about borrowing money from banks, you must review the 6% *method* of computing interest (see page 251), for which you learned the following rule:

To find the interest at 6% *for* 60 *da., move the decimal point in the principal two places to the left; to find the interest for* 6 *da., move the decimal point three places to the left.*

1. Find the interest on \$400 at 6% for 72 da.

Change 72 da. to 60 da. + 6 da. + 6 da. 60 da. is \$4.00; for 6 da. it is \$.40; for 72 da. it is \$4.00 + \$.40 + \$.40, as shown at the right.

Always change the time into parts of, or multiples of, 60 da. or of 6 da. Thus, 30 da. = $\frac{1}{2}$ of 60 da.; 120 da. = 2×60 da.; 2 da. = $\frac{1}{3}$ of 6 da.; 54 da. = 60 da. − 6 da., etc. (see page 251).

The interest on \$400 for

\$4.00 Int. for 60 da.
.40 Int. for 6 da.
.40 Int. for 6 da.
\$4.80 Int. for 72 da.

2. Find the interest on \$240 for 90 da. at 7%; at 5%.

The interest for 90 da. at 6% is \$3.60. The interest for 90 da. at 1% is $\frac{1}{6}$ of \$3.60, or \$.60. Hence the interest for 90 da. at 7% is \$3.60 + \$.60, or \$4.20.

Likewise, the interest on \$240 for 90 da. at 5% is \$3.60 − \$.60, or \$3.

\$2.40
1.20
\$3.60 Int. at 6%
.60 Int. at 1%
\$4.20 Int. at 7%

Exercises

By the 6% *method, find the interest at* 6%, 7%, *and* 5% *on:*

1. \$200 for 60 da.
2. \$350 for 36 da.
3. \$800 for 90 da.
4. \$480 for 54 da.
5. \$900 for 80 da.
6. \$125 for 60 da.
7. \$950 for 12 da.
8. \$785 for 42 da.
9. \$370 for 96 da.
10. \$250 for 75 da.
11. \$500 for 66 da.
12. \$100 for 30 da.
13. \$280 for 90 da.
14. \$360 for 72 da.
15. \$120 for 78 da.
16. \$450 for 120 da.
17. \$150 for 152 da.
18. \$2400 for 93 da.

FINDING THE EXACT DAYS BETWEEN TWO DATES

Suppose you wish to borrow money from a bank from July 9 until Sept. 1. To find the interest, you must first count the exact number of days from July 9 to Sept. 1, thus:

From July 9 to July 31 there are	22 da.
In the whole month of August there are	31 da.
Sept. 1, the last day of the loan, counts as . . .	1 da.
Hence from July 9 to Sept. 1 there are	54 da.

In finding the exact number of days between two dates, count the last day, but not the first day.

In computing interest on loans, most banks count the exact number of days between dates and use 360 da. as a year.

Exercises

Find the exact number of days from:

1. Apr. 6 to Oct. 3
2. Aug. 1 to Dec. 18
3. Apr. 9 to Aug. 30
4. Mar. 3 to Aug. 18
5. Mar. 15 to Aug. 8
6. Mar. 28 to May 2
7. Mar. 2 to July 3
8. July 5 to Dec. 17
9. June 3 to Aug. 15
10. Aug. 4 to Sept. 16
11. Sept. 15 to Nov. 5
12. May 11 to Sept. 12

13. Mr. Lunt borrowed \$90 at 6% from May 1 to June 30. For how many days did he borrow it? What was his interest?

14. Mrs. Case borrowed \$1000 from Mar. 1 to May 12. How many days did she keep the money? Find the interest at 6%.

Counting exact days between dates and letting 360 *da.* = 1 *yr., find the interest at* 6%. *Use the* 6% *method:*

15. \$200; June 1 to Aug. 15
16. \$525; May 12 to Aug. 4
17. \$460; Apr. 10 to June 3
18. \$100; Mar. 4 to Apr. 3
19. \$650; Oct. 2 to Dec. 7
20. \$500; Sept. 16 to Dec. 15

A TABLE FOR FINDING SIMPLE INTEREST

Most banks find interest by using a table like that below.

Table for Finding Simple Interest at 6%

Based on 360 days per year

Days	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000
1	.017	.033	.05	.067	.083	.10	.117	.133	.15	.167
2	.033	.067	.10	.133	.167	.20	.233	.267	.30	.333
3	.050	.100	.15	.200	.250	.30	.350	.400	.45	.500
4	.067	.133	.20	.267	.333	.40	.467	.533	.60	.667
5	.083	.167	.25	.333	.417	.50	.583	.667	.75	.833
6	.100	.200	.30	.400	.500	.60	.700	.800	.90	1.000
7	.117	.233	.35	.467	.583	.70	.817	.933	1.05	1.167
8	.133	.267	.40	.533	.667	.80	.933	1.067	1.20	1.333
9	.150	.300	.45	.600	.750	.90	1.050	1.200	1.35	1.500
10	.167	.333	.50	.667	.833	1.00	1.167	1.333	1.50	1.667
20	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00	3.333
30	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50	5.000
40	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00	6.667
50	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50	8.333
60	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00	10.000
70	1.167	2.333	3.50	4.667	5.833	7.00	8.167	9.333	10.50	11.667
80	1.333	2.667	4.00	5.333	6.667	8.00	9.333	10.667	12.00	13.333
90	1.500	3.000	4.50	6.000	7.500	9.00	10.500	12.000	13.50	15.000
100	1.667	3.333	5.00	6.667	8.333	10.00	11.667	13.333	15.00	16.667

1. At 6%, find the interest for 50 da. on $200; on $20.

Find 50 da. in the left-hand column of the table, and lay a ruler or the edge of a sheet of paper under the 50 and horizontally across the table. The number just above the ruler in the column marked $200 is 1.667. Hence the interest on $200 for 50 da. at 6% is $1.667, or $1.67.

Since $20 is $\frac{1}{10}$ of $200, the interest on $20 is $\frac{1}{10}$ of $1.667, which is obtained by moving the decimal point in $1.667 one place to the left. This gives $.1667, or $.17, as the interest on $20 for 50 da. at 6%.

2. Find the interest on $220 for 50 da. at 6%.

Since $220 = $200 + $20, add the interest on $200 for 50 da. and the interest on $20 for 50 da., as found in ex 1. This gives $1.667 + $.1667, or $1.8337. Keep the extra decimal places in $1.667 and $.1667 until you get the sum, $1.8337. Then express the result as $1.83.

USING THE INTEREST TABLE

All loans below are at 6% simple interest.

1. Find from the interest table the interest on a 10-day loan of \$400; of \$600; of \$800; of \$1000. Do you have to change the position of your guiding paper in this example?

2. What is the interest on \$600 for 20 da.? on \$50 for 20 da.? on \$650 for 20 da.?

3. Tell how to find the interest on \$730 for 30 da.

4. Find the interest for 40 da. on \$300; on \$30; on \$3; on \$333; on \$3000.

Using the table, find the interest, to the nearest cent, on:

5. \$600 for 6 da. **8.** \$50 for 30 da. **11.** \$330 for 60 da.

6. \$200 for 8 da. **9.** \$80 for 40 da. **12.** \$580 for 40 da.

7. \$900 for 7 da. **10.** \$90 for 80 da. **13.** \$140 for 20 da.

14. Using the table, find the interest on \$840 for 53 da.

Think of \$840 as \$800 + \$40, and of 53 da. as 50 da. + 3 da.

First find the interest for 50 da. on \$800 and on \$40, as shown at the right. Then find the interest for 3 da. on \$800 and on \$40.

Int. for	50 da. on	\$800 =	\$6.667
Int. for	50 da. on	40 =	.333
Int. for	3 da. on	800 =	.40
Int. for	3 da. on	40 =	.02
			\$7.420

Add these four results, which gives \$7.42 as the total interest.

15. Find the interest for 25 da. on \$700; on \$500; on \$900.

16. Find the interest on \$200 for 34 da.; for 93 da.

Using the table, find the interest, to the nearest cent, on:

17. \$550 for 48 da. **21.** \$200 for 166 da. **25.** \$325 for 90 da.

18. \$620 for 54 da. **22.** \$810 for 110 da. **26.** \$675 for 60 da.

19. \$380 for 42 da. **23.** \$950 for 170 da. **27.** \$148 for 50 da.

20. \$470 for 33 da. **24.** \$830 for 275 da. **28.** \$442 for 90 da.

29. Check your answers in ex. 25 to 28 by the 6% method.

BORROWING FROM A BANK

Fred Shaw is known by The Colonial Bank to be reliable. Therefore he is able to borrow from this bank $350 at 6% by writing this **promissory note:**

$350 00/100 Denver, Colo., Mar. 4, 19~

Three months after date I promise to pay

to the order of The Colonial Bank

Three hundred fifty and 00/100 Dollars

Value received

No. 148 Due June 4, 19~ Fred Shaw

Interest is not mentioned in this note because The Colonial Bank collects the interest *in advance*. Interest paid to a bank in advance is called **bank discount.**

Since the money is borrowed on Mar. 4 for 3 mo., it is due on June 4. It is exactly 92 da. from Mar. 4 to June 4. By the interest table you find that the interest on $350 for 92 da. at 6% is $5.37. The bank *discounts the note* by deducting this $5.37 in advance from the $350. Mr. Shaw receives the difference, $344.63, which is called the **proceeds.**

Exercises

1. How much does Mr. Shaw pay the bank on June 4?

2. Make out a note, dated Aug. 1, to any bank, promising to pay $90 in 60 da. Find the bank discount at 6%. How much do you get on Aug. 1? How much do you pay in 60 da.?

At 6%, find the bank discount and the proceeds on these loans:

3. $600 from Apr. 2 to May 2

4. $120 from Mar. 8 to Apr. 9

5. $180 from Aug. 15 to Nov. 15

6. $550 from Mar. 22 to July 23

7. $1400 for 60 da.

8. $1600 for 30 da.

9. $1250 for 30 da.

10. $2500 for 90 da.

11. Write the notes in ex. 7 to 10, supplying names, etc.

GIVING SECURITY FOR A NOTE

A man whose financial responsibility is not well known must give security to a bank for the money he borrows or else have some one in good standing indorse his note. Stocks and bonds are often given as security, or **collateral,** for a note.

$1500.00 Los Angeles, Aug. 16, 1928

On demand FOR VALUE RECEIVED I

promise to pay to The First National Bank or order, at said Bank, fifteen hundred and no/100 DOLLARS, with interest at the rate of 6% per annum, payable quarterly on the first days of January, April, July, and October, having deposited with said Bank as collateral security for the payment of this note the following property:

2 California Railroad $1000 - 5% bonds due 1944

having a present market value of $2020 * * * * I hereby give to said Bank full power to sell the whole or any part of said collaterals, on the non-performance of any of the promises herein contained.

* * * * * * * *

John Sherman

The above **collateral note** for $1500 is secured by bonds worth $2020. Because bonds change in value, banks usually lend only 80% or less of the value of the collateral.

This note is payable *on demand* and is called a **demand note.** If the bank should need the money or lose confidence in John Sherman, it could demand payment at once. Otherwise he can keep the money until he is ready to repay it. Most demand notes are secured by collateral or by indorsement. Since you do not know when the demand note will be paid, the interest is not collected in advance; it is usually paid quarterly, interest being charged for the exact number of days in the quarter.

1. Tell on what dates interest is due on the above note. Why is the interest not paid in advance?

2. How many days are there from Aug. 16 to Oct. 1? At 6%, how much interest is paid on Oct. 1?

3. How much interest is due Jan. 1, 1929? If the $1500 is repaid Mar. 4, 1929, how much interest is due then?

DISCOUNTING A TRADE ACCEPTANCE

Burton Heath sells Floyd Grady $597 worth of goods and sends this draft, or **trade acceptance,** with the shipment:

TRADE ACCEPTANCE

No. 177 Chicago, June 14, 1928 $597.00/100

Sixty days after date pay to the order of Ourselves

Five hundred ninety seven and 00/100 Dollars

The obligation of the purchase of goods ACCEPTED June 18, 1928 Payable at City Bank, Seattle, Wash. acceptor arises out of the from the drawer.

To Floyd Grady, Seattle Due Aug. 13, 1928 Date Payable at Signed Burton Heath

When "Accepted" is written across the face of the acceptance, it becomes a promissory note which Mr. Grady promises to pay in 60 da.; but, unlike other notes, it draws no interest. After acceptance, Mr. Grady returns the paper to Mr. Heath.

If Mr. Heath needs the money on June 22, which is 52 da. before Mr. Grady has agreed to pay it, he may take the acceptance to a bank and *discount* it, receiving $597 less the bank discount. The bank discount on $597 for 52 da. at 6% is $5.17; hence Mr. Heath receives $597 − $5.17, or $591.83. The bank now holds the acceptance until Aug. 13, when Mr. Grady pays $597 to the bank.

Exercises

1. On Mar. 11 George Wills of Cincinnati sent Arthur Moon of Buffalo a trade acceptance for $384, payable in 90 da. Write out the acceptance and accept it for Mr. Moon. When is the acceptance due? If Mr. Wills sells it to a bank on Mar. 21, how much does he get for it if it is discounted at 6%?

2. Write a trade acceptance for $214, drawn on Glenn Owen, dated Sept. 5, and due in 30 da. Accept it for Mr. Owen. If on Sept. 15 you discount the acceptance at a bank at 5%, how much do you get for it? How much does Mr. Owen pay the bank?

DISCOUNTING AN INTEREST-BEARING NOTE

Owen Judd borrowed money from his friend, David Ross. He gave his friend this promissory note:

$800 00/100 Nashville, Tenn., Mar. 21, 1928

Ninety days after date I promise to pay

to the order of David Ross

Eight hundred and 00/100 Dollars

with interest at 6%

Due June 19, 1928 Owen Judd

This note states that the rate of interest is 6%. When the note is due, Mr. Ross will get the \$800 principal + \$12 interest, or \$812 in all. Why is interest mentioned on this note, but not on the note to a bank shown on page 343?

Suppose that on Apr. 26 Mr. Ross needs the \$800 very much. Mr. Judd does not have to pay the note until June 19. Therefore Mr. Ross indorses the note and sells it to a bank on Apr. 26. The total value of the note, including interest, will be \$812 on June 19; but the bank does not give Mr. Ross that amount. Since the bank is really lending Mr. Ross this \$812 from Apr. 26 to June 19 (54 da. in all), it deducts in advance the bank discount on \$812 for 54 da. at 6%, which is \$7.31. Hence Mr. Ross gets \$812 − \$7.31, or \$804.69, for the note. On June 19, when the note is due, Mr. Judd will pay \$812 to the bank. June 19 is the **day of maturity** of this note.

In figuring its discount, the bank finds the exact number of days to the day of maturity; but it counts 360 da. as 1 yr.

To find the bank discount on an interest-bearing note, first find the amount (interest + principal) to be paid on the note when it is due; then compute the bank discount on this amount, at the bank's rate, from the day on which the note is discounted until the day on which it is due.

DISCOUNTING NOTES AND TRADE ACCEPTANCES

Write notes bearing interest at 6% for the following, supplying names. Find the total amount due on each on the day of maturity, counting exact days between dates:

	Amount of Loan	Date of Loan	Date Due		Amount of Loan	Date of Loan	Date Due
1.	$480	June 1	Aug. 6	**5.**	$1200	Mar. 1	June 29
2.	$520	Apr. 4	Apr. 14	**6.**	$1600	May 2	Oct. 29
3.	$600	Mar. 18	June 17	**7.**	$2000	Apr. 21	May 11
4.	$850	Mar. 24	June 22	**8.**	$3600	Mar. 20	Dec. 31

9. The note in ex. 1 was sold to a bank on July 1. How many days did the bank hold it? Find the bank discount on it at 7%. How much did the bank pay the man who sold it? How much did the bank receive on Aug. 6?

10. Find the proceeds on the note in ex. 6 if it was discounted by a bank on July 1 at 6%. How much did the bank receive for it at maturity?

11. If the notes in ex. 3, 4, 5, 7, and 8 are each discounted on May 1 at 6%, find the proceeds on each.

12. John Miller sends Hugh White a trade acceptance for $450 on May 6, payable in 90 da. On what date is it due?

13. If Mr. Miller sold the trade acceptance (ex. 12) on June 20 to a bank which discounted it at 6%, how much did he get? How much did Mr. White pay the bank on Aug. 4?

14. Find the date of the day of maturity if it comes 90 da. after Mar. 2; 60 da. after Apr. 5; 30 da. after Oct. 15; 90 da. after May 1; 60 da. after Sept. 3.

15. Name two sources from which a business man may borrow money and five ways in which he can send money to Denver. Name two commercial forms which help him collect the bill for a shipment to Seattle, and tell the difference between them.

FINDING THE RATE OF INTEREST

Unless otherwise specified, the rate of interest charged in borrowing money always means the rate for one year.

1. John borrowed \$120 from Mr. Fox. He paid it back in 1 yr. with \$18 interest. What rate of interest did he pay?

Since John paid \$18 interest on \$120 for 1 yr., you must find what per cent \$18 is of \$120. Since \$18 is 15% of \$120, John paid 15% interest.

2. Alice borrowed \$80 from Mr. Wood. She repaid it in 3 mo. with \$5 interest. What rate of interest did she pay?

If the interest is \$5 for 3 mo., this is at the rate of \$20 interest for 1 yr. Since \$20 is 25% of \$80, Alice paid interest at the rate of 25% yearly.

To find the yearly rate of interest, first find the amount of interest that would be charged if the loan were for 1 *year; then find what per cent this amount is of the principal.*

Exercises

1. John Carter borrowed \$150 from a man who charged him \$4.50 a month interest. How much interest did he pay in a year? What yearly rate was he paying?

2. Mr. Mace paid \$18 quarterly interest on a note for \$1200. Find the annual interest; the rate of interest.

3. A man gave his brother \$288 for the use of \$800 for 3 yr. Find the interest for 1 yr. and the yearly rate.

Find the yearly rate of interest on the following:

	Principal	Time	Interest		Principal	Time	Interest
4.	\$300	1 yr.	\$30	**9.**	\$1500	3 mo.	\$30
5.	\$450	1 yr.	\$27	**10.**	\$2000	1 mo.	\$80
6.	\$510	1 yr.	\$85	**11.**	\$9000	1 mo.	\$225
7.	\$350	3 mo.	\$35	**12.**	\$1750	2 yr.	\$245
8.	\$125	6 mo.	\$15	**13.**	\$3600	3 yr.	\$540

IMPROVEMENT TEST No. 24

Test 24 A — Addition (Time 3 min.)

1.	2.	3.	4.
296.41	906.32	21.312	95.826
373.85	84.98	39.595	9.712
69.92	424.76	68.471	31.482
847.26	630.33	5.046	97.678
53.18	575.65	16.717	73.981
408.95	25.44	25.877	46.789
218.64	894.19	35.835	2.028

Test 24 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
2684.51	69.7348	7.00000	111.300
1132.84	8.9364	2.53691	2.243
5.	**6.**	**7.**	**8.**
866.315	58.2842	522.621	7369.80
141.467	5.2667	55.273	684.48
9.	**10.**	**11.**	**12.**
76309.7	5129.23	81.6731	877.654
58524.9	2741.89	19.0475	18.459

Test 24 C — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.
600	.051	5.47	549
.59	18	0.6	.900
5.	**6.**	**7.**	**8.**
863	.009	1.39	.018
.98	.07	.65	500
9.	**10.**	**11.**	**12.**
2.74	.906	62	8.07
17	12	.001	4000

Test 24 D — Division (Time 3 min. after copying)

Find each quotient correct to the nearest hundredth:

1. $5.8\overline{)3.909}$ **2.** $42\overline{)199.9}$ **3.** $.77\overline{)70.8}$

INSTALLMENT BUYING

Buying on the Installment Plan

1. The Miller family want to buy some furniture. It will cost \$88 if they pay all cash or \$98 if they pay \$18 cash and the rest in installments of \$16 a month.

How many months will it take the Miller family to pay for the furniture if they buy it on the installment plan?

2. If the Millers already have \$18 and can save \$16 a month, how long would it take them, to the nearest half month, to save enough to pay cash for the furniture? How much sooner can the family have the use of the furniture if they buy it on the installment plan?

3. Some people find it easier to pay a small amount weekly on furniture which they already have than to save up enough money in advance to pay all cash for the furniture. Why?

4. When a store sells on the installment plan, it has the extra expenses of investigating its customer's reputation, of collecting the weekly or monthly payments, and of taking the risk that some customer may fail to pay all his installments. Is the extra \$10 which is charged on the installment plan in ex. 1 all extra profit to the company? Is this extra \$10 an extra cost to the customer who buys on installments?

5. A farmer can buy a double harness by paying \$54.75 cash or by paying \$5 down and \$5 a month for 11 mo. How much more is the installment price than the cash price?

6. An electric washing machine is sold for \$20 down and payments of \$7 a month for 1 yr. A discount of $12\frac{1}{2}\%$ is offered for cash payment. How much can Mrs. Thomas save by paying cash when she buys the machine?

7. A piano can be purchased for \$420 cash or on installments as follows: \$10 down and \$12 monthly for 38 mo. Find the difference between the cash and the installment price.

RATE OF INTEREST IN INSTALLMENT BUYING

Mrs. Ames wants some new dining-room furniture. Its cash price is \$120. Mrs. Ames has not the \$120 and she cannot borrow it from a bank because she has nothing to offer as security. But the company will sell her the furniture for \$135 and let her pay \$30 down and \$15 a month for the next 7 mo. Thus the company, by letting Mrs. Ames retain the use of part of the price of the furniture, is really lending her money. Find the yearly rate of interest on this loan.

First let us see how much the company really lends Mrs. Ames. If Mrs. Ames buys the furniture for \$135 on Feb. 1 and pays \$30 down, she owes \$135 − \$30, or \$105, during February. On Mar. 1 she pays another \$15 and thus reduces the loan during March to \$90. On Apr. 1 she pays another \$15; therefore she owes only \$75 during April. How much does she owe during May? June? July? August?

Feb.	\$105 for 1 mo.
Mar.	90 for 1 mo.
Apr.	75 for 1 mo.
May	60 for 1 mo.
June	45 for 1 mo.
July	30 for 1 mo.
Aug.	15 for 1 mo.
Total	\$420 for 1 mo.

The table above shows the amounts which Mrs. Ames owes for periods of 1 mo. each. Adding these amounts, you find that she owes in all the equivalent of \$420 for 1 mo.

Since Mrs. Ames pays \$135 for furniture whose cash price is \$120, she pays \$15 more than the cash price. This \$15 may be considered as interest on the money she owes. Hence she pays \$15 interest on a total loan of \$420 for 1 mo. To find the rate of interest, you must first find the amount of interest that would be charged for 1 yr., as shown on page 348. Interest of \$15 a month on \$420 is the same as interest of \$180 a year on \$420. Hence the rate of interest which Mrs. Ames pays on the \$420 is $\frac{180}{420}$, or about 43%. Is 43% a high yearly rate of interest to pay on a loan?

INTEREST IN INSTALLMENT BUYING

1. The cash price of a phonograph is \$118. Charles buys it for \$145 on the installment plan, paying \$25 down and \$15 monthly for 8 mo. Show, by a table like that on page 351, that the company lends him the equivalent of \$540 for 1 mo. How much interest does Charles pay for 1 mo.? Find the equivalent yearly interest and the yearly rate of interest.

Find, to the nearest whole per cent, the rate of interest charged under the installment plan on the following:

2. *Electric Washing Machine*

Cash Price	\$75
Installment Price	\$85
\$5 cash, \$10 monthly	

3. *Great Authors Series*

Cash Price	\$18
Installment Price	\$20
Terms, \$2 down and \$3 a month	

4. *Higrade Gas Stove*

Cash Price	\$44
Installment Price	\$50
\$10 cash, \$5 a month	

5. *Athletes' Dumbbells*

Cash Price	\$25
Installment Price	\$30
Terms, \$15 down, \$3 weekly for 5 weeks	

6. Mrs. Roe can buy a fur coat on the installment plan for \$150 on these terms: \$15 down and \$15 a month for 9 mo. If she pays all cash, she gets a discount of 10% from the price of \$150. What is the cash price? Find, to the nearest whole per cent, what annual rate of interest Mrs. Roe pays if she buys the coat on the installment plan.

7. At the Star Store a vacuum cleaner can be purchased for \$65 cash, or for \$12.50 down and \$5 a month for 12 mo. At Green's Department Store the same kind of vacuum cleaner can be purchased for \$65 cash or for \$12 down and \$10 a month for 6 mo. The installment plan is equivalent to paying what rate of interest at the Star Store? at Green's Store? Which store makes the better offer?

PROBLEMS

1. Bertha Foster bought a rebuilt typewriter for \$50, paying \$5 down and \$5 a month. After 2 mo. she did not need the typewriter any more, so she stopped her monthly payments and the company took back the typewriter. How much did it cost her to have the typewriter 2 mo.?

2. Bertha (ex. 1) could have rented a similar typewriter at \$2 a month. How much cheaper would this have been than buying it on the installment plan and failing to pay after 2 mo.? Is it wise to buy on the installment plan if you are not sure that you will make all the payments?

3. If Bertha had used the typewriter for 3 yr., how much cheaper would buying it on the installment plan have been than renting it (see ex. 1 and 2)? Who would have owned the typewriter at the end of the 3 yr. in each case?

4. Clyde Smith found that he could buy a radio set for \$150 cash or for \$175 on the following terms: \$25 down and \$25 a month for the next 6 mo. If he bought the set on the installment plan, what yearly rate of interest would he have to pay?

5. Clyde's uncle offered to lend him \$125 for 5 mo. at 6%. If Clyde borrows this amount from his uncle to buy the radio set for cash, how much cheaper will this be for Clyde than buying on the installment plan?

Clyde would only have to borrow \$125 from his uncle, because even on the installment plan he would have to pay \$25 down.

6. On Jan. 1 Harvey Mason borrowed \$100 from the Bell Loan Company, agreeing to pay it back at the rate of \$10 monthly for 10 mo., beginning Feb. 1. The company charged him \$6 interest and \$2 for investigating his reputation. So far as Harvey was concerned, this was equivalent to a total interest charge of \$8. What rate of interest did Harvey really pay?

As on the installment plan, he owed \$100 for 1 mo., \$90 for 1 mo., etc.

THE COST OF INSTALLMENT BUYING

1. How do the rates of interest on installment buying, as found on pages 352 and 353, compare with the rates usually charged for loans by banks?

2. Does a bank which lends money to a well-known man on good security take as great a risk as a furniture store which sells to comparative strangers on the installment plan? Does the bank or the store have more work in investigating its customers, in collecting the payments, and in bookkeeping?

3. Many states have laws that allow interest rates of from 1% to 3% a month on small loans (loans of not over $300). To what yearly rates are these rates equal? Which rates found on pages 352 and 353 are within the limits permitted for small loans? Which rates are higher?

A Summary of Installment Buying

You have seen that the installment plan is often a help to the purchaser in enabling him to have the use of articles sooner than if he had to pay cash. It also forces him to save regularly in order to make regular payments. These advantages are so apparent that about $\frac{1}{7}$ of the retail buying in the United States is now done on the installment plan.

But if a person can pay cash, it is much cheaper to do so, because it costs the merchant more to sell on the installment plan. The customer should consider installment buying as the equivalent of a small loan and see what yearly rate of interest he will have to pay. If the rate is too high, he will be wise to wait until he has saved enough to pay cash for the article. The rate is usually higher for luxuries like jewelry than it is for necessities like household goods. While the customer has to pay a high rate of interest in installment buying, the merchant does not make so high a rate on his money. Why not?

MR. COX BORROWS MONEY TO BUY A HOUSE

In order to buy a house, Mr. Cox borrows \$6000 from an insurance company. He must pay back \$7 a month for each \$1000 borrowed. He must also pay interest each month on what he still owes, at an annual rate of 6%.

Since he borrows \$6000, Mr. Cox must pay back $6 \times \$7$, or \$42, a month. At the end of the first month he pays \$42 plus interest on \$6000 for 1 mo. at 6%, which is \$30. Hence he pays $\$42 + \$30$, or \$72, the first month.

By the 60-day method, the interest on \$6000 for 2 mo. at 6% is \$60. For 1 mo. it is \$30.

Mr. Cox now owes $\$6000 - \$42$, or \$5958. The second month he again pays \$42 plus interest on \$5958 for 1 mo. at 6%, which is \$29.79. Hence he pays \$71.79 the second month.

The principal is now $\$5958 - \$42$, or \$5916, and interest for 1 mo. on this amount is \$29.58. Hence at the end of the third month the payment is $\$42 + \$29.58$, or \$71.58.

As the principal decreases \$42 each month, the interest and the total payment also decrease, as shown below:

	Principal	Payment on Principal	Interest on Principal	Total Payment
For 1st month	\$6000	\$42	\$30.00	\$72.00
For 2d month	5958	42	29.79	71.79
For 3d month	5916	42	29.58	71.58
For 4th month	5874	42	29.37	71.37

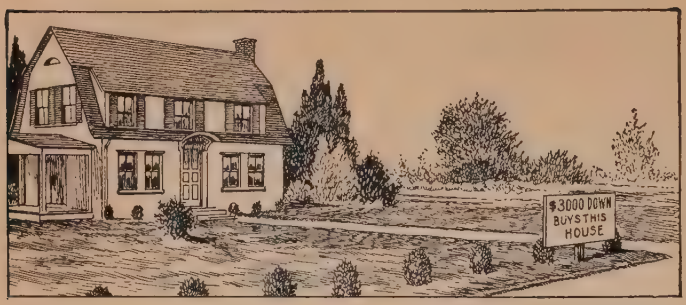
Exercises

1. Continue the above table for the next 12 months.

2. How much will Mr. Cox pay back on the principal in 4 yr.? What will be the principal after the 48th payment? What will be the interest on the 49th payment?

3. Find the principal after the 138th payment. Then make a table like that above, showing the payments after the 138th month. How long does it take to pay the entire debt?

OWN YOUR OWN HOME, PAY LIKE RENT



Mr. Howe sells a house to Mr. Lane for $9000. Mr. Lane pays $3000 cash and gives Mr. Howe a **mortgage** for the other $6000, agreeing to pay it off in monthly installments with interest at 6%. Mr. Lane agrees to pay $90 a month in all. Part of this $90 is interest and the rest goes toward reducing the mortgage. Hence instead of paying rent for a house, Mr. Lane is using the $90 a month to buy his own house.

A mortgage gives Mr. Howe the right to sell the house if Mr. Lane does not make his payments.

The interest for the first month on the $6000 which Mr. Lane owes is $30. Hence, of his first payment of $90, $30 is interest and the other $60 is a reduction of the debt. He then owes $6000 − $60, or $5940.

The second month Mr. Lane again pays $90, out of which $29.70 is deducted as interest for 1 mo. on $5940, leaving $60.30 to be applied for reducing the principal. The principal now becomes $5940 − $60.30, or $5879.70.

The table below shows how this plan of payment works.

	Principal	Total Payment	Interest on Principal	Reduction on Principal
1st month	$6000.00	$90	$30.00	$60.00
2d month	5940.00	90	29.70	60.30
3d month	5879.70	90	29.40	60.60
4th month	5819.10	90	29.10	60.90

BUYING HOUSES BY MONTHLY PAYMENTS

1. As the principal in the table on page 356 is reduced each month, how does the amount of interest change? Does the reduction on the principal each month increase or decrease? Is there any change in Mr. Lane's total payment each month? How does the plan on page 356 differ from that on page 355?

2. The interest for the third month, in the table on page 356, is \$29.40. On what amount is this interest computed? Show how to find this interest by the 6% method.

3. The interest for the fourth month is \$29.10. On what amount is this computed? Show how it is computed.

4. Copy the table given on page 356 and continue it for 20 mo. more. Compute all interest to the nearest cent.

5. What will the principal be at the end of 2 yr. (at the beginning of the 25th month)? Is the principal being reduced more or less quickly now than during the first year?

6. Mr. Adams bought a house for \$12,000. He paid \$4000 cash; the other \$8000 he paid by monthly payments of \$75, which included interest at 6%. Make a table like the one on page 356, showing the first 12 payments.

7. Alice Jones bought a bungalow for \$2000. She paid \$500 down and the balance like rent. Her monthly payment, including interest at 6%, was \$25. Make a table like the one on page 356, showing the first 12 payments.

8. Mr. Todd bought a house for \$5000, paying \$2000 down. The monthly payment, including interest at 6%, was \$60. Make a table like that on page 356 for the first 6 payments.

9. In ex. 8, suppose that each monthly payment of \$60 was applied entirely to reduce the principal and that each month's interest was paid in addition to the \$60, as on page 355. Make a table showing the first 6 payments.

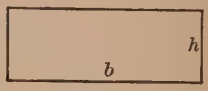
MEASUREMENTS

Review of Areas

You must now review the rules and formulas which you learned last year (pages 294 to 311) for finding the areas of rectangles and other figures. Remember that a formula is a brief way of writing a rule.

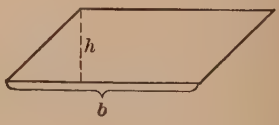
In the rules below, the *base* (b) of a figure is the side on which it stands and the *height* (h) is perpendicular to the base. In the formula $A = bh$, for the area of a rectangle, bh means $b \times h$ and stands for *base* $\times$ *height*.

Rectangle: *Area* = *base* $\times$ *height*
or $A = bh$



If the base is 10 in. and the height is 4 in., then $b = 10$ and $h = 4$. Hence $A = 10 \times 4$, or 40. The area is 40 sq. in.

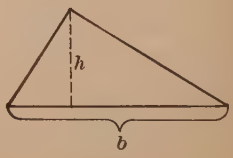
Parallelogram: *Area* = *base* $\times$ *height*
or $A = bh$



If the base is 14 in. and the height is 6 in., then $b = 14$ and $h = 6$. Hence $A = 14 \times 6$, or 84. The area is 84 sq. in.

Triangle: $Area = \dfrac{base \times height}{2}$

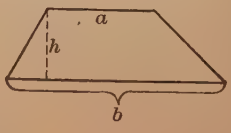
or $A = \dfrac{bh}{2}$, or $\frac{1}{2}bh$



If the base is 12 in. and the height is 8 in., then $b = 12$ and $h = 8$. Hence $A = \frac{1}{2} \times 12 \times 8$, or 48. The area is 48 sq. in.

Trapezoid: $Area = \dfrac{height \times sum\ of\ bases}{2}$

or $A = \dfrac{h(a + b)}{2}$, or $\frac{1}{2}h(a + b)$



If the bases of the trapezoid are 16 in. and 12 in. and the height is 10 in., then $A = \frac{1}{2} \times 10 \times (16 + 12)$, or 140. The area is 140 sq. in.

USING RECTANGLES AND PARALLELOGRAMS

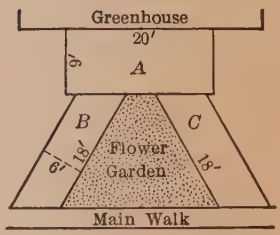
1. Mr. Turner wants to make a rectangular lawn 24 ft. wide and 66 ft. long. How many square yards of sod will he need for it? How much will it cost at 35¢ a square yard?

2. A baseball diamond is 90 ft. square. What decimal part of an acre, to the nearest hundredth, does it cover?

3. Mrs. Carter's kitchen floor is 15 ft. square. She is having it covered with linoleum which comes only in widths of 6 ft. and 9 ft. If she buys it all in the 6-foot width, how many running feet of linoleum will she need? How can it be cut without waste? If she buys some of both widths, how many running feet of each will she need?

4. How much must Mrs. Carter (ex. 3) pay for linoleum for her kitchen if it costs $1.85 per square yard and if there is an extra charge of $5 for laying it?

5. Mr. Fuller is laying three new pieces of cement walk, *A*, *B*, and *C*, in the park. *B* and *C* are each 6 ft. wide. The other dimensions are given in the picture. What kinds of figures are *B* and *C*? Find the total cost of laying these walks at $2.10 per square yard.



Find the areas of rectangles having these dimensions:

6. 8 ft. × 6 ft.

7. 5 yd. × 1 yd.

8. $6\frac{1}{2}$ ft. × $3\frac{1}{2}$ ft.

9. $7\frac{1}{4}$ yd. × $1\frac{3}{4}$ yd.

10. 4.5″ × 3.1″

11. 7.2″ × 7.2″

Find the areas of the following parallelograms. The first number gives the base and the second number, the height:

12. 7 in.; 6 in.

13. 8 in.; $2\frac{1}{2}$ in.

14. $5\frac{1}{2}$ ft.; 3 ft.

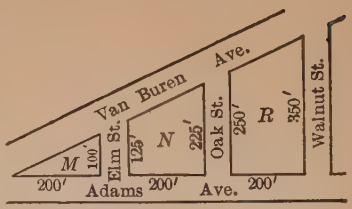
15. $1\frac{1}{2}$ yd.; $1\frac{1}{4}$ yd.

16. 18 ft.; 7.3 ft.

17. 9.2 ft.; 6.8 ft.

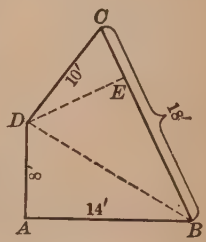
USING TRIANGLES AND TRAPEZOIDS

1. On this map, Elm, Oak, and Walnut streets are each perpendicular to Adams Avenue. What kind of figure is block M? block N? block R? Find the areas of M, N, and R, using the dimensions given on the map.



In the trapezoid N, $h = 200$ ft., $b = 225$ ft., and $a = 125$ ft. Substituting these values in $A = \frac{1}{2}\,h(a + b)$, you get $A = \frac{1}{2}\times 200\times 350$, which gives the area in square feet. What is the area of N?

2. This picture shows the sail of Bert's catboat, drawn to the scale of $\frac{1}{16}'' = 1'$. Show that this sail is not a trapezoid. Find the area of the sail by adding the areas of triangles ABD and BCD.



Line DE shows the height of triangle BCD. Measure this line carefully and find the number of feet it represents, from the scale $\frac{1}{16}'' = 1'$. What line do you take as the base of triangle BCD?

Angle A is a right angle, hence line AD is the height of triangle BAD.

Find the areas of the following triangles. The first number gives the base and the second, the height:

3.	5 ft.; 3 ft.	**6.**	$6\frac{1}{2}$ in.; 4 in.	**9.**	2.5′; 3.1′
4.	7 ft.; 4 ft.	**7.**	$4\frac{1}{4}$ ft.; 3 ft.	**10.**	6.6′; 2.8′
5.	6 ft.; 5 ft.	**8.**	$5\frac{3}{4}$ yd.; 2 yd.	**11.**	5.8′; 2.5′

Find the areas of the following trapezoids:

	Bases	Height		Bases	Height
12.	5 yd., 7 yd.	3 yd.	**16.**	75 ft., 49 ft.	60 ft.
13.	8 yd., 5 yd.	$6\frac{1}{3}$ yd.	**17.**	$15\frac{1}{2}$ ft., $10\frac{1}{2}$ ft.	7 ft.
14.	18 ft., 16 ft.	20 ft.	**18.**	$9\frac{1}{2}$ ft., $11\frac{1}{2}$ ft.	$8\frac{1}{2}$ ft.
15.	50 ft., 45 ft.	35 ft.	**19.**	$14\frac{1}{4}$ ft., $15\frac{3}{4}$ ft.	10 ft.

IMPROVEMENT TEST No. 25

Test 25 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$7\frac{2}{3}$	$3\frac{3}{4}$	$3\frac{1}{3}$	$9\frac{1}{4}$	$6\frac{1}{6}$
$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{2}{5}$	$4\frac{1}{2}$	$5\frac{1}{2}$
$3\frac{2}{3}$	$1\frac{3}{8}$	$5\frac{1}{3}$	$3\frac{1}{5}$	$3\frac{3}{4}$
$5\frac{5}{6}$	$5\frac{5}{16}$	$6\frac{1}{10}$	$1\frac{3}{10}$	$8\frac{5}{12}$

Test 25 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$6\frac{1}{3}$	$9\frac{5}{8}$	5	$8\frac{3}{4}$
$3\frac{2}{3}$	$1\frac{7}{16}$	$4\frac{9}{10}$	$5\frac{2}{3}$
5.	**6.**	**7.**	**8.**
$7\frac{4}{5}$	$4\frac{1}{2}$	3	$2\frac{5}{6}$
$6\frac{7}{10}$	$2\frac{5}{6}$	$1\frac{3}{4}$	$\frac{1}{4}$
9.	**10.**	**11.**	**12.**
$9\frac{3}{8}$	$8\frac{3}{8}$	$1\frac{1}{2}$	$6\frac{2}{3}$
$2\frac{2}{3}$	$6\frac{1}{2}$	$1\frac{2}{5}$	$2\frac{3}{5}$

Test 25 C — Multiplication (Time 4 min. after copying)

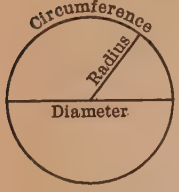
1.	2.	3.
625	$48\frac{2}{3}$	488
$31\frac{1}{4}$	75	$18\frac{3}{8}$
4. $14\times\frac{2}{3}$	**5.** $\frac{8}{15}\times\frac{9}{10}$	**6.** $1\frac{1}{3}\times\frac{9}{16}$
7. $\frac{5}{12}$ of 64	**8.** $\frac{4}{15}\times3\frac{3}{4}$	**9.** $7\frac{4}{5}\times3\frac{5}{6}$

Test 25 D — Division (Time 4 min. after copying)

1. $5\frac{1}{5}\div\frac{4}{5}$	**2.** $\frac{5}{8}\div\frac{2}{5}$	**3.** $28\div\frac{7}{8}$
4. $9\frac{1}{3}\div\frac{1}{2}$	**5.** $\frac{1}{2}\div1\frac{1}{3}$	**6.** $8\frac{1}{4}\div3\frac{3}{4}$
7. $7\frac{1}{4}\div\frac{5}{8}$	**8.** $\frac{3}{16}\div1\frac{1}{5}$	**9.** $16\frac{1}{2}\div3$
10. $6\frac{2}{3}\div15$	**11.** $\frac{9}{10}\div6\frac{3}{4}$	**12.** $17\frac{1}{4}\div9$

MEASURING CIRCLES

You have learned this rule for the circumference of a circle:



$$\textit{Circumference} = 3\tfrac{1}{7} \times \textit{Diameter}$$

or $C = \pi D$

Here the symbol π (pronounced "pie") stands for $3\frac{1}{7}$, or 3.14.

Since a diameter equals two radii ($D = 2r$), the above formula may also be written thus:

$$C = 2\pi r$$

You have also learned this rule for the area of a circle:

$$\textit{Area of circle} = 3\tfrac{1}{7} \times \textit{square of the radius}$$

or $A = \pi r^2$

Here r^2 is read "r square" and means $r \times r$.

Exercises

In the following exercises let $\pi = 3\frac{1}{7}$.

1. The wheels on Fred's bicycle have a diameter of 28 in. How many feet does the bicycle travel while each wheel makes one complete revolution?

2. A circus ring has a radius of 21 ft. A pony running around the outside edge travels how far in one trip around it? How many times must the pony run around the ring to go 1 mi.?

3. A forest ranger watches for forest fires from a tower on a mountain. He can see 35 mi. in all directions. How many square miles of land does he guard?

Find circumferences and areas of circles with radii of:

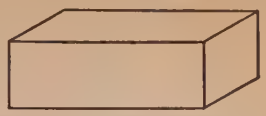
4. 7 in.	**6.** 28 yd.	**8.** 1 yd.	**10.** $1\frac{1}{6}$ ft.
5. 14 ft.	**7.** 25 ft.	**9.** 49 in.	**11.** $4\frac{2}{3}$ ft.

Find circumferences and areas of circles with diameters of:

12. 14 in.	**14.** 84 ft.	**16.** 154 ft.	**18.** $8\frac{2}{5}$ rd.
13. 10 mi.	**15.** 18 rd.	**17.** 100 ft.	**19.** $5\frac{3}{5}$ mi.

THE VOLUME OF A RECTANGULAR SOLID

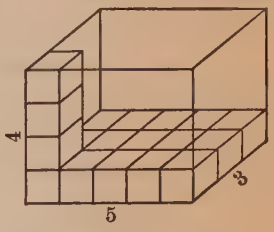
A solid shaped like a box is called a **rectangular solid** or **a rectangular prism** because its six faces are all rectangles. If its six faces are all squares, it is a **cube.** A cube 1 in. long, 1 in. wide, and 1 in. high contains 1 *cubic inch* and is called a 1-inch cube. What is a cubic foot? a cubic yard?



RECTANGULAR PRISM

CUBE

The **volume** of a solid is the number of cubic units it contains. This picture shows a box 5 in. long, 3 in. wide, and 4 in. high. It would take 5×3, or 15, 1-inch cubes to cover the bottom of the box, and 4×15, or 60, 1-inch cubes to fill the box. Hence the volume of the box is 5×3×4 cu. in., or 60 cu. in. in all.



Volume of rectangular solid = length × width × height

This rule may be written as a formula, thus:

$$V = lwh$$

In finding the volume (V), first express all dimensions in inches or all in feet or all in yards, etc.

Exercises

Find the volumes of boxes with these dimensions:

1. 7 in.×5 in.×3 in.

2. 8 in.×8 in.×8 in.

3. 6 ft.×5 ft.×2 ft.

4. 1 ft.×6 in.×4 in.

5. $3\frac{1}{2}$ ft.×2 ft.×2 ft.

6. $7\frac{1}{2}$ ft.×5 ft.×16 in.

7. A box 15 in. long, 12 in. wide, and 12 in. high is filled with cans of cocoa. If each can is 5 in. by 3 in. by 2 in., how many cans does the box contain?

FINDING VOLUMES

Review this table of *cubic measure:*

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Exercises

1. A cubic foot is how many inches long? how many inches wide? how many inches high? Show that 1 cu. ft. contains 1728 cu. in.

2. Show that 1 cu. yd. contains 27 cu. ft.

3. A schoolroom is 35 ft. by 30 ft. by 12 ft. How many cubic feet of air space does it contain? If 200 cu. ft. of air space are necessary for each pupil, for how many pupils is this room large enough?

4. How many cubic yards of earth, to the nearest whole number, will be removed in digging a tunnel 50 ft. long, 10 ft. wide, and 9 ft. high? Find the cost at 65¢ per cubic yard.

5. Granite weighs 170 lb. per cubic foot. How much does a granite block $6\frac{1}{2}$ ft. square and 3 ft. high weigh?

6. A swimming pool is 60 ft. long and 24 ft. wide, and is filled to an average depth of 4 ft. 3 in. How many cubic feet of water does it contain? How much does it cost to fill the pool, as above, if the water costs 27¢ per 100 cu. ft.?

7. An aquarium 18 in. long and 12 in. wide is 11 in. high. How many cubic inches of water does it hold when filled to within 4 in. of the top? A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of the water in the aquarium?

8. How many truck loads of earth, to the nearest whole load, are removed in digging a ditch 513 ft. long, $1\frac{1}{2}$ ft. wide, and $1\frac{3}{4}$ ft. deep, if 5 cu. yd. make a load for a truck?

MEASURING LIQUIDS

Memorize these useful equivalents in *liquid measure:*

1 gal. = 231 cu. in.
$31\frac{1}{2}$ gal. = 1 barrel
$7\frac{1}{2}$ gal. = about 1 cu. ft.

Exercises

1. Bob wants a can which will hold 3 gal. of oil. He finds one can which measures 7 in.×10 in.×6 in., another which is 8 in.×12 in.×$6\frac{1}{2}$ in., and a third which is 9 in.×11 in.×7 in. Which can should he take?

In ex. 1 to 3, use 1 gal. = 231 cu. in.; in ex. 4 and 5, use $7\frac{1}{2}$ gal. = 1 cu. ft.

2. Will a rectangular can which measures $3\frac{1}{2}$ in. by $2\frac{3}{4}$ in. by 4 in. hold more or less than a pint of olive oil? how many cubic inches more or less?

3. Ann has an aquarium 28 in. long, 18 in. wide, and 13 in. high. How many gallons of water will it hold when filled to within 2 in. of the top?

4. A swimming pool 90 ft. long and 35 ft. wide is filled with water to an average depth of 6 ft. How many gallons of water does it hold?

5. During a storm $\frac{3}{4}$ in. of rain fell. How many cubic feet of rain fell on a flat roof 48 ft. by 35 ft.? If all this rain ran off the roof into a cistern, how many gallons of water went into the cistern? how many barrels of water?

Tell how many gallons these rectangular cans or tanks hold:

6. 4 ft.×2 ft.×5 ft.
7. $1\frac{1}{2}$ ft.×4 ft.×7 ft.
8. $3\frac{1}{2}$ ft.×5 ft.×4 ft.
9. $3\frac{1}{2}$ in.×6 in.×11 in.
10. 6 in.×$8\frac{3}{4}$ in.×11 in.
11. 60 ft.×30 ft.×$5\frac{1}{2}$ ft.

HOW ACCURATELY CAN YOU MEASURE A GALLON?

You will now learn why, for most purposes, it is sufficiently accurate to say that $7\frac{1}{2}$ gal. = 1 cu. ft. instead of using the exact equivalent, which is 1 gal. = 231 cu. in.

1. How many cubic inches are there in 1 gal.? in $7\frac{1}{2}$ gal.? Show that $7\frac{1}{2}$ gal. contain $4\frac{1}{2}$ cu. in. more than 1 cu. ft.

When you say that $7\frac{1}{2}$ gal. = 1 cu. ft., you make an error of $4\frac{1}{2}$ cu. in. to a cubic foot, or $\frac{4.5}{1728}$ of a cubic foot. Dividing both numerator and denominator of this large fraction by 4.5, or $4\frac{1}{2}$, you get $\frac{1}{384}$, which is roughly equal to $\frac{1}{400}$. An error of $\frac{1}{400}$ cu. ft. is called an error of 1 in 400 and shows how accurately the measurement has been made.

2. Jack has a rectangular can which measures 6 in. by $5\frac{1}{2}$ in. at the bottom, and which is 7 in. high. Show that it will hold exactly 1 gal. of oil if it is filled to the top.

If Jack fills his can to within $\frac{1}{16}$ in. of the top, how many cubic inches less than 1 gal. does it contain? Show that it is short about $\frac{1}{100}$ gal. and that the error is about 1 in 100.

3. Show that Jack makes an error of about 1 in 200 if he fills the can to within $\frac{1}{32}$ in. of the top, and an error of about 1 in 400 if he fills it to within $\frac{1}{64}$ in. of the top.

Now look at your ruler. Are $\frac{1}{32}$ in. and $\frac{1}{64}$ in. very small? Can you measure accurately to $\frac{1}{64}$ in. with your ruler?

4. In the ordinary measuring of liquids, do you think that dealers get as close to an exact gallon as Jack did in ex. 3? Do you think the error in measuring liquids is generally nearer to 1 in 100 or to 1 in 400? Is an error of 1 in 100 a larger or a smaller error than one of 1 in 400?

In using $7\frac{1}{2}$ gal. = 1 cu. ft., you have an error of only 1 *in* 400, *while most measurements of liquids in everyday life give an error greater than* 1 *in* 400.

MEASURING BINS

Memorize these useful equivalents in *dry measure:*

1 bu. = about 2150 cu. in.
1 bu. = about $1\frac{1}{4}$ cu. ft.
.8 bu. = about 1 cu. ft.
1 T. hard coal = about 35 cu. ft.

A bushel contains exactly 2150.42 cu. in. This exact value is not used in everyday measurements because it is impossible to measure so accurately with ordinary instruments. In finding the capacities of small boxes, you may let 2150 cu. in. = 1 bu.; but in measuring bins, it is more practical to use $1\frac{1}{4}$ cu. ft. = 1 bu., or 1 cu. ft. = .8 bu.

Exercises

1. If you know that $1\frac{1}{4}$ cu. ft. = 1 bu., how can you show that 1 cu. ft. = $\frac{4}{5}$ bu., or .8 bu.?

2. A corn bin 10 ft. long, 8 ft. wide, and 5 ft. deep will hold how many bushels of corn when filled to a height of $2\frac{1}{2}$ ft.? when filled to a height of 4 ft. 3 in.?

3. Mr. Dane's coal bin is 7 ft. wide and $9\frac{1}{2}$ ft. long. How many tons of hard coal does it contain when filled to a depth of 5 ft.? when filled to a depth of 6 ft.?

4. Mr. Ford is building a bin 6 ft. long and 5 ft. wide. He wants it to hold 120 bu. of feed. How many feet high should he make the bin?

Find the capacity of each of these bins in bushels:

5. $10' \times 8\frac{1}{2}' \times 6'$
6. $12' \times 8\frac{1}{4}' \times 5'$
7. $7\frac{3}{4}' \times 7\frac{1}{2}' \times 4'$
8. $9\frac{1}{3}' \times 7\frac{1}{2}' \times 6'$
9. $9\frac{1}{2}' \times 9' \times 5'$
10. $6\frac{3}{4}' \times 5' \times 3\frac{1}{2}'$
11. $7' \times 6\frac{1}{4}' \times 4\frac{1}{2}'$
12. $10\frac{1}{2}' \times 8\frac{2}{3}' \times 5'$
13. $6.5' \times 4.5' \times 5'$
14. $3.5' \times 1.5' \times 4'$
15. $6' \times 2.6' \times 6.5'$
16. $7' \times 5.7' \times 8.5'$

SOME DIFFICULTIES IN MEASURING GRAIN

In these problems, consider 2150 *cu. in. as* 1 *exact bushel.*

1. How many cubic inches are there in $1\frac{1}{4}$ cu. ft.? In using $1\frac{1}{4}$ cu. ft. for 1 bu., there is an error of about how many cubic inches in each bushel? Is this an error of about 1 in 100 or 1 in 200 or 1 in 400?

Now let us see whether the error in the ordinary measuring of grain is as small as 1 in 200.

2. Paul has a box which measures, on the inside, $12\frac{1}{2}'' \times 16''$ on the bottom, and $10\frac{3}{4}''$ in height. What is its volume in cubic inches? You see that this box holds just 1 bushel.

If Paul fills this box with grain to within $\frac{1}{4}$ in. of the top, what is the volume of the part of the box that is not yet filled? How many cubic inches less than a full bushel has Paul? Is his error about 1 in 40 or 1 in 50 or 1 in 100?

3. If Paul fills the box to within $\frac{1}{8}$ in. of the top, the exact error is 1 in how many? If he fills it to within $\frac{1}{16}$ in. of the top, is the error greater or less than 1 in 200?

4. If Paul fills the box to the top and then shakes it, so that the grain settles $\frac{3}{16}$ in. below the top, is the error considerably more or less than 1 in 200? What error did you find in ex. 1?

Since the error in measuring grain is seldom less than 1 *in* 200, *it is sufficiently accurate to let* $1\frac{1}{4}$ *cu. ft.* = 1 *bu.*

5. A farmer has boxes with the following inside measurements. Tell which box holds nearest to an exact bushel:

(*a*) $15'' \times 16'' \times 9''$
(*b*) $16'' \times 16'' \times 8\frac{3}{8}''$
(*c*) $14'' \times 14'' \times 11''$
(*d*) 14 in. $\times$ 15 in. $\times 10\frac{1}{4}$ in.
(*e*) $13\frac{1}{2}$ in. $\times 13\frac{1}{4}$ in. $\times$ 12 in.
(*f*) 14 in. $\times$ 14 in. $\times 10\frac{31}{32}$ in.

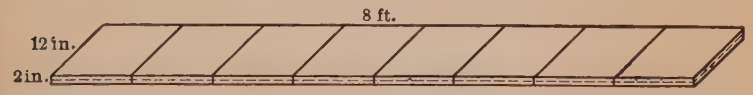
6. Does a box measuring 10 in. $\times$ 10 in. $\times 10\frac{3}{4}$ in. or one measuring $10\frac{1}{2}$ in. $\times 10\frac{1}{2}$ in. $\times 9\frac{3}{4}$ in. hold nearer to an exact half bushel? The dimensions given are inside measurements.

MEASURING LUMBER

Lumber is measured by a unit called the **board foot,** which is 1 ft. long, 1 ft. wide, and 1 in. or less thick, as shown in *A*.

The board shown in *B* is 1 in. thick, 1 ft. wide, and 4 ft. long. Since it is as wide and as thick as a board foot and is 4 times as long, it contains 4 board feet.

If the board in *B* were only $\frac{7}{8}$ in. thick, it would also contain 4 bd. ft. Can you tell why?



The plank above is 2 in. thick, 12 in. wide, and 8 ft. long. To find how many board feet it contains, imagine it divided into two layers, each 1 in. thick, 1 ft. wide, and 8 ft. long. Then each layer contains 8 bd. ft. and the entire plank contains 2×8 bd. ft., or 16 bd. ft.

An easier way to find *the number of board feet* is to multiply and divide as follows:

$$\frac{\textit{Thickness in inches} \times \textit{width in inches} \times \textit{length in feet}}{12}$$

By using this rule, you can quickly find, as shown at the right, that a plank 2 in. thick, 12 in. wide, and 8 ft. long contains 16 bd. ft.

$$\frac{2 \times \cancel{12} \times 8}{\cancel{12}} = 16$$

In applying the above rule, be sure that the width is expressed in inches. Since the width of boards is usually given in inches, this rule enables you to multiply all the dimensions in the form in which they are given. Use cancellation whenever possible.

FINDING THE NUMBER OF BOARD FEET

A piece of lumber is $\frac{7}{8}$ in. thick, 6 in. wide, and 8 ft. long. How many board feet does it contain?

A thickness of less than 1 in. is counted as 1 in. Applying the rule on page 369, you get $\frac{1\times6\times8}{12}$, or 4, as the number of board feet.

If the thickness of the lumber is 1 in. or more, use the exact thickness in finding the number of board feet. Thus, a plank $2\frac{1}{2}$ in. thick, 6 in. wide, and 8 ft. long contains $\frac{2\frac{1}{2}\times6\times8}{12}$ bd. ft., or 10 bd. ft.

Exercises

Find the number of board feet in each of the following:

1. A piece $\frac{3}{4}$ in. thick, 4 in. wide, 6 ft. long

2. A piece 5 in. thick, 6 in. wide, 8 ft. long

3. A piece 6 in. thick, 7 in. wide, 10 ft. long

4. A piece $\frac{7}{8}$ in. thick, 6 in. wide, 14 ft. long

5. A piece $3\frac{1}{2}$ in. thick, 8 in. wide, 12 ft. long

6. A piece $1\frac{1}{2}$ in. thick, 8 in. wide, 16 ft. long

7. For a cedar chest Ned used 4 boards each 1″ thick, 9″ wide, and 10′ long. How many board feet in all was that?

To find the number of board feet in 4 boards, multiply the number of board feet in 1 board by 4, thus: $\frac{4\times1\times9\times10}{12}$.

8. A manual training teacher ordered 42 pieces of maple lumber $2\frac{1}{2}$ in. thick, 10 in. wide, and 10 ft. long. How many board feet did he order?

9. Tell how many board feet this order calls for in all:

26 beams 8 in. thick, 12 in. wide, 16 ft. long
48 rafters 2 in. thick, 4 in. wide, 16 ft. long
144 boards $\frac{7}{8}$ in. thick, 10 in. wide, 12 ft. long
24 planks $2\frac{1}{2}$ in. thick, 10 in. wide, 18 ft. long

BUYING LUMBER

1. Lumber is usually bought by the 1000 bd. ft. (M). A fraction of a board foot counts as an extra board foot.

A board 2 in.$\times$8 in.$\times$14 ft. contains $18\frac{2}{3}$ bd. ft. If one such board is purchased, it counts as 19 bd. ft. If 20 such boards are purchased, multiply by 20 before changing the fraction of a board foot to another board foot, thus: $\frac{20\times2\times8\times14}{12}$. Then 20 boards contain $373\frac{1}{3}$ bd. ft., or 374 bd. ft.

2. Find the cost of 12 pieces of lumber, each 2 in. thick, 8 in. wide, and 10 ft. long, at \$50 per M.

You find that 160 bd. ft. of lumber are purchased. The cost is \$50 per M, which means \$50 for each 1000 bd. ft. Then 160 bd. ft. will cost $\frac{160}{1000}$ of \$50, which is the same as .160$\times$\$50, or \$8.

Exercises

1. How much will a girder 8 in. thick, 12 in. wide, and 32 ft. long cost at \$64 per M?

Find the cost of the following:

2. 4 pieces, $\frac{3}{4}''$ thick, 4″ wide, 14′ long, at \$40 per M

3. 6 pieces, $1\frac{1}{2}''$ thick, 6″ wide, 8′ long, at \$47 per M

4. 5 pieces, 2″ thick, 8″ wide, 10′ long, at \$56 per M

5. To repair his barn, Mr. Blake buys 6 boards, each 1 in. thick, 8 in. wide, and 16 ft. long. They are listed on the bill as 6 boards, $1''\times8''-16'$, at \$60 per M. Find the total cost.

6. Find the amount of the following bill of lumber:

56 planks, $2''\times12''-16'$, at \$66.85 per M
240 boards, $\frac{7}{8}''\times8''-12'$, at \$68.20 per M
100 boards, $\frac{3}{4}''\times9''-12'$, at \$58.50 per M

7. Find the total cost of the following at \$52.75 per M:

10 pieces, $1\frac{1}{2}''\times6''-14'$
32 pieces, $2\frac{1}{2}''\times6''-16'$
17 pieces, $\frac{7}{8}''\times12''-12'$
24 pieces, $1''\times10''-14'$

BUYING LUMBER BY THE LINEAL FOOT

Mr. Gray orders for his living room 68 ft. of oak molding at \$3.75 per 100 *lineal feet.* Find the total cost.

Finishing lumber, moldings, and trim are often sold by the lineal foot instead of by the board foot. The number of lineal feet in a piece of lumber is the same as the number of feet in its length. Since Mr. Gray's molding is 68 ft. long, it contains 68 lineal feet, no matter how thick or how wide it is. At \$3.75 per 100 lineal feet, the molding costs $.68 \times \$3.75$, or \$2.55.

Exercises

1. Mr. Lee bought 50 ft. of finishing lumber 1″ thick and 4″ wide at \$2.90 per 100 lineal feet. How much did it cost?

2. Find the cost of oak baseboard for a room $14' \times 18'$, deducting for 2 doors, each 3 ft. wide. The baseboard costs \$16.80 per 100 lineal feet.

3. Find the cost of oak picture molding for the room in ex. 2 at \$4.20 per 100 lineal feet. As the molding goes all around the room, do not deduct for doors or windows.

4. Find the cost of oak picture molding for 2 rooms, each $12' \times 15'$, at \$3.95 per 100 lineal feet.

5. At \$5.75 per 100 lineal feet, find the cost of 40 ft. of $1'' \times 6''$ finishing lumber. Find the cost of 48 ft.; of 70 ft.; of 82 ft.; of 144 ft.; of 212 ft.

6. Find the total amount of the following bill:

38 ft. of porch rails at $6\frac{1}{2}¢$ per lineal foot
18 ft. of baluster stock at $4\frac{1}{2}¢$ per lineal foot
68 ft. of baseboards at \$7.40 per 100 lineal feet
72 ft. of picture molding at \$1.44 per 100 lineal feet
82 ft. of finishing lumber at \$5.80 per 100 lineal feet
28 ft. of finishing lumber at \$6.40 per 100 lineal feet

IMPROVEMENT TEST No. 26

Test 26 A — Reduction (Time 3 min.)

Change to exact decimals:

1. 99% **2.** 3% **3.** 200% **4.** 0.1% **5.** $\frac{1}{2}$%
6. 10% **7.** 9% **8.** 106% **9.** 5.5% **10.** $\frac{1}{4}$ of 1%

Change to common fractions in lowest terms:

11. 60% **12.** 1% **13.** $12\frac{1}{2}$% **14.** 16% **15.** 95%

Change to per cents:

16. $\frac{5}{6}$ **17.** 9 **18.** .08 **19.** .215 **20.** .092
21. $\frac{9}{10}$ **22.** .5 **23.** $.06\frac{1}{4}$ **24.** .006 **25.** 2.25

Test 26 B — Finding Per Cents (Time 4 min. after copying)

Find answers correct to the nearest cent:

1. 5% of \$34.50
2. $\frac{1}{5}$% of \$590
3. 135% of \$150
4. 82.7% of \$400
5. $6\frac{1}{4}$% of \$3470
6. 200% of \$550.95
7. $66\frac{2}{3}$% of \$175.11
8. $\frac{1}{10}$ of 1% of \$1258

Test 26 C — Comparing Numbers (Time 4 min. after copying)

What numbers should be put in place of the dots?

1. 69 is . . .% of 1150
2. 32 is . . .% of 6400
3. 203 is . . .% of 725
4. 8856 is . . .% of 7380
5. 1560 is . . .% of 2340
6. 6806 is . . .% of 3320

Test 26 D — Per Cent of Increase (Time 3 min. after copying)

What per cent greater is the second number than the first?

1. 3140; 3611 **2.** 5862; 11,724 **3.** 5600; 21,560

NOTE. For scoring Test 26 A, see the note to Test 21 A on page 305.

HOW TO FIND THE VOLUME OF A PRISM

You learned on page 363 that a solid like *A* is called a rectangular prism. In a **prism** the upper and lower bases are parallel, and the sides are all rectangles.

A

B

C

D

If a prism has squares for all its bases and sides, as in *B*, it is called a **cube.** If its bases are triangles, as in *C*, it is called a **triangular prism.** A prism may also have bases of other shapes, as in *D*. In each of the above prisms the *height is perpendicular to the base.*

On page 363 you learned this rule:

Volume of rectangular prism = length × width × height

By studying prism *A* above, you see that *the length × the width* is the same as *the area of the base.* Hence the rule may also be written thus:

Volume of prism = area of base × height

When you study geometry, you will learn that this new rule may be used to find the volume of any prism, whether its base is a rectangle, a triangle, a trapezoid, or a figure of some other shape.

If the base of the triangular prism shown at the right has an area of 5 sq. in. and the height of the prism is 4 in., then the volume of the prism is 5×4 cu. in., or 20 cu. in.

What would the volume of this triangular prism be if its height were 7 in. and the area of its base, 8 sq. in.?

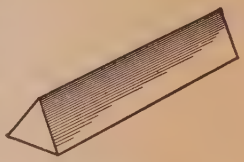
FINDING VOLUMES OF PRISMS

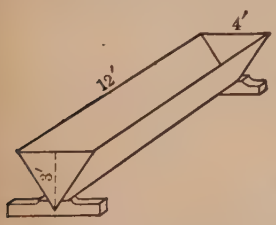
1. Find the volume of a rectangular prism whose base has an area of 28 sq. in. and whose height is 7 in.

2. Find the volume of a triangular prism whose base has an area of $4\frac{1}{2}$ sq. ft. and whose height is 6 ft.

3. Mary has a glass prism 7 in. high. Its base has an area of $1\frac{1}{2}$ sq. in. Find its volume.

4. When a prism is lying on its side, as shown here, think of the ends as the bases and of the length of the prism as its height. What is the volume of this prism if it is 14 in. long and if its end has an area of 2 sq. in.?

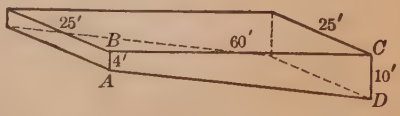




5. In the water trough shown at the left, first find the area of the triangle which forms the end (base); then find the volume of the trough.

6. In ex. 5, find how many gallons of water the trough will hold, using 1 cu. ft. = $7\frac{1}{2}$ gal.

7. The swimming tank shown at the right is 60 ft. long and 25 ft. wide. It is 10 ft. deep at one end and 4 ft. deep at the other end. Find its volume in cubic feet. How many gallons of water will it hold when filled to the very top?



Think of the trapezoid *ABCD*, whose height is 60 ft., as the base of the prism and find its area. Then 25 ft. is the height of the prism.

8. In ex. 7, how many gallons of water will the tank hold when filled to within 1 ft. of the top? How much will the water cost at 32¢ per 100 cu. ft.?

FINDING THE VOLUME OF A CYLINDER

This tin can is a cylinder. Peaches, tomatoes, and corn are usually canned in cylindrical cans. The bases of a cylinder are circles.

You know that the volume of a prism equals the area of its base multiplied by its height. You may use the same rule for the volume of a cylinder:

Volume of cylinder = area of base × height

Since the base is a circle, its area $= \pi r^2$. The rule above may therefore be written as a formula, thus:

$$\textit{Volume of cylinder} = \pi r^2 h$$

If the above can is 5 in. high and its base has a diameter of 4 in., then its radius is 2 in. To find the volume (V) of the can, substitute these values in the formula, as follows:

$$V = 3\tfrac{1}{7} \times 2 \times 2 \times 5 = 62\tfrac{6}{7}$$

Hence the volume of the can is $62\frac{6}{7}$ cu. in.

Exercises

Find the volumes of these cylinders, letting $\pi = 3\frac{1}{7}$:

	Area of Base	Height		Radius of Base	Height
1.	8 sq. in.	4 in.	**5.**	2 ft.	6 ft.
2.	7 sq. in.	6 in.	**6.**	3 in.	8 in.
3.	9 sq. ft.	$5\frac{1}{2}$ ft.	**7.**	$1\frac{1}{2}$ in.	4 in.
4.	5 sq. ft.	$2\frac{1}{2}$ ft.	**8.**	7 in.	45 in.

9. A can of chicken soup is 4 in. high. It has a diameter of 3 in. Find its volume in cubic inches.

10. A cylindrical gasoline tank is 7 ft. in diameter and 5 ft. high. Find its volume. How many gallons of gasoline, to the nearest whole gallon, will it hold?

FINDING VOLUMES OF CYLINDERS

In these problems, let $\pi = 3\frac{1}{7}$.

1. The stone pillars in front of a building are cylinders 35 ft. high and 3 ft. in diameter. How many cubic feet of stone does each pillar contain? What is the weight of each pillar if the stone weighs 140 lb. per cubic foot?

2. A cylindrical pail is 4 in. in radius and 11 in. high. What is its volume? How many gallons of water (to the nearest tenth of a gallon) will it hold?

Since the volume is found in cubic inches, use 1 gal. = 231 cu. in. If the volume were found in cubic feet, as in ex. 5 below, you would use 1 cu. ft. = $7\frac{1}{2}$ gal.

3. How many gallons of water will a cylindrical hot-water tank hold if it is 60 in. long and 14 in. in diameter? if it is 70 in. long and 16 in. in diameter?

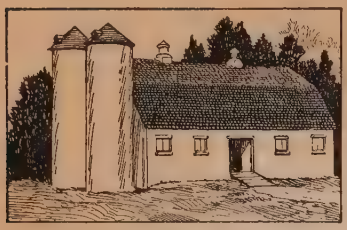
4. A cylindrical gas tank is 90 ft. in diameter and 112 ft. high. How many cubic feet of gas can it hold? Find the value of this gas at \$1.25 per 1000 cu. ft.

5. How many gallons of water will a cylindrical cistern 6 ft. in diameter contain if it is filled to a depth of 10 ft. 6 in.? (First express all dimensions as feet.)

6. A street sprinkling wagon has a cylindrical tank 4 ft. in diameter and 10 ft. long. Find its volume. How many gallons of water will it hold?

7. A cylindrical gasoline can has a diameter of 7 in. It is 18 in. high. Find out how many gallons of gasoline this can will hold.

CANNING FOOD FOR COWS



Every modern dairy farm has a building, called a *silo,* in which finely cut green corn or other green food is tightly packed to preserve it. This food, called *silage* or *ensilage,* is fed to the cows during the winter in place of grass and other fresh foods. To give it greater strength and capacity and to avoid corners where the silage is apt to spoil, the silo is usually built in the form of a tall cylinder.

Finding the Capacity of Silos

Find answers to the nearest whole number. Let $\pi = 3\frac{1}{7}$.

1. The inside diameter of a silo is 10 ft. Find its volume if its height is 20 ft.; 25 ft.; 30 ft.; 35 ft.

2. Find the volume of the silage in a silo 12 ft. in diameter and filled to a height of 25 ft.; of 30 ft.; of 40 ft.

3. Find how many cubic feet of silage there are in a silo 14 ft. in diameter when the depth of the silage is 30 ft.; when the depth is 35 ft.; 40 ft.; in a silo 16 ft. in diameter when the depth is 36 ft.; when the depth is 38 ft.; 42 ft.

4. Mr. Fain has a silo 16 ft. in diameter and 45 ft. high. It is entirely filled with silage. How many cows can he feed for 180 da. if each cow needs 1 cu. ft. of silage per day?

5. Mr. Jones has a silo 18 ft. in diameter and 40 ft. high. How many days' food will it hold for 64 cows, allowing 1 cu. ft. a day for each cow?

6. A farmer builds two silos, each 10 ft. in diameter and 30 ft. high, in place of one silo 14 ft. in diameter and 30 ft. high. How much does this increase his silo volume?

KEEPING SILAGE FROM SPOILING

Find all answers to the nearest whole number.

To prevent silage from spoiling, it is necessary to remove an average of 3 in. daily from the top.

1. How many cubic feet of silage are there in a 3-inch layer from the top of a silo 14 ft. in diameter? Allowing 1 cu. ft. of silage to each cow, how many cows would it take to use such a layer every day?

2. Would 30 cows, each eating 1 cu. ft. a day, be able to eat a 3-inch layer of silage daily from a silo 12 ft. in diameter? Would 20 cows be able to do this?

3. Mr. Lee feeds his 20 cows each 1 cu. ft. of silage a day. Is a silo 10 ft. in diameter or one 20 ft. in diameter more economical for him to build?

Other Problems

1. If silage averages 40 lb. per cubic foot, how many acres of corn, yielding 15 tons of silage per acre, are needed to fill a silo 12 ft. in diameter and 40 ft. high?

2. A farm bulletin gives the following daily silage rations: dairy cows, 40 lb.; calves, 20 lb.; sheep, 4 lb. How many tons of silage are needed to feed 18 dairy cows for 220 da., 10 calves for 180 da., and 60 sheep for 180 da.?

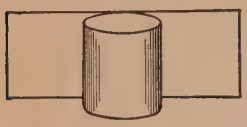
3. A farmer uses the upper 14 ft. of a concrete silo 16 ft. in diameter for a water tank, putting a concrete floor under the tank. How many gallons of water will this tank hold?

4. A coal company constructs 6 silo-like buildings, each 18 ft. in diameter and 40 ft. high, for holding coal. Find the total volume in cubic feet. If 1 cu. ft. of hard coal weighs 57 lb., how many tons of coal will the 6 buildings hold?

5. Two cylindrical cans have the same height, but the diameter of one is twice that of the other. Compare their volumes.

FINDING THE AREA OF A CYLINDRICAL SURFACE

The entire surface of a cylinder is made up of the curved surface plus the areas of the two bases, which are circles. You will now learn how to find the area of the curved surface.



If the label on a can of corn exactly covers the curved surface, its area equals that of the curved surface. By moistening the label, you can remove it and spread it out flat to form a rectangle. The height of this rectangle equals the height of the can and its base equals the circumference of the can. Since the curved surface of the can is equal in area to the rectangle, you have this rule:

Area of curved surface = circumference of base × height

Since *circumference* $= 2\pi r$, you may write this formula:

$$\textit{Area of curved surface} = 2\pi rh$$

Exercises

1. A can of corn 4 in. high has a radius of $1\frac{1}{2}$ in. How large a label is needed to cover the curved surface?

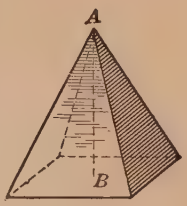
Substituting in the formula, you get Area $= (2\times 3\frac{1}{7}\times 1\frac{1}{2}\times 4)$ sq. in.

2. At 18¢ per square yard, find the cost of painting the 2 smokestacks of a ship if each stack is 9 ft. in diameter and 35 ft. high; if each stack is 12 ft. in diameter and 40 ft. high.

Find the curved surfaces of the following cylinders:

	Circumference of Base	Height of Cylinder		Radius of Base	Height of Cylinder
3.	15 in.	6 in.	**7.**	7 in.	20 in.
4.	16 ft.	$1\frac{1}{2}$ ft.	**8.**	5 in.	21 in.
5.	65 in.	40 in.	**9.**	6 ft.	25 ft.
6.	145 in.	30 in.	**10.**	$3\frac{1}{2}$ in.	10 in.

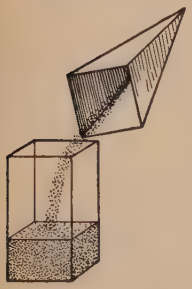
THE VOLUME OF A PYRAMID



The solid shown here is a **pyramid.** The sides of a pyramid are all triangles. Its base may be a triangle, a rectangle, or a figure of some other shape.

The point A is the **vertex.** The line AB, which is perpendicular to the base, shows the *height* of the pyramid.

Take a hollow prism and a pyramid, each having the same base and the same height. Fill the pyramid with sand and pour the sand into the prism. The contents of the pyramid will fill exactly $\frac{1}{3}$ of the prism; hence the volume of the pyramid is $\frac{1}{3}$ of the volume of the prism. But the volume of a prism equals the area of the base times the height; hence



Volume of pyramid $= \frac{1}{3} \times$ *area of base* $\times$ *height*

Exercises

Find the volumes of the following pyramids:

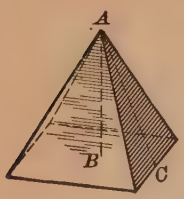
	Area of Base	Height		Area of Base	Height
1.	6 sq. in.	5 in.	**4.**	7 sq. in.	3 in.
2.	10 sq. ft.	4 ft.	**5.**	15 sq. ft.	6 ft.
3.	14 sq. ft.	$4\frac{1}{2}$ ft.	**6.**	20 sq. ft.	$6\frac{1}{2}$ ft.

7. The largest Egyptian pyramid, Cheops, is 461 ft. high. Its square base measures 746 ft. on a side. How many cubic feet does this pyramid contain?

8. At the county fair some prize apples are piled in a pyramid 4 ft. high and covering 25 sq. ft. How many bushels of apples are there in the pile if 1 bu. $= 1\frac{1}{4}$ cu. ft.?

FINDING THE LATERAL AREA OF A PYRAMID

The lateral area of the pyramid shown here is the area of its sides. These sides are all triangles of the same size. Hence to find the lateral area of the pyramid, you find the area of one triangle and multiply this by the number of triangles.



To find the area of one triangle, multiply half the height of the triangle by its base. In doing this, notice that the height of the triangle, AC, is *not* the same as the height of the pyramid, AB. AC is called the **slant height** of the pyramid to distinguish it from AB, the *real height* of the pyramid. The height of each triangle is the *slant height* of the pyramid.

Exercises

1. Find the lateral area of a pyramid whose base is a square measuring 8 in. on a side, and whose slant height is 12 in.

2. The roof of this garage is a pyramid with 4 equal sides, each measuring 14 ft. in slant height and 20 ft. at the base. How many gallons of paint (to the nearest whole gallon) will it take to paint this roof if 1 gal. covers 200 sq. ft.?

3. If each side of the roof of the garage in ex. 2 had a base of 30 ft. and a slant height of 21 ft., how many gallons of paint would it take to paint it?

4. Last night John dreamed that he was painting the Great Pyramid of Egypt, which has 4 equal sides, each having a slant height of 593 ft. and a base of 746 ft. If its sides were entirely smooth, what would its lateral area be? If John took 1 hr. to paint 100 sq. ft. and worked 8 hr. a day, how many days would he take to paint the Great Pyramid?

THE VOLUME OF A CONE

The solid shown here is a **cone.** An ice-cream cone gets its name from its cone shape.

The base of a cone is a circle. The point A is the *vertex* and the line AB, which is perpendicular to the base, shows the *height* of the cone.

The rule for finding the volume of a cone is just like the rule for the volume of a pyramid:

$$\textit{Volume of cone} = \tfrac{1}{3} \times \textit{area of base} \times \textit{height}$$

The base of a cone, being a circle, has an area of πr^2. Hence you may write the rule as a formula, thus:

$$\textit{Volume of cone} = \tfrac{1}{3}\pi r^2 h$$

Exercises

1. A pile of gravel falls naturally into the shape of a cone. If a pile is $4\frac{1}{2}$ ft. high and 12 ft. in diameter, how many cubic yards of gravel does it contain?

2. Edna bought an ice-cream cone which had a diameter of 2 in. at the top and was $3\frac{1}{2}$ in. deep. If it was exactly filled, how many cubic inches of ice cream did it hold? How many such cones could be filled with 1 gal. of ice cream?

3. How many bushels of grain are there in a conical pile of grain 14 ft. in diameter and $4\frac{1}{2}$ ft. high?

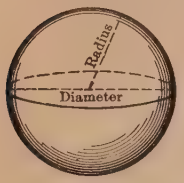
4. A conical pile of coal is 60 ft. in diameter and 21 ft. high. How many cubic feet of coal does it contain? How many tons is that, to the nearest whole ton, if 1 T. = 35 cu. ft.?

Find the volumes of cones with the following dimensions:

	Radius of Base	Height		Radius of Base	Height
5.	6 in.	15 in.	**7.**	28 ft.	45 ft.
6.	21 in.	30 in.	**8.**	70 in.	150 in.

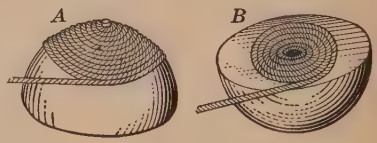
THE AREA OF A SPHERE

The **sphere** is a very common figure. Tennis balls and baseballs are spheres. Can you name other common objects that are spheres?



The radius of a sphere is a line from its center to any point on its surface. The diameter is twice the length of the radius.

Cut a sphere, such as a wooden ball, into halves. Each half is a **hemisphere.** Wind heavy cord around one of the hemispheres, as in *A*, and around the circular base of the other hemisphere, as in *B*.



It takes twice as much cord to cover the hemisphere in *A* as to cover the circle in *B*. The area of the circle in *B* is πr^2. Hence the area of the hemisphere in *A* must be $2\,\pi r^2$ and the area of the whole sphere is $4\,\pi r^2$.

$$\textit{Area of sphere} = 4\,\pi r^2$$

Exercises

Find the areas of spheres with the following radii:

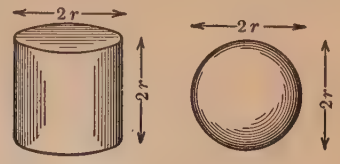
1. 7 in.	**3.** 5 in.	**5.** 21 in.	**7.** 35 ft.	**9.** 116 in.
2. $3\frac{1}{2}$ in.	**4.** 9 ft.	**6.** 14 in.	**8.** 42 ft.	**10.** 350 in.

11. The earth is almost a perfect sphere. If its radius is about 4000 mi. long, what is the approximate area of the entire surface of the earth?

12. The gilded dome on top of a state capitol is in the shape of a hemisphere with a diameter of 21 ft. What is the total area of the surface of this dome? How much does it cost to gild this dome at 75¢ per square foot?

FINDING THE VOLUME OF A SPHERE

Ted has a wooden cylinder whose base has a diameter of 4 in. and whose height is 4 in. He also has a sphere of the same kind of wood, with a diameter of 4 in. Hence the sphere has the same height as the cylinder.



Ted finds that the sphere weighs 4 oz. and the cylinder, 6 oz. Hence the sphere weighs $\frac{2}{3}$ as much as the cylinder. Since both objects are made of the same kind of wood, the volume of the sphere must be $\frac{2}{3}$ the volume of the cylinder.

The formula for the volume of a cylinder is $V = \pi r^2 h$. In Ted's cylinder, the height equals the diameter; that is, $h = 2r$. Hence the formula for the volume of Ted's cylinder may be written $V = \pi r^2 \times 2r$, or $2\pi r^3$. Remember that $r^3 = r \times r \times r$.

Since the volume of the sphere is $\frac{2}{3}$ the volume of the cylinder, or $\frac{2}{3} \times 2\pi r^3$, you have this formula:

$$\textit{Volume of sphere} = \tfrac{4}{3}\pi r^3$$

Exercises

1. A solid rubber ball has a diameter of 3 in. How many cubic inches of rubber does it contain?

Since the diameter is 3 in., the radius is $\frac{3}{2}$ in. Substituting in $V = \frac{4}{3}\pi r^3$, you get $V = \frac{4}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$, or $14\frac{1}{7}$. Hence the ball contains $14\frac{1}{7}$ cu. in.

Find the volumes of spheres having these radii:

2. 14 in.	**5.** 21 in.	**8.** 3.5 ft.	**11.** 1 ft.	**14.** $3\frac{1}{2}$ in.
3. 28 in.	**6.** 10 in.	**9.** 4.2 ft.	**12.** $\frac{3}{4}$ in.	**15.** $5\frac{1}{4}$ in.
4. 30 in.	**7.** 12 in.	**10.** 2.8 ft.	**13.** $\frac{7}{8}$ in.	**16.** $4\frac{3}{8}$ in.

17. Find the volume of a croquet ball whose diameter is 4 in.; of a spherical balloon whose diameter is 15 in.

FINDING VOLUMES OF SPHERES

1. A granite sphere 2 ft. in diameter rests on the top of a granite column. How much does this sphere weigh if 1 cu. ft. of granite weighs 168 lb.?

2. How much would a granite sphere 4 ft. in diameter weigh (see ex. 1)? A sphere 4 ft. in diameter is how many times as heavy as one 2 ft. in diameter?

3. A kettle used for boiling maple sirup is a hemisphere 2 ft. in diameter. How many gallons will it hold?

4. A stack of hay is approximately a hemisphere 18 ft. in diameter. How many cubic feet of hay does it contain? If 1 ton of hay occupies 500 cu. ft., about how many tons of hay are there in this stack?

5. What is the volume of an orange whose diameter is $2\frac{1}{2}$ in.? of one whose diameter is $3\frac{1}{4}$ in.? If the oranges are equally juicy, the second orange will give about how many times as much juice as the first?

6. Helen says that the second orange (ex. 5) gives twice as much juice as the first one. Joe says that if the first orange is $2\frac{1}{2}$ in. in diameter, the second orange would have to be 5 in. in diameter to give twice as much juice. Is Joe right?

7. In the lunchroom of a large school, soup is made in a kettle that consists of a cylindrical part 18 in. in diameter and 18 in. deep, with a hemisphere 18 in. in diameter for a bottom. How many gallons does this kettle hold when filled to within 6 in. of the top? when filled to within 1 ft. of the top?

IMPROVEMENT TEST No. 27

Test 27 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
251	916	697	626	188
949	274	875	274	335
654	835	604	695	204
833	312	192	863	813
864	379	178	239	345
676	138	790	137	176
802	565	821	967	422
520	384	919	594	898
694	207	415	357	845
956	943	368	556	194
282	579	847	773	840
727	730	624	459	236

Test 27 B — Subtraction (Time 3 min.)

1.	2.	3.
6730724	8914468	5713531
1253365	1188887	1557244
4.	**5.**	**6.**
9758006	6823257	8572751
2773464	3457397	3112857
7.	**8.**	**9.**
6009948	4207862	9925161
1880469	2745565	6387342

Test 27 C — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.	5.
954	829	706	583	673
285	139	975	781	624

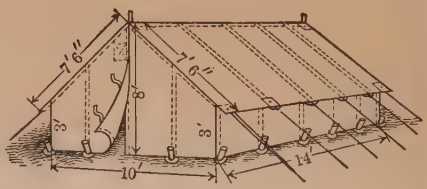
Test 27 D — Division (Time 3 min. after copying)

1. $65\overline{)33735}$ 2. $27\overline{)18333}$ 3. $83\overline{)29631}$

4. $59\overline{)51035}$ 5. $47\overline{)36848}$

MAKING A TENT WATERPROOF

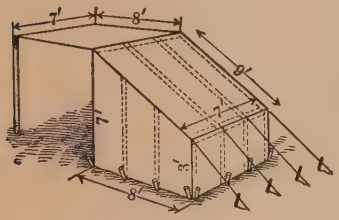
1. Mr. Bruce is going camping. He buys a tent of the shape and dimensions shown at the right. What is the total area in square feet of the outer surface of this tent?



The front of the tent consists of two trapezoids. Is the back the same?

2. Mr. Bruce (ex. 1) applies waterproof liquid to his tent. If 1 qt. of the liquid covers 25 sq. ft., how many quarts of it does he need? At 60¢ a quart, how much does it cost?

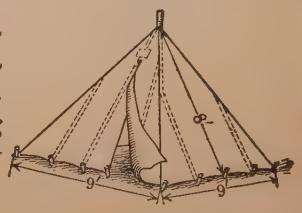
The waterproof liquid is sold only in whole quarts or gallons.



3. Mr. Lee has an automobile tent like the one shown here. How many square feet must he cover with waterproof liquid to make the tent waterproof? Find how many quarts he will need (see ex. 2), and the cost at 60¢ a quart.

4. How much would Mr. Lee (ex. 3) save or lose by buying the waterproofing in gallon cans at $2 a gallon?

5. Bob's tent is shaped like a pyramid. Each of its faces has a base of 9 ft. and a height of 8 ft. How many quart cans of waterproofing would he need to cover it? how many gallon cans?



6. Using the prices in ex. 3 and 4, how much would Bob save or lose if he bought the waterproofing by the gallon?

THE BUSINESS DEALINGS OF A STOREKEEPER

1. On Aug. 14 Paul Barron of Baltimore orders from James R. Cook of Chicago 15 doz. notebooks @ \$4.60, 36 boxes of writing paper @ \$.35, 48 school bags @ \$1.25, and 72 packages of crêpe paper @ \$.15. His discounts are 25%, 10%. Find the net amount and write the invoice.

2. With the goods mentioned in ex. 1, Mr. Cook sends a trade acceptance, dated Aug. 18 and payable in 60 da. Write the acceptance and accept it for Mr. Barron. If Mr. Cook discounts it on Aug. 30 at 6%, how much does he get for it?

3. Mr. Barron buys from Albert Wills of New York 60 dictionaries @ \$1.89 and 25 encyclopedias @ \$5.92, with discounts of $12\frac{1}{2}$%, 5%. The terms are 2% 30 da. Mr. Barron pays at once by a check drawn on the Merchants Bank of Baltimore. Write the check.

4. Mr. Wills has an account in the Grant Bank of New York, which does business with the Colonial Bank of Baltimore. Diagram the travels of the check in ex. 3 until it reaches the Baltimore Clearing House. What happens to it there?

5. Suppose that Mr. Barron bought a bank draft at the Merchants Bank of Baltimore, drawn on the Central Bank of New York, to pay the bill in ex. 3. Make out the draft, indorse it for Mr. Barron, and diagram its travels.

6. Louis Marsh of Denver owes Mr. Barron \$356, which is long past due. By what kind of draft can Mr. Barron try to collect the money? Write the draft and describe its travels.

7. On Oct. 3 Mr. Barron borrows \$535 at 6% from the Merchants Bank of Baltimore, giving a demand note for it. Write the note. Mr. Barron repays the money on Nov. 8. How much does he pay the bank on that date?

8. In what ways is it a help to Mr. Barron to have a checking account in the Merchants Bank?

REVIEW

Express the following in round numbers correct to the nearest thousand. Then find, to the nearest tenth of 1%, what per cent the first number is of the second:

1. 524,532; 6,786,104

2. 294,380; 8,168,666

3. 936,195; 4,462,423

4. 17,001; 2,240,990

5. 3,390,271; 1,105,899

6. 8,987,656; 9,001,715

Find the interest at 6%, first by the 6% method, then by the table on page 341. Use exact days between dates:

7. \$800; Apr. 5 to May 11

8. \$450; Aug. 8 to Oct. 25

9. \$120; Mar. 4 to July 17

10. \$5000; June 10 to Aug. 15

11. \$2225; July 18 to Aug. 17

12. \$7500; Sept. 23 to Dec. 22

Find the proceeds on the following notes:

	Amount of Loan	Date of Loan	Date Due	Rate of Interest	Date Discounted	Rate of Discount
13.	\$240	Jan. 3	Feb. 2	6%	Jan. 9	6%
14.	\$3500	Mar. 12	June 10	6%	Apr. 29	6%
15.	\$8840	Oct. 18	Dec. 18	6%	Nov. 28	7%

Find the yearly rate of interest on the following:

	Principal	Time	Interest		Principal	Time	Interest
16.	\$50	2 mo.	\$1.50	**19.**	\$1500	2 yr.	\$225
17.	\$82	1 mo.	\$2.05	**20.**	\$2000	3 yr.	\$495
18.	\$75	3 mo.	\$6.25	**21.**	\$2500	$1\frac{1}{2}$ yr.	\$675

Find the volumes and the curved surfaces of these silos:

22. $D = 12'$; $h = 35'$

23. $D = 16'$; $h = 40'$

Find the areas and the volumes of these spheres:

24. $r = 6.3$ in.

25. $r = 10\frac{1}{2}$ in.

26. $r = 1$ ft. 9 in.

27. Give the formula for the volume of a rectangular solid, a pyramid, a cone, a cylinder, a sphere, and a triangular prism.

CHAPTER VI

KEEPING MONEY IN A SAVINGS BANK

On pages 256 to 260 you learned that a savings bank, besides being a safe place to keep money, also pays interest on money which is left there long enough.

Most savings banks pay interest semiannually (twice a year) or quarterly (four times a year). If this interest is left in the bank, it also draws interest. Interest paid on the principal and also on the interest left in the bank is called **compound interest.** Interest paid only on the original principal is **simple interest.** The principal plus the interest is called the **amount.**

On Jan. 1, 1928, Leo puts $400 in a bank which pays 4% interest compounded semiannually. Hence he gets $\frac{1}{2}$ of 4%, or 2%, interest every half year. On July 1, 1928 his interest is 2% of $400, or $8. On Jan. 1, 1929 his interest is 2% of $408, or $8.16. How much in all has he in the bank on Jan. 1, 1929?

$400.00 Prin., Jan. 1, 1928
8.00 Int., July 1, 1928
$408.00 Prin., July 1, 1928
8.16 Int., Jan. 1, 1929
$416.16 Prin., Jan. 1, 1929

Exercises

1. What is Leo's interest on July 1, 1929? on Jan. 1, 1930?

Compute the interest of July 1 not on $416.16, but on $416, since, *in computing interest, savings banks disregard cents in the principal.*

2. Miss Adams invests $800 at 6% compounded semiannually. What rate of interest is that every 6 mo.? Find her interest and her new principal for the first 8 half years.

3. Fred has $200 in a bank at 4% compounded quarterly. What per cent does it earn each quarter? Find Fred's new principal at the end of 3 mo.; at the end of 1 yr.

A TABLE FOR FINDING COMPOUND INTEREST

Table Showing Amount of $1 at Compound Interest

Years or Periods	1%	2%	3%	4%	5%	6%
1	1.01000	1.02000	1.03000	1.04000	1.05000	1.06000
2	1.02010	1.04040	1.06090	1.08160	1.10250	1.12360
3	1.03030	1.06121	1.09273	1.12486	1.15763	1.19102
4	1.04060	1.08243	1.12551	1.16986	1.21551	1.26248
5	1.05101	1.10408	1.15927	1.21665	1.27628	1.33823
6	1.06152	1.12616	1.19405	1.26532	1.34010	1.41852
7	1.07214	1.14869	1.22987	1.31593	1.40710	1.50363
8	1.08286	1.17166	1.26677	1.36857	1.47746	1.59385
9	1.09369	1.19509	1.30477	1.42331	1.55133	1.68948
10	1.10462	1.21899	1.34392	1.48024	1.62889	1.79085
11	1.11567	1.24337	1.38423	1.53945	1.71033	1.89830
12	1.12683	1.26824	1.42576	1.60103	1.79586	2.01220
13	1.13809	1.29361	1.46853	1.66507	1.88565	2.13293
14	1.14947	1.31948	1.51259	1.73168	1.97993	2.26090
15	1.16097	1.34587	1.55797	1.80094	2.07893	2.39656
16	1.17258	1.37279	1.60471	1.87298	2.18287	2.54035
17	1.18430	1.40024	1.65285	1.94790	2.29202	2.69277
18	1.19615	1.42825	1.70243	2.02582	2.40662	2.85434
20	1.22019	1.48595	1.80611	2.19112	2.65330	3.20714
24	1.26973	1.60844	2.03279	2.56330	3.22510	4.04893
28	1.32129	1.74102	2.28793	2.99870	3.92013	5.11169
35	1.41660	1.99989	2.81386	3.94609	5.51602	7.68609

Using the Table

1. Find the amount of $1 in 2 yr. at 4% compounded yearly.

Hold a ruler horizontally under the 2 in the column marked "Years or Periods." The number just above the ruler in the 4% column is 1.08160. Hence the amount of $1 in 2 yr. at 4% is $1.08.

2. With interest compounded annually, find the amount to which $1 grows in 2 yr. at 5%; in 5 yr. at 6%; in 20 yr. at 4%; in 15 yr. at 3%; in 35 yr. at 6%; in 18 yr. at 5%, 4%, 3%, and 6%.

3. Find the amount of $50 in 7 yr. at 4% compounded yearly.

Find the amount of $1 for 7 yr. and multiply the result by 50.

4. Find the amount of $125 in 10 yr. at 4% compounded annually; of $250 in 18 yr. at 5% compounded annually.

COMPOUNDING INTEREST SEMIANNUALLY

Edna has \$200 in the bank at 4% interest compounded semiannually. To what amount will the \$200 grow in 2 yr.?

Since the yearly rate is 4%, the rate for each half year (or for each interest period) is 2%. In 2 yr. there are 4 half years, or 4 interest periods. Hence in the table on page 392, hold a ruler under 4 in the column marked "Years or Periods" and find the number just above the ruler in the 2% column. This gives \$1.08243 as the amount of \$1. Multiplying \$1.08243 by 200, you get \$216.49 as Edna's amount in 2 yr. Why do you look in the 2% column rather than in the 4% column?

Exercises

1. Peter puts \$50 in a bank at 4% interest compounded semiannually. How long is each interest period? How many interest periods are there in 3 yr.? What is the rate for a period? Find the amount to which the \$50 will grow in 3 yr.

Find the amounts, by the table, at 6% compounded semiannually:

	Principal	Time		Principal	Time		Principal	Time
2.	\$400	2 yr.	**7.**	\$550	6 yr.	**12.**	\$385	10 yr.
3.	\$300	3 yr.	**8.**	\$275	5 yr.	**13.**	\$270	12 yr.
4.	\$910	1 yr.	**9.**	\$900	8 yr.	**14.**	\$480	14 yr.
5.	\$650	4 yr.	**10.**	\$157	9 yr.	**15.**	\$868	14 yr.
6.	\$380	7 yr.	**11.**	\$750	3 yr.	**16.**	\$452	$17\frac{1}{2}$ yr.

17. Mrs. Dean invested \$480 at 4%, compounded twice a year. How much did she have after 2 yr.? after 4 yr.? 6 yr.? 8 yr.? 9 yr.? 10 yr.? How much of each amount was interest?

18. Lucy puts \$100 in a savings bank at 4% compounded quarterly. How long is each interest period? How many such periods are there in 5 yr.? What is the rate of interest for a period? Find the amount Lucy has in the bank after 5 yr.

19. Find the amount to which \$760 invested at 4% compound interest, payable quarterly, will grow in 6 yr.

WHAT A MAN LOSES BY HOARDING HIS MONEY

Use the interest table on page 392 for these problems.

1. Many people hide or bury their money for safe keeping. Can you tell a safer and wiser thing to do with it?

2. The peasants of one country have about $400,000,000 in gold hidden on their farms. To about what amount would this sum grow in a bank in 10 yr. at 4% compounded yearly? How much interest could it thus earn for the peasants in 10 yr.?

3. One peasant kept $100 buried for $17\frac{1}{2}$ yr. Show that if he had invested it at 4% compounded semiannually, it would have grown to almost twice its original value in $17\frac{1}{2}$ yr.

4. Another man had $600 hidden in a box. His neighbor had $450. Show that each sum would practically double in $17\frac{1}{2}$ yr. if invested at 4% compounded semiannually.

5. If the peasant in ex. 3 invested his $100 at 5% compounded annually, find his amount after 14 yr. How does this amount compare with his original principal? Would the amount be even greater if interest were compounded semiannually?

6. Suppose that the peasant in ex. 3 found a good investment paying 6% interest compounded every 6 mo. Show that his money would be more than doubled in 12 yr.

With interest compounded semiannually, money doubles in $17\frac{1}{2}$ yr. at 4%, in 14 yr. at 5%, and in 12 yr. at 6%.

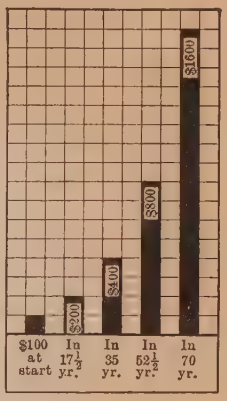
7. Jack is now $10\frac{1}{2}$ yr. old. If he puts $100 in a bank at 4%, compounded twice a year, to how much will it amount when he is 28 yr. old? Since this amount will double again in $17\frac{1}{2}$ yr., how much will it be when he is $45\frac{1}{2}$ yr. old?

8. Lena has $50 in gold in her jewel box. How much interest does she lose on it in 12 yr. by not putting it in a savings bank at 4% compounded semiannually?

HOW MONEY GROWS AT COMPOUND INTEREST

Albert wins \$100 for an essay when he is 14 yr. old. He puts the money in a bank at 4% interest compounded semiannually. In $17\frac{1}{2}$ yr. the investment will double and he will have \$200 instead of \$100. In another $17\frac{1}{2}$ yr. this \$200 will also double and he will have \$400. If he then leaves this \$400 in the bank for $17\frac{1}{2}$ yr., it will grow to \$800. How many more years will it take for this \$800 to double?

In $17\frac{1}{2}$ years \$100 grows to \$200.
In 35 years \$100 grows to \$400.
In $52\frac{1}{2}$ years \$100 grows to \$800.
In 70 years \$100 grows to \$1600.
In $87\frac{1}{2}$ years \$100 grows to \$3200.



How Money Grows at Compound Interest

Exercises

1. Continue the table above to show what \$100 grows to in 105 yr.; in $122\frac{1}{2}$ yr.; in 140 yr.; in $157\frac{1}{2}$ yr.; in 175 yr.

2. Show that if a man had invested \$140 in 1789 and if it had been compounded semiannually at 4%, it would have grown to \$35,840 by 1929.

3. John has been spending 5¢ a day for candy. At the age of 13 he saves this money for one year (365 da.) and invests it at 4% compounded semiannually. How much will this money amount to when John is $30\frac{1}{2}$ yr. old? 48 yr. old? $65\frac{1}{2}$ yr. old?

4. Abe spent \$15 for a toy train when he was 11 yr. old. How much would this money have grown to in a savings bank at 4% compounded semiannually by the time he was 46 yr. old?

At 4% interest compounded semiannually, show what the following will grow to in 35 yr.; in 70 yr.; in 175 yr.:

5. \$1 **6.** \$5 **7.** \$20 **8.** \$35 **9.** \$200 **10.** \$250

LETTING YOUR MONEY WORK ITS HARDEST

Have you a 10-dollar bill which you are keeping in your toy bank? If so, you are letting it waste energy with which it could earn money for you.

Tim had two 10-dollar bills. He lent Bill No. 1 at 6% simple interest and invested Bill No. 2 at 6% interest compounded semiannually.

1. How much simple interest did Bill No. 1 earn in 1 yr.? in 2 yr.? in 3 yr.? in 5 yr.?

2. From the table on page 392, show that at compound interest Bill No. 2 earned $3.44 in 5 yr. How much more did this 10-dollar bill earn in 5 yr. than Bill No. 1?

3. Show that Bill No. 2 was worth more than twice its original value in 12 yr. How much more does $10 at compound interest earn in 12 yr. than $10 at simple interest?

4. Tim's brother kept his money at home in a box. By not having his money earn all it could, how much did he lose in 12 yr. on one 10-dollar bill? on ten 10-dollar bills?

Assume that Tim's money could have earned interest at the rate of 6%, compounded semiannually.

5. On July 2, 1828, Dirk Van Dorn deposited $15 in a savings bank of New York. On July 1, 1928, when his great-grandson inherited the account, the $15, at an average of 5% interest compounded annually, had grown to $1972.52. How much of this amount was interest? How much less would the $15 have earned in the same period at 5% simple interest?

6. How much would Dirk have lost for his heirs if he had kept the $15 at home instead of investing it as he did?

7. Show that $1000 at 6% compounded semiannually earns $43.92 more in 5 yr. than $1000 at 6% simple interest.

8. In 7 yr., how much more will $1500 earn at 4% interest compounded quarterly than at 4% simple interest?

IMPROVEMENT TEST No. 28

Test 28 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
9837	2369	6407	2603	7516
6895	3576	9869	7367	6982
9374	3347	4778	3457	5151
7063	4897	6987	8565	4771
6970	3464	8452	4527	6622
6499	8885	7633	4722	6488
9666	4547	9992	6994	4868
7785	5559	8983	1588	2748
8888	5157	5299	1328	8493

Test 28 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
601074	754312	945228	333621
215316	578643	346667	136033
5.	**6.**	**7.**	**8.**
703659	600717	712802	952062
284665	355388	468235	156582
9.	**10.**	**11.**	**12.**
820837	961365	840411	953908
581379	847889	467732	567669

Test 28 C — Multiplication (Time 3 min. after copying)

1.	2.	3.
61759	52601	40289
67054	2189	1765

Test 28 D — Division (Time 4 min. after copying)

Find each result to the nearest hundredth:

1. $235\overline{)76895}$ **2.** $798\overline{)719929}$ **3.** $674\overline{)209899}$

HELPING YOUR SAVINGS TO GROW

You have seen that money left in the bank grows quickly. It grows even more quickly if you add to it regularly.

Table Showing Amount if $1 Per Year is Deposited

This table gives the total amount (with compound interest) accumulated *at the end of each year* if $1 is deposited *at the beginning of each year.*

Years or Periods	2%	$2\frac{1}{2}$%	3%	4%	5%	6%
1	1.0200	1.0250	1.0300	1.0400	1.0500	1.0600
2	2.0604	2.0756	2.0909	2.1216	2.1525	2.1836
3	3.1216	3.1525	3.1836	3.2465	3.3101	3.3746
4	4.2040	4.2563	4.3091	4.4163	4.5256	4.6371
5	5.3081	5.3877	5.4684	5.6330	5.8019	5.9753
6	6.4343	6.5474	6.6625	6.8993	7.1420	7.3938
7	7.5830	7.7361	7.8923	8.2142	8.5491	8.8975
8	8.7546	8.9545	9.1591	9.5828	10.0266	10.4913
9	9.9497	10.2034	10.4639	11.0061	11.5779	12.1808
10	11.1687	11.4835	11.8078	12.4864	13.2068	13.9716
18	21.8405	22.9460	24.1169	26.6712	29.5390	32.7600
20	24.7833	26.1833	27.6765	30.9692	34.7193	38.9927
30	41.3794	45.0003	49.0027	58.3283	69.7608	83.8017
34	48.9945	53.9282	59.4621	72.6522	89.3203	110.4348
40	61.6100	69.0876	77.6633	98.8265	126.8398	164.0477
50	86.2710	99.9215	116.1808	158.7738	219.8154	307.7561

Studying the Table

1. If you deposit $1 in a bank at the beginning of each year at 4% compounded annually, the interest period is 1 yr. At the end of 1 yr. the principal and interest total $1.04. Where is this shown in the above table?

2. If, at the beginning of the second year, you add $1 to the $1.04, making $2.04, your interest for this year is 4% of $2.04, or $.0816, and your amount at the end of the second year is $2.04 + $.0816, or $2.1216. Where is this shown in the table?

3. If you deposit $1 at the beginning of each year at 4% compounded yearly, find from the table your amount at the end of 3 yr.; of 4 yr.; of 5 yr.; of 6 yr.; of 7 yr.; of 30 yr.

WATCHING YOUR SAVINGS GROW

1. If at the beginning of each year you put \$25 in a bank that pays 4% interest compounded annually, what amount will you have at the end of 5 yr.? at the end of 20 yr.? of 30 yr.?

For each period, find from the table on page 398 the amount if \$1 is deposited every year. Then multiply the result by 25.

2. The day John Allen was born his father began to invest \$100 a year at 6% compounded annually, to build up a fund for John's college education. If John went to college when he was 18 yr. old, how large was the fund by that time?

Find the amounts if the following sums are deposited yearly with interest compounded annually:

	Deposit	Rate	Time		Deposit	Rate	Time
3.	\$25	5%	4 yr.	**6.**	\$100	5%	9 yr.
4.	\$50	4%	6 yr.	**7.**	\$250	6%	30 yr.
5.	\$60	5%	7 yr.	**8.**	\$275	6%	40 yr.

DEPOSITING MONEY EVERY HALF YEAR

1. Will Davis deposited \$10 every half year at 4% compounded semiannually. Hence he had two interest periods each year and received 2% each period. Find, from the table on page 398, how much he had at the end of 1 yr.

2. How many interest periods did Will have in 2 yr.? Find the amount of his deposits at the end of 2 yr.; at the end of 3 yr.; 4 yr.; 5 yr.; 10 yr.; 15 yr.; 20 yr.; 25 yr.

3. How much did Will's amount grow from the end of the 5th to the end of the 10th year? from the end of the 10th to the end of the 15th? from the 15th to the 20th? the 20th to the 25th? In which 5-year period was its growth greatest?

4. Clara said that Will's money grew almost twice as fast from the 20th to the 25th year as it did from the 5th to the 10th year. Was she right?

DEPOSITING MONEY EVERY HALF YEAR

1. If Elsie deposits \$10 at the beginning of each half year at 4% compounded semiannually, she has \$111.69 in 5 yr., \$247.83 in 10 yr., \$1163.33 in 30 yr., \$1529.78 in 35 yr., \$1976.47 in 40 yr., and \$2521.00 in 45 yr. How much do her savings grow from the 30th to the 35th year? from the 35th to the 40th year? from the 40th to the 45th year? The growth for each of these 5-year periods is roughly how many times as great as that from the 5th to the 10th year?

Compound interest makes savings grow faster and faster the longer they are left in the bank. During each 5 years the gain is much greater than during the preceding 5 years.

2. How much has Rose after 5 yr. if she deposits \$15 every 6 mo. at 5% interest compounded semiannually?

3. How much does Rose (ex. 2) deposit in 5 yr.? How much of her total amount at the end of 5 yr. is interest?

4. Bert's grandfather has smoked three 15-cent cigars a day for over 50 yr. How much could he have saved in a year (365 da.) by not smoking? If he had deposited this amount once a year for 50 yr., how much would it have grown to in 50 yr. at 4% compounded annually?

5. Anna spends 10¢ a day for candy. How much does she spend for candy in 6 mo. of 30 da. each? If she deposited this sum every 6 mo. at 5% compounded semiannually, how much would she have 10 yr. after the first deposit?

Find the amounts if these sums are deposited every 6 mo. and the interest is compounded semiannually:

	Deposit	Rate	Time		Deposit	Rate	Time
6.	\$20	6%	4 yr.	**9.**	\$100	5%	17 yr.
7.	\$85	5%	5 yr.	**10.**	\$400	6%	10 yr.
8.	\$70	4%	15 yr.	**11.**	\$550	4%	20 yr.

STOCKS AND BONDS

How a Corporation Is Formed

A dramatic club needed \$400 its first year for costumes and other expenses. To raise the money, the members formed the Woodside Dramatic Company and sold **shares** in this *company* at \$1 a share. \$1 was the *par value* of these shares.

1. How many shares did the company have to sell, at \$1 a share, in order to raise \$400?

2. Albert Clark bought 20 shares. How much did he have to pay for them? How much did Marion Peck have to pay for 15 shares? Albert and Marion and the other people who owned shares were **shareholders** in the company.

3. The first year the company made a net profit of \$32, which was divided among the shareholders in the form of **dividends**. Since there were 400 shares, the dividend on each share was $\frac{1}{400}$ of \$32, or \$.08. Show that this was a dividend at the rate of 8% on each \$1 share.

4. If Mary Reed had 2 shares, find her dividend. Find the dividend on 5 shares; on 14 shares; 25 shares; 35 shares.

5. Because the profits were good, Joseph wanted to buy some of Albert's shares. Albert charged him \$1.20 for each share. The value of the shares was then above the par value, or *above par*. How much above par was each of these shares?

6. The third year the company's performances were not successful and they did not make any profit. Would people want to buy more shares that year or would they try to sell the shares they had? How would this affect the prices of the shares? When \$1 shares are sold for less than \$1, they are said to be *below par*.

A company like the Woodside Dramatic Company is called a **corporation.** Shares in a corporation are often called **stocks.**

CORPORATIONS

A large business is usually owned and run in a similar way to the Woodside Dramatic Company, but on a larger and more complex scale. For example, a man starting an automobile factory is seldom either able to pay its entire cost alone or willing to take such a large risk without assistance from others. He therefore organizes a corporation, which is chartered by the state government. The total par value of the shares issued by a corporation is called its **capital stock.**

Most shares of stock have a par value of $100, although a few corporations issue shares with a par value of $50 or $25 or even $1. The shareholders, or stockholders, who own the stock, become owners of the corporation and each receives a *stock certificate* to show the number of shares he owns. The stockholders elect a *board of directors* to manage the corporation. In this election, each stockholder has one vote for every share of stock he owns in the corporation.

Hence a large factory or a railroad is owned and run by the coöperative support of many people. The Pennsylvania Railroad is owned by about 140,000 stockholders, including residents of every state of the Union. The American Telephone and Telegraph Company is owned by over 400,000 people and is worth almost $2,000,000,000.

Explain why a very large business is owned by many people. What other large corporations do you know about?

The profits of a corporation, after a sum has been set aside for emergencies, are divided among the stockholders as **dividends.** If a corporation with a capital stock of $500,000 declares a dividend of $25,000 in a year, its rate of dividend is $25,000 ÷ $500,000, or 5%, *of the par value of the stock.* Hence a man who owns 2 of the $100 shares will receive 5% of $200, or $10, in dividends. How much would a man owning 5 of the $100 shares receive in dividends?

FINDING DIVIDENDS

All shares in the following have a par value of \$100.

1. A corporation has a capital stock of \$800,000. How many shares has it if each share has a par value of \$100?

2. Last year the corporation in ex. 1 distributed \$56,000 in dividends. What rate of dividend did it pay?

3. In ex. 2, how much was the dividend on 1 share of stock? How much was Mr. Ray's dividend if he owned 120 shares?

4. If the capital stock of a company consists of 5250 shares, find the total par value of its capital stock. How much did it pay in dividends in a year if the rate was 5%?

5. Mr. Toole bought 25 shares of stock in the Holsted Fruit Company at \$100 per share. His dividends for the first year were \$112.50. Find the per cent of his profit.

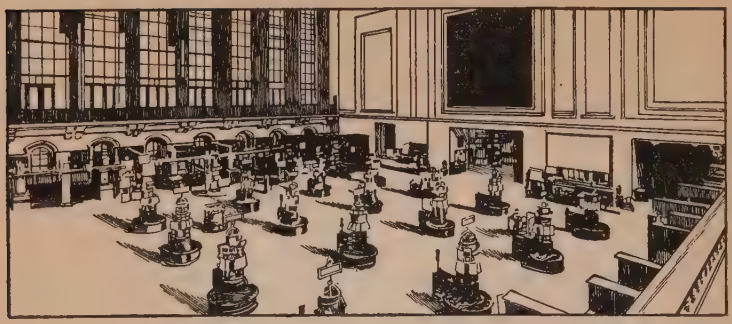
Find the rate of dividend; then find the amount of dividend on 50 *shares of stock:*

	Capital Stock	Dividends		Capital Stock	Dividends
6.	\$250,000	\$10,000	**10.**	\$300,000	\$18,000
7.	\$585,000	\$40,950	**11.**	\$890,000	\$35,600
8.	\$780,000	\$42,900	**12.**	\$1,200,000	\$72,000
9.	\$950,000	\$47,500	**13.**	\$1,325,000	\$99,375

Preferred Stock

Some corporations have two kinds of stock, *preferred stock* and *common stock*. The preferred stock receives a fixed rate of dividend, like 6% or 7%, and the remaining profits, if any, are divided among the holders of the common stock. The dividends on the common stock may rise or fall with the prosperity of the business, while the dividends on the preferred stock continue at the guaranteed rate. The preferred stock of a successful corporation is usually considered a very desirable investment.

THE STOCK EXCHANGE



The above picture shows the interior of the New York Stock Exchange, where the sale of stocks averages over 1,500,000 shares daily. Certain groups of stocks are sold at each of the trading posts which you see on the floor of the exchange. Philadelphia, Chicago, San Francisco, and other large cities also have stock exchanges.

A man who wishes to buy or sell stocks usually does so through a **broker** who is a member of a Stock Exchange. The broker buys the stock for his customer at the Stock Exchange and charges him a small fee, called **brokerage.**

The price at which a stock sells is its **market value.** If a reliable corporation is paying a dividend of 6% or more, the market value of a share of its stock is usually over $100, or *above par.* If the company is insecure and the dividend is low or if no dividend at all is being paid, the market value of each share will be under $100, or *below par.*

The prices at which important stocks are selling are printed each day in the newspapers. In large cities the newspapers usually devote several pages a day to information about stocks and bonds because they are so important in the business of the modern world. Look for the stock news in your newspaper.

STUDYING STOCK QUOTATIONS

Here are some *stock quotations* from a newspaper. For each stock the total number of shares sold during the day is given, as well as the price per share at the opening of the exchange, the highest and lowest prices during the day, and the price at the close of the exchange. The number in parentheses after each stock shows its last rate of dividend; pf. means preferred stock. The par value of each share is \$100.

STOCK QUOTATIONS

SALES	STOCK AND DIVIDEND RATE	OPEN	HIGH	LOW	CLOSE
2,800	Atlantic Telephone (9)	$167\frac{1}{2}$	$168\frac{3}{4}$	$167\frac{1}{2}$	$167\frac{5}{8}$
8,900	Brazil Rubber	$50\frac{1}{8}$	$51\frac{1}{4}$	$49\frac{1}{2}$	51
2,600	Goodwin Steel, pf. (7)	133	133	$132\frac{1}{2}$	$132\frac{5}{8}$
19,500	Marx Motors (4)	$63\frac{7}{8}$	$64\frac{3}{4}$	$63\frac{3}{8}$	$63\frac{1}{2}$
4,000	Pacific R. R. (5)	$88\frac{3}{8}$	$88\frac{3}{4}$	$87\frac{7}{8}$	$88\frac{1}{2}$
300	Standard Silk (8)	$146\frac{1}{2}$	148	$146\frac{1}{2}$	$147\frac{1}{4}$
4,600	Winthrop Canning	$14\frac{3}{4}$	15	$14\frac{1}{4}$	$14\frac{1}{4}$

Exercises

1. The first sales of Atlantic Telephone stocks were at $167\frac{1}{2}$, or \$167.50 per share. What were the highest and the lowest prices paid for this stock during the day?

2. What was the last rate of dividend on Atlantic Telephone stock? Why does this stock sell above par?

3. What other stocks are selling above par? Why are people willing to pay so much for these stocks?

4. The Winthrop Canning and Brazil Rubber stocks are not paying dividends. How has this affected their prices? Judging from their prices, which of these stocks is more likely to begin paying dividends in the near future?

5. Show that the stocks paying the lowest dividends are selling at the lowest prices.

BUYING AND SELLING STOCKS

A person wishing to buy or sell stocks listed on the stock exchange has to employ a stock broker to carry out the transaction for him. The broker charges a fee called **brokerage,** which is a certain amount per share and varies with the market price of the stock. The fees charged by brokers on the New York Stock Exchange are shown in the table at the right.

Market Price per Share	Broker's Fee per Share
\$10 to \$25	$12\frac{1}{2}$¢
\$25 to \$50	15¢
\$50 to \$75	$17\frac{1}{2}$¢
\$75 to \$100	20¢
\$100 to \$200	25¢

Exercises

1. Mr. Rees purchased 10 shares of Standard Silk stock at $146\frac{3}{4}$. How much did he pay in all?

The price per share is \$146.75. Hence 10 shares cost $10 \times \$146.75$, or \$1467.50. Since the price is between 100 and 200, the brokerage is 25¢ per share, or \$2.50 for 10 shares. Mr. Rees paid \$1467.50 + \$2.50, or \$1470.

2. Mr. Lane's broker sells 50 shares of Pacific R. R. stock for him at $88\frac{3}{8}$. How much does Mr. Lane receive?

$88\frac{3}{8}$ means $\$88.37\frac{1}{2}$. Hence 50 shares sell for $50 \times \$88.37\frac{1}{2}$, or \$4418.75. The brokerage is 20¢ per share, or \$10 in all. This is deducted from the amount paid to Mr. Lane by the broker. Hence Mr. Lane gets \$4408.75.

Tell how much it costs, including brokerage, to buy:

3. 100 shares of Marx Motors at $64\frac{3}{4}$.

4. 50 shares of Brazil Rubber at $50\frac{1}{4}$.

5. 15 shares of Goodwin Steel, preferred, at $132\frac{5}{8}$.

6. 200 shares of Central R. R., preferred, at $114\frac{7}{8}$.

If a broker sells the following for you, tell what net amount he will send you after the sale:

7. 40 shares at $48\frac{5}{8}$

8. 20 shares at $146\frac{3}{4}$

9. 80 shares at $101\frac{1}{8}$

10. 100 shares at $79\frac{1}{4}$

11. 300 shares at $27\frac{5}{8}$

12. 600 shares at $169\frac{1}{2}$

SELLING STOCKS AT A PROFIT

Many persons buy real estate in a growing city and sell it several years later at a good profit. A person may also buy the stock of a well-managed corporation in the hope that, as the corporation develops, the price of its stock will increase. By selling the stock when it reaches a higher price, the owner makes a profit. Such a transaction is entirely legitimate; but it requires both good judgment and foresight in selecting the stock.

Some people, however, buy stocks with the hope of selling them a few days later at a higher price, relying on the daily fluctuations in the prices of stocks. Such people have not the patience to wait several years for a substantial growth in the corporation whose stocks they are purchasing; they want quick profits, hence they take a chance. Such buying and selling is *speculation* and in most cases it eventually results in heavy losses for the speculator.

Avoid speculation. It is dangerous and leads to disaster!

Exercises

1. Mr. Marvin bought 25 shares of Interstate R. R. stock at $98\frac{3}{4}$. Several years later, owing to prosperous railroad conditions, he was able to sell it at $133\frac{1}{2}$. What profit did he make on the sale?

He paid $25 \times \$98.75$ for the stock + \$5.00 brokerage. When he sold it he received $25 \times \$133.50 - \$6.25$ brokerage. Find Mr. Marvin's profit. Also show how the brokerage fee is computed in each case.

2. While Mr. Marvin held the stock, he received a total of \$21 in dividends on each share. Find his total dividends. What was his total profit, including his profit in ex. 1?

3. Mr. Roe bought 100 shares of oil stock at 33. Next day the value dropped and he sold the stock at $19\frac{1}{2}$. Find his total loss, including brokerage. Was Mr. Roe speculating?

SELLING STOCKS AT A PROFIT OR LOSS

Find the profit or loss on the following, using the table on page 406 to find the brokerage fees:

1. 600 shares bought at 74 and sold at 74.
2. 200 shares bought at $69\frac{1}{4}$ and sold at $83\frac{1}{2}$.
3. 500 shares bought at $86\frac{7}{8}$ and sold at $73\frac{1}{4}$.
4. 150 shares bought at $97\frac{1}{2}$ and sold at $112\frac{3}{4}$.
5. 100 shares bought at $112\frac{5}{8}$ and sold at 104.

BUYING STOCKS AS AN INVESTMENT

People with much money to invest often buy high-grade stocks because such investments may yield 6% interest or more, while a savings bank pays only 4% to $4\frac{1}{2}$%.

1. Mr. Forrest buys 1 share of Ralco 7% preferred stock at 112 as an investment. What rate of income does Mr. Forrest receive on this investment?

Since dividends are paid on the par value of a stock, a 7% stock pays \$7 a year dividend. Hence Mr. Forrest's investment of \$112 yields \$7 a year. By finding what per cent \$7 is of \$112, show that his money is earning $6\frac{1}{4}$%. The brokerage may here be disregarded since it is so small.

Find the rate of income on stocks purchased at these prices and paying dividends at the rates stated. Count \$100 *as the par value and disregard brokerage:*

2. \$80, 4%
3. \$80, 5%
4. \$80, 6%
5. \$80, 7%
6. \$60, 3%
7. \$50, 5%
8. \$96, 6%
9. \$125, 5%
10. \$180, 9%
11. \$150, 12%
12. \$125, 10%
13. \$125, $7\frac{1}{2}$%

Find the rate of income, to the nearest tenth of 1%, *on stocks whose prices and rates of dividend are as follows:*

14. \$104, 6%
15. \$112, 7%
16. \$82, 7%
17. \$96, 4%
18. \$90, 5%
19. \$83, 5%
20. \$85, 7%
21. \$92, 6%

BONDS

If a city wishes to build a new school, it can borrow money by issuing *bonds*. A corporation may also issue bonds to raise money with which to extend its business.

A **bond** is a kind of promissory note in which a government or a corporation promises to pay a certain sum at a stated time. Bonds usually run from 5 yr. to 50 yr. The face value of a bond is usually $1000, although bonds in denominations of $500 and smaller amounts are also issued. Every bond has a fixed rate of interest, like 4% or 5%, payable semiannually. Bonds are often secured by a mortgage on the corporation's property. There is no such security for stocks.

If a railroad wants to borrow $2,000,000 to make improvements, it may issue 2000 bonds of $1000 each due in 30 yr. and paying 5% interest. The interest is paid on the face value; hence the yearly interest in this case is $50, payable in semiannual installments of $25.

The interest on a bond is often paid by means of small **coupons** attached to the bond and dated 6 mo. apart. On a 5% bond for $1000, each coupon is like a check for $25. When a coupon is due, the owner cuts it off the bond and usually deposits it in his bank. The bank then collects it for him. In the case of a *registered bond*, the corporation has the owner's name and sends him the interest by check instead of paying it by coupon.

Bonds of the highest grade, like United States government bonds, may pay only 3% to 4%. A first-class railroad bond pays from 4% to 5%, while the bonds of a less dependable corporation may have to pay 6% or 7% to attract buyers.

Important bonds, like stocks, are sold on the stock exchange and their prices are given daily in the newspapers. A bond listed: "Newtown 4s, 1953" is a Newtown bond paying 4% interest and due in 1953. On the New York Stock Exchange about $15,000,000 worth of bonds are sold daily.

THE INTEREST ON BONDS

Give the rate of interest, the date due, and the amount of interest paid each 6 *mo. on these* $1000 *bonds:*

1. Minneapolis $3\frac{1}{2}$s, 1954
2. Penn. Steel 5s, 1963
3. Hudson R. R. 7s, 1940
4. Warren Silk 6s, 1936
5. United Sugar $6\frac{1}{2}$s, 1936
6. U. S. Liberty 1st 4s, 1947

7. Find the interest rate and the date due of some of your city's bonds. For what purpose were the bonds issued?

8. Mr. Field owns $8000 worth of the Colorado Electric Company's 5% coupon bonds. How much interest from these bonds does he deposit in his bank every 6 mo.?

9. In 1926 an oil company sold $120,000,000 worth of 20-year 5% bonds. Find the yearly interest on these bonds. When will the company pay back the $120,000,000?

10. Rosedale County needs $250,000 to improve its roads. If it raises this money by 4% bonds, interest payable semi-annually, how much in all will the county pay out each 6 months in interest on these bonds?

11. In what way does the rate of interest on a bond help to show how safe the bond is?

12. A bond may run for 20 yr., 50 yr., or more. How does it differ in this respect from a promissory note?

13. Does a 6% preferred stock always pay a 6% dividend if the profits of the company are sufficient to cover it?

14. The common stock of the Akron Motor Company paid 6% dividend last year. Does the company guarantee to pay the same rate of dividend each of the following years? Does common stock always pay the same rate of dividend?

15. Does a 6% bond always pay the same rate of interest each year? Is this rate of interest guaranteed?

16. Why are bonds usually a safer investment than stocks?

WHY THE PRICES OF BONDS FLUCTUATE

Since bond quotations show the price in dollars of a $100 bond, the price of a $1000 bond is 10 times the quotation. Thus, a $100 bond quoted at $102\frac{3}{4}$ costs $102.75 and a $1000 bond quoted at $102\frac{3}{4}$ costs $10 \times \$102.75$, or $1027.50. Similarly, a $1000 bond quoted at $87\frac{1}{2}$ costs $10 \times \$87.50$, or $875.

On the New York Stock Exchange, the broker's fee for purchasing $1000 bonds is usually $2 per bond. Hence for one $1000 bond quoted at $87\frac{1}{2}$, the price is $875 + $2, or $877.

The change, or *fluctuation,* in the market prices of bonds is due to these factors:

1. If the corporation that issued the bonds becomes less prosperous, the price of the bonds will drop *below par* because the public has less confidence in them.

2. A rise or fall in the rates of interest paid on other loans will make bonds, on which the interest rate is fixed, correspondingly less or more desirable. If money is scarce and many people wish to borrow, the prevailing rates of interest will be high and it will be easy to get 6% or 7% interest on a well-secured loan. A bond paying only 5% will then have to be offered at a price below par, possibly as low as 83, to attract customers. A 5% bond purchased at 83 yields an income of about 6%. On the other hand, when there are many wishing to lend money and few wishing to borrow, the prevailing interest rates will be low and it will be necessary to pay a price above par to buy a high-grade 5% bond.

In general, the prices of bonds go down when prevailing interest rates are high and go up when interest rates are low.

3. The price of a $1000 bond gets nearer and nearer to $1000 as the date of maturity approaches. For example, if you buy at 90 a good 4% bond which is due in 1940, the price of the bond will gradually rise to 100 as 1940 approaches. Can you explain why? Remember that a bond is a kind of promissory note.

BUYING BONDS

1. A \$1000 bond is quoted at $85\frac{1}{2}$. Find the cost, including brokerage, of 1 bond; of 5 bonds; of 15 bonds.

2. Find the cost of buying a \$1000 Utah Railway 5% bond quoted at $98\frac{1}{2}$. How much yearly interest does this bond pay?

Remember that interest is always computed on the par value of a bond.

Find the cost, including brokerage, of buying the following bonds. In each problem also find the total yearly interest:

3. One \$1000 Union Telephone 5s, 1946, at 103.

4. One \$1000 French Government 7s, 1949, at $105\frac{1}{2}$.

5. One \$1000 Robbins Chocolate $5\frac{1}{2}$s, 1940, at $102\frac{1}{2}$.

6. Five \$1000 U. S. Treasury $4\frac{1}{4}$s, 1947, at 107.

7. Five \$1000 Michigan Motors 6s, 1940, at $91\frac{1}{2}$.

8. Twenty \$1000 Gulf R. R. 4s, 1948, at $95\frac{1}{2}$.

9. Two \$1000 Columbia Trolley 4s, 1998, at $94\frac{5}{8}$.

10. Give several possible reasons why Columbia Trolley bonds (ex. 9) were selling at less than Union Telephone bonds (ex. 3).

11. On Nov. 13 Mr. Howe buys from Mr. Long a 6% \$1000 bond at 101 with interest coupons payable on January 1 and July 1. Since Mr. Howe will receive the entire \$30 interest on the coupon for Jan. 1, he must pay Mr. Long the interest at 6% for the 135 da. from July 1 to Nov. 13, during which time Mr. Long held the bond. This is called **accrued interest**. How much is the accrued interest on this bond?

How much does Mr. Howe pay for the bond in all, including accrued interest and brokerage?

Compute interest by the 6% method or use the table on page 341.

12. Find the accrued interest on a 4% \$1000 bond bought on June 18 if the interest is payable Apr. 1 and Oct. 1. How much does the buyer pay in all for the bond if he buys it at $88\frac{1}{2}$?

FINDING THE RATE OF INCOME ON A BOND

1. Mr. Tate buys a 5% $1000 bond at 93. What rate of income does he get on his investment?

Since the interest is computed on the par value, Mr. Tate receives 5% of $1000, or $50, yearly interest. Since he has paid only $930 for the bond, he earns $\frac{50}{930}$, or 5.4%, every year on his investment. This 5.4% is called the **current yield** of the bond.

Brokerage is here disregarded because it is so small, compared with the price of the bond, as hardly to affect the rate of income.

$$
\begin{array}{r}
.0537 = 5.4\% \\
\$930\overline{)\$50.0000} \\
46\ 50 \\
\hline
3\ 500 \\
2\ 790 \\
\hline
7100 \\
6510
\end{array}
$$

2. Find the current yield on a 6% bond bought at 120.

The interest is 6% of $1000, or $60 a year. Since the price of the bond is $1200, the current yield on the price is $\frac{60}{1200}$, or 5%.

The price paid for a bond does not affect the yearly amount of interest. It does affect the rate of income earned on the money invested.

Exercises

Disregard brokerage in the following examples.

1. Mr. Harris buys a 5% $1000 bond at 89. How much does he pay for it? How much interest does he get yearly? Find, to the nearest tenth of 1%, the current yield of the bond.

Find the current yield, to the nearest tenth of 1%, *if* $1000 *bonds are purchased as follows:*

2. A 4% bond at 87

3. A 5% bond at 96

4. A $3\frac{1}{2}$% bond at 79

5. A $4\frac{1}{2}$% bond at 92

6. A 6% bond at 103

7. A 7% bond at 103

8. A $6\frac{1}{2}$% bond at 111

9. A $5\frac{1}{2}$% bond at 120

10. Mr. Locke can buy a 4% $1000 bond at 85 or a $4\frac{1}{2}$% $1000 bond at par. He considers both investments equally safe. On which bond would the current yield be higher? how much higher, to the nearest tenth of 1%?

FINDING THE YIELD TO MATURITY

In 1928 Mr. Lynn bought a 5% $1000 bond at 93. In 1935, when the bond is due, he will receive $1000 for it. To what yearly rate of income will his earnings then be equal?

The bond cost $930 and pays $50 interest yearly. In 7 yr. Mr. Lynn will receive $1000 for the bond, which is $70 more than he paid for it. This $70 is an average of $10 extra profit yearly. Hence his yearly income amounts to $50 interest + $10 extra profit, or $60. Therefore he has a yearly profit of $60 on an investment of $930. Since $60 is 6.5% of $930. his income is equal to 6.5% yearly.

Since 6.5% is the average yearly yield to the bond's date of maturity, it is called the **yield to maturity.** How does this differ from the current yield as found on page 413?

Exercises

1. Mrs. Shaw buys an 8% bond, due in 5 yr., at 115. Show that the yield to maturity on her investment is 4.3%.

At the end of 5 yr. she will receive $1000 for a bond that cost her $1150. Hence she will lose $150 in 5 yr., or an average of $30 a year. Subtracting this $30 from her yearly interest of $80, you find that she really receives $50 a year on an investment of $1150. What per cent is $50 of $1150?

Find the yield to maturity on these $1000 *bonds:*

2. A 5% bond bought at 90 with 5 yr. still to run.

3. A 4% bond bought at 80 with 20 yr. still to run.

4. A $4\frac{1}{2}$% bond bought at 98 with 2 yr. still to run.

5. A 6% bond bought at 110 with 10 yr. still to run.

6. An 8% bond bought at 118 with 9 yr. still to run.

7. A 4% bond bought Jan. 1, 1928 at 90, due Jan. 1, 1938.

8. A 5% bond bought Jan. 1, 1928 at 95, due Jan. 1, 1933.

9. Is the yield to maturity more or less than the current yield on a bond bought below par? on a bond bought above par?

NOTE. The above method gives the approximate yield to maturity. Bond dealers and bankers use *bond tables* for more exact values.

BUILDING AND LOAN ASSOCIATIONS

Saving in a Building and Loan Association

Many people invest savings in a building and loan association, which lends the money to help other people build their own homes. There are over 12,000 building and loan associations in this country, with about 10,000,000 members.

These associations are also called *savings and loan associations*, *homestead associations*, *building associations*, and *coöperative banks*.

Exercises

1. Mr. Hunt buys 10 *shares* in a building and loan association. These will be worth \$200 each in about 139 mo. He pays for them at the rate of \$1 a month on each share. How much does Mr. Hunt pay on the 10 shares in 1 mo.? in 1 yr.? in 5 yr.? in 10 yr.? in 139 mo.?

2. In 139 mo. Mr. Hunt's deposits, with compound interest, equal \$200 per share, and the company then pays him this amount. How much does he receive in all? How much of this total amount represents deposits? How much is interest?

3. Mr. Hunt would have about \$1750 at the end of 139 mo. if he had deposited \$10 a month in a savings bank at 4% compounded semiannually. How much less would this have been than the amount that he receives from the building and loan association?

4. Some building and loan associations sell \$100 shares at the rate of \$1 a month per share. How much must Mr. Low pay each month to buy 15 of these shares?

5. The \$100 shares in ex. 4 *mature*, or become worth \$100 each, in about 80 mo. How much does Mr. Low pay in 80 mo. on 15 shares? How much of the total amount which he receives at the end of 80 mo. is interest?

6. \$100 shares are also sold at 50¢ per month, but such shares require about 139 mo. to mature. What would be the monthly payment on \$3200 worth of such shares?

BORROWING FROM BUILDING AND LOAN ASSOCIATIONS

Mr. Trent built a house with the help of the Franklin Building and Loan Association. The house cost $5000. Mr. Trent furnished $1800 and the association lent him $3200, with the property as security. On this loan he paid 7% interest, or $18.67 a month. He also bought 32 $100 shares in the Franklin Association, on which he paid 50¢ a share monthly. Hence he paid the association $34.67 in all each month.

$18.67 Monthly Interest
16.00 Payment on Shares
$34.67 Monthly Payment

In about 139 mo. the 32 shares were worth $100 each, and Mr. Trent turned them over to the association to pay his debt. Thus, by paying only $34.67 a month, he canceled his loan and the house was entirely his.

New York State, Ohio, and certain other associations use a system of repaying the loan similar to that outlined on page 356.

Exercises

1. Why was Mr. Trent's plan wiser than renting a house?

2. Mr. Dane borrows $6600 from the Lincoln Building Association. At 7%, how much interest must he pay monthly? How many $200 shares must he buy to cover the debt?

3. If Mr. Dane (ex. 2) pays $1 a month per share, how much, including interest on his loan, must he pay to the association in 1 mo.? in the 139 mo. required for the shares to mature? This is how much more than the amount he borrowed?

4. Who gets the shares in ex. 3 at the end of 139 mo.?

5. Mr. Lee borrows $2400 from the Economy Building Association at 7% interest. To repay the debt he buys $100 shares at 50¢ a month per share. How many shares does he buy? How much does he pay the association each month, including payments on the shares and interest?

BUILDING YOUR OWN HOME

1. Mr. Gray wishes to build a house. He therefore borrows $4200 from the Prudence Savings and Loan Association. Find the monthly interest on this loan at 6%.

2. Mr. Gray (ex. 1) buys $100 shares for $4200, on which he pays $1 a month per share. What is his total payment, including interest on the loan, in 1 mo.? in 80 mo.?

3. In about 80 mo. Mr. Gray's shares mature and are used to pay off his debt. How much more than the $4200 loan has he paid in all? Then how much did the loan cost him?

4. If Mr. Gray had borrowed the $4200 for 80 mo. from a bank at 6%, how much interest would he have paid in the 80 mo. ($6\frac{2}{3}$ yr.)? How much more is this than the cost of borrowing the $4200 from a savings and loan association (ex. 3)?

The reason the total interest is more if Mr. Gray borrows from a bank is because he keeps the $4200 for the whole 80 mo. instead of repaying it in monthly installments. The shares that Mr. Gray buys (ex. 2 and 3) are the equivalent of repaying the association's loan in monthly payments.

5. Mr. Collins wishes to build a $10,000 house. He has only $3500 in cash now. He can borrow 50% of the value of the property from a bank or 65% to 70% of its value from a building association. Where will it be better for him to borrow the money?

6. Mr. Davis borrows $9000 at 7% from a homestead association. To repay it, he buys shares of $100 each, at $1 a month per share. How much does he pay in all each month?

7. If the shares reach their full value in 80 mo., how much more than the loan does Mr. Davis (ex. 6) pay the association? Then what is the total cost of the loan?

A man who borrows from a building and loan association decreases his debt with each payment on his shares, which is much easier than repaying the entire loan at once. Likewise, the association's risk on the loan becomes less and less each month.

INSURANCE

Insuring a House Against Fire

In a recent year fire destroyed the homes of about 1,150,000 people in the United States. Many of these people had all their money invested in their homes and could not rebuild them without outside help.

The wisest of these people had insured their houses against fire. To insure a house against fire, a man pays to an *insurance company* a small sum of money called a **premium.** The company then gives him a written agreement, or **policy,** promising to pay for damage that may be done to his property by fire, smoke, or lightning.

Mr. Austin has his house insured for $8000; hence $8000 is the **face** of his policy, and the company will give him $8000 if the house is completely burned down. Mr. Austin's premium is at the rate of $.24 per $100. Since $8000 is 80×$100, the premium is 80×$.24, or $19.20. By paying $19.20, Mr. Austin insures his house for 1 yr.

To find the premium, multiply the rate per $100 *by the number of times* $100 *is contained in the face of the policy.*

Exercises

1. Mr. Smith insures his house for $10,000. At a yearly rate of $.22 per $100, what is his premium for 1 yr.?

Find the yearly premium on these insurance policies:

	Face	Rate per $100		Face	Rate per $100
2.	$6000	$.30	**6.**	$15,000	$.21
3.	$4300	$.28	**7.**	$34,000	$.80
4.	$7500	$.60	**8.**	$12,800	$1.05
5.	$5400	$2.75	**9.**	$23,700	$1.20

WHY FIRE INSURANCE RATES VARY

The insurance company can pay for the damage caused by fires because most buildings on which insurance is paid are never burned. By studying for many years the fire losses of different kinds of buildings, the company determines the rate for each type of building. The rate on a wooden house, a garage, or any building without fire protection and far from a fire house is higher than the rate on a well-protected and fireproof building.

This table shows the yearly rates for different buildings:

Kind of Building	Place Located	Rate per $100	Kind of Building	Place Located	Rate per $100
Brick dwelling	City	$.25	Frame church	Country	$1.50
Frame dwelling	City	.50	Frame hotel	Country	2.20
Brick garage	City	.35	Grist mill	Village	3.65
Frame garage	City	.80	Power mill	Village	4.22
Farmhouse	Near town	.58	Frame school	Country	1.40
Barn	Near town	.82	Soap factory	Village	6.50

Exercises

1. Which of the above kinds of buildings has the highest rate of insurance? Explain why. Give reasons why a frame hotel in the Adirondack Mountains pays a higher rate than a brick dwelling in Detroit.

2. Would a frame dwelling in a city pay a higher or a lower rate than one on a country road? Why?

Find the premiums on these policies at the above rates:

	Building	Face		Building	Face
3.	Brick garage	$1500	**7.**	Brick dwelling	$14,500
4.	Farmhouse	$7000	**8.**	Soap factory	$51,000
5.	Frame garage	$1200	**9.**	Frame school	$10,000
6.	Barn	$2200	**10.**	Power mill	$13,250

11. Name several uses to which a building might be put which would cause it to have a high rate of fire insurance.

INSURING A HOUSE FOR THREE OR FIVE YEARS

Mr. Law insures his house for \$14,000. The agent says that the rate for a 1-year policy is 30¢ per \$100; for a 3-year policy, it is $2\frac{1}{2}$ times as much; and for a 5-year policy, it is 4 times as much. Hence Mr. Law would pay $140 \times 30¢$, or \$42, for a 1-year policy. He would pay $2\frac{1}{2} \times \$42$, or \$105, for a 3-year policy. How much would a 5-year policy cost him?

Exercises

1. For 3 yr. Mr. Law buys 1-year policies. How much would he save by buying one 3-year policy?

2. How much cheaper would it be for Mr. Law to buy one 5-year policy than to buy five 1-year policies?

3. For 30 yr. Mr. Day renewed annually a \$31,600 insurance on his home at 34¢ per \$100. How much would he have saved in this time by buying 3-year policies? 5-year policies?

4. For 30 yr. Mr. Day insured his house furnishings annually for \$5000, at 38¢ per \$100. How much would he have saved by buying 5-year policies?

Find the premiums at the given yearly rates per \$100:

	Face	Rate	Term		Face	Rate	Term
5.	\$2000	\$ .15	1 yr.	**10.**	\$15,500	\$ .30	3 yr.
6.	\$8700	\$ .42	3 yr.	**11.**	\$11,450	\$ .62	3 yr.
7.	\$3750	\$ .60	3 yr.	**12.**	\$24,500	\$3.75	5 yr.
8.	\$9460	\$1.00	5 yr.	**13.**	\$63,800	\$1.88	5 yr.
9.	\$2350	\$4.22	5 yr.	**14.**	\$17,850	\$6.75	5 yr.

15. A cotton merchant had a 3-year insurance policy for \$46,000 on his stock, at a yearly rate of \$4.50 per \$100. Find his premium. If \$2000 worth of his cotton was destroyed by fire and the company paid the damage, how much did the company lose or gain on this policy?

IMPROVEMENT TEST No. 29

Test 29 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$1\frac{5}{8}$	$2\frac{1}{3}$	$5\frac{1}{6}$	$2\frac{2}{5}$	$8\frac{5}{8}$
$4\frac{1}{8}$	$7\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{4}{5}$	$7\frac{1}{3}$
$2\frac{3}{4}$	$9\frac{1}{4}$	$8\frac{5}{6}$	$1\frac{1}{2}$	$5\frac{1}{3}$
$9\frac{1}{16}$	$6\frac{1}{2}$	$3\frac{1}{6}$	$2\frac{7}{10}$	$6\frac{3}{8}$

Test 29 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$4\frac{1}{5}$	$6\frac{5}{12}$	$7\frac{1}{2}$	$8\frac{5}{6}$
$2\frac{9}{10}$	$3\frac{7}{12}$	$\frac{1}{3}$	$5\frac{1}{2}$
5.	**6.**	**7.**	**8.**
$3\frac{3}{8}$	8	$9\frac{4}{5}$	$7\frac{1}{2}$
$2\frac{5}{16}$	$7\frac{3}{8}$	$3\frac{1}{3}$	$2\frac{15}{16}$
9.	**10.**	**11.**	**12.**
$6\frac{1}{4}$	9	$8\frac{1}{6}$	$6\frac{2}{3}$
$2\frac{1}{5}$	$8\frac{5}{12}$	$1\frac{1}{4}$	$3\frac{5}{8}$

Test 29 C — Multiplication (Time 4 min. after copying)

1.	2.	3.
\$1.53	\$2.76	\$1.90
$18\frac{1}{2}$	$11\frac{3}{4}$	$24\frac{2}{3}$

4. $\frac{3}{8}\times 1\frac{5}{6}$	**5.** $18\times\frac{5}{6}$	**6.** $4\frac{1}{2}\times 37$
7. $\frac{5}{12}\times\frac{9}{10}$	**8.** $98\times 1\frac{1}{8}$	**9.** $4\frac{1}{6}\times 6\frac{9}{10}$

Test 29 D — Division (Time 4 min. after copying)

1. $\frac{7}{8}\div 2$	**2.** $5\frac{1}{4}\div 7$	**3.** $29\div\frac{5}{6}$
4. $\frac{7}{10}\div\frac{7}{10}$	**5.** $2\frac{5}{8}\div\frac{14}{15}$	**6.** $3\frac{1}{2}\div 4\frac{1}{4}$
7. $8\div 3\frac{1}{5}$	**8.** $8\frac{3}{4}\div 15$	**9.** $9\frac{3}{4}\div 1\frac{1}{2}$
10. $6\div 2\frac{5}{8}$	**11.** $3\frac{1}{3}\div 16$	**12.** $12\frac{1}{4}\div 1\frac{3}{4}$

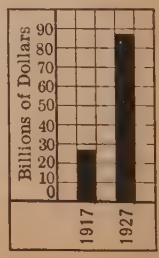
THE IMPORTANCE OF LIFE INSURANCE

Mr. Scott worked in an automobile factory. Besides his wife, there were four children in the family, all attending elementary school. When Mr. Scott was 35 yr. old a life insurance agent called upon him one day and tried in vain to persuade him to have his life insured for $10,000. Five years later Mr. Scott died suddenly.

After his death, Mrs. Scott found it very hard to get along. She got a position as clerk in a store to help support her children; but as she did not earn very much, her son Jack, who was 14 yr. old, had to leave school and go to work. The Scotts had no relatives to help them and it was a hard struggle. A few years later Mrs. Scott became ill from overwork and her son Ben, who was then 13 yr. old, also had to leave school in order to go to work.

All this distress could have been avoided if Mr. Scott had insured his life for $10,000. The life insurance company would then have paid Mrs. Scott $10,000 when Mr. Scott died, which would have kept all the children in school and prevented much hardship and worry.

People are rapidly realizing that life insurance is a very valuable form of protection, and each year much more life insurance is being sold in the United States than the year before. In 1917 the total amount of life insurance in force in this country was about $27,000,000,000, while in 1927 the total amount was about $87,400,000,000. This growth is shown by the graph at the right. The bar for 1927 is about how many times as long as the bar for 1917?



GROWTH OF LIFE INSURANCE IN THIS COUNTRY

At the present time, about half of all the men, women, and children in the United States carry some form of life insurance.

BUYING AN ORDINARY LIFE POLICY

The kind of life insurance policy most frequently purchased is called an **ordinary life policy.** This type of policy is the most popular because it is the cheapest form of regular life insurance.

Suppose Mr. White, who is 30 yr. old, buys an ordinary life policy for $10,000. In this policy the life insurance company promises to pay $10,000 to Mr. White's wife if Mr. White should die. Mrs. White is the **beneficiary,** Mr. White is the **policyholder,** and $10,000 is the **face** of the policy.

Mr. White pays a yearly premium of $22.85 for each $1000 of insurance. Since the policy is for $10,000, his total yearly premium is $10 \times \$22.85$, or $228.50. This premium must be paid each year as long as Mr. White lives. For this reason an ordinary life policy is sometimes called a **continuous premium policy.**

The amount of the premium for an ordinary life policy depends on the age of the person *at the time he is insured,* as shown at the right. Thus, if a person is 35 yr. old when he takes out an ordinary life policy, he pays $26.35 per year for each $1000 insurance.

ORDINARY LIFE POLICY

Age	*Premium per* $1000
25	$20.14
30	22.85
35	26.35
40	30.94
45	37.08
50	45.45

Exercises

Using the rates above, find the yearly premiums for these ordinary life policies, purchased at the given ages:

1. $1000, age 25
2. $4000, age 25
3. $3000, age 40
4. $5000, age 50
5. $7000, age 30
6. $8000, age 45
7. $9000, age 35
8. $2000, age 50
9. $10,000, age 30
10. $12,000, age 45
11. $25,000, age 35
12. $42,000, age 40

13. Why must a man who takes out a policy at age 45 pay a higher premium than one who insures his life at age 30?

14. Why is it best to insure your life at an early age?

OTHER KINDS OF LIFE INSURANCE POLICIES

You have seen that the holder of an *ordinary life policy* must pay the premium each year as long as he lives.

On a **limited-payment policy** the premiums are required only for a definite number of years, like 20 yr. The insurance continues in force, however, until the policyholder dies, when the face of the policy is paid to the beneficiary.

Limited-payment policies cost much more than ordinary life policies. Such policies are preferred by those who wish to pay all their premiums before their earning power is decreased by old age.

On an **endowment policy,** the premiums are also paid for a definite number of years, like 20 yr. If the policyholder dies within the 20 yr., the face of the policy is paid to the beneficiary; but if he is living at the end of that period, the full amount is paid to him and the insurance ceases.

Endowment policies combine life insurance with a system of saving money to provide for old age. If a man lives to the end of the 20-year period, he usually gets back more money than he has paid in premiums, since the money has earned interest during that time. Although the premiums paid on an endowment policy do not yield as much, from the savings standpoint alone, as equal deposits in a savings bank for the same number of years, the endowment policy gives the additional benefit of life insurance for the period it is in force.

On a **term policy,** premiums are paid for a short term of years, like 5 yr., 10 yr., or 15 yr., and the company pays insurance only if the policyholder dies within that period. At the end of the term the insurance ceases.

Term insurance is purchased when a person wants extra protection for a short time. For example, a man may carry an ordinary life policy for $15,000 all his life; but during the 10 yr. that his children are in school, he may also take out a 10-year term policy for $10,000 to make certain that the education of his children will be completed in case he dies during that period. The premium rates on term policies are lower than the rates on the other kinds of policies, since the company is taking the risk for a shorter period.

A person applying for life insurance has to pass a medical examination. Unless his health is good, he cannot be insured.

EVERYDAY PROBLEMS IN LIFE INSURANCE

The premium rates per $1000 vary according to the kind of policy and the age at which it is taken out, as shown below:

Age	Ordinary Life	20-Payment Life	20-Year Endowment	10-Year Term
25	$20.14	$30.12	$48.15	$11.40
30	22.85	32.87	48.83	12.10
35	26.35	36.22	49.85	13.19
40	30.94	40.38	51.48	15.02
45	37.08	45.73	54.22	18.40
50	45.45	52.87	58.81	24.50
55	56.93	62.68	66.36	34.80

1. From the table, find the premium per $1000 on a 20-payment life policy taken out at age 25; at age 30; at age 35.

2. Find the premium per $1000 on a 20-year endowment policy taken out at the age of 30 yr.; 40 yr.; 45 yr.; 50 yr.

3. At age 35, the 20-year endowment premium rate is about how many times as much as the ordinary life rate?

Using the rates above, find the yearly premium for these policies if purchased at the given ages:

4. A 10-year term policy for $15,000 issued at age 40.

5. A 20-payment life policy for $3000 issued at age 30.

6. An ordinary life policy for $25,000 issued at age 25.

7. A 20-year endowment policy for $5000 issued at age 35.

Reread pages 423 *and* 424 *before answering ex.* 8 *to* 10.

8. Why is the ordinary life policy the most popular?

9. Why are the premiums on a 20-payment life policy higher than those on an ordinary life policy? Why do some persons prefer the 20-payment policy?

10. How does a 20-year endowment policy differ from a 20-payment life policy? Why are the endowment rates higher?

EVERYDAY PROBLEMS IN LIFE INSURANCE

Use the premium rates given on page 425.

1. Mr. Stark is 30 yr. old. He wishes to protect his family with as much life insurance as possible. He can afford to pay about $200 a year in premiums. Roughly, what amount of insurance can he buy for $200 yearly if he selects the ordinary life policy? the 20-payment life policy? the 20-year endowment? Which do you think it advisable for Mr. Stark to buy?

2. Give several reasons why it is unwise to wait until old age before taking out life insurance. Remember that a person who applies for life insurance must take a medical examination.

3. A motion-picture company took out an ordinary life policy for $1,000,000 on the life of its manager, upon whose business ability the success of the company depended. In case of the manager's death, the policy was payable to the company. What yearly premium did the company pay if the manager was 35 yr. old when the policy was issued? Why did the company carry such a large insurance on his life?

Life Insurance in the United States

1. Of the 120,000,000 inhabitants of the United States, about 62,000,000 carry life insurance. About what per cent of the inhabitants have their lives insured?

2. The total amount of life insurance in force in this country in 1927 was about $87,400,000,000. If 62,000,000 people were insured, find the average amount of insurance carried by each.

3. Find the per cent of increase in the amount of life insurance in force from 1917 to 1927 (see bottom of page 422).

4. One prominent man in the United States carries $7,500,000 life insurance; another carries $4,500,000 insurance. Find each man's yearly premium if their insurance is in ordinary life policies taken out at the age of 35 yr.

HOW THE COMPANY USES THE PREMIUMS

The premium on an ordinary life policy at age 35 is $26.35 per $1000. The company uses this premium as follows:

1. *Mortality cost.* About $9 of this $26.35 is set aside to pay the insurance of policyholders who die during the year.

From a careful study of past records of deaths, mortality tables have been made, which show how many people may be expected to die each year.

2. *Expenses.* About $5 of the premium is used to pay overhead expenses, like rent, salaries, and commissions.

3. *Reserve.* The rest of the premium, about $12.35, forms a reserve fund which is credited to the policyholder each year like a savings bank deposit.

$9.00	**Mortality Cost**
5.00	**Expenses**
12.35	**Reserve**
$26.35	**Premium**

This reserve, which usually earns 3% compound interest, is regarded as the property of the policyholder. If he gives up his policy, this reserve fund, which is then called the *cash value* of the policy, is returned to him. A policyholder can also borrow from the company an amount equal to his reserve fund, which is then known as the *loan value* of his policy.

Table of Cash or Loan Values

The table at the right gives the cash or loan values on an ordinary life policy issued at age 35. At the end of the fifth year the cash or loan value is $68.16. Hence the policyholder will receive $68.16 if he discontinues the policy at the end of 5 yr., or he may borrow $68.16 from the company at that time if he continues paying the premiums. This $68.16 equals the reserve fund of the policy.

Cash or Loan Value per $1000 Insurance	
End of 5th yr.	$ 68.16
End of 10th yr.	146.01
End of 15th yr.	233.28
End of 20th yr.	327.58
End of 25th yr.	425.49
End of 30th yr.	522.92

At the top of the page you found that $12.35 is put into the reserve fund each year. Notice that $68.16 is a little more than 5×$12.35, the extra amount being compound interest earned on this reserve fund.

BORROWING FROM AN INSURANCE COMPANY

Use the cash or loan values at the bottom of page 427.

1. At 35 yr. of age Mr. Baron purchased an ordinary life policy for \$5000. At the end of 10 yr. he needed some money. How much could he borrow from the life insurance company?

The loan values on page 427 are for a \$1000 policy. For a \$5000 policy they are 5 times as much.

2. Mr. Baron borrowed \$725 from the life insurance company and was charged interest at 6%. How much interest did he pay at the end of the first year?

3. If Mr. Baron had died before paying off his loan, the company would have paid the \$5000 insurance less the amount of the loan and the interest due on it. If Mr. Baron had died 6 mo. after making the loan, how much insurance would the company have paid to his beneficiary?

4. Mr. Grady has a \$15,000 policy issued at age 35. How much would the company lend him at the end of the 5th year?

Life Insurance Dividends

On page 427 you saw that life insurance premiums are used to pay the mortality cost and the company's expenses, and to provide a reserve fund. It often happens that fewer people die during the year than the company expected and the *mortality cost* is therefore less than was estimated. The company may also save on *expenses*. On the *reserve fund* the interest may be over 4%, although only 3% was counted on.

The profit arising from these savings and from the extra interest above 3% is divided among the policyholders in the form of **dividends** at the end of each policy year. If a person whose premium is \$26.35 receives a dividend of \$5.31 at the end of the year, his insurance actually costs him only \$26.35 − \$5.31, or \$21.04, for that year.

THE ACTUAL COST OF LIFE INSURANCE

1. Ten years ago at age 35 Mr. Bell took out an ordinary life policy for \$1000, with an annual premium of \$26.35. His dividends have averaged \$5.26 a year. How much has he paid each year, on the average, to carry this insurance? Find his total net payments for the 10 yr.

2. From the table on page 427, tell what cash value Mr. Bell (ex. 1) receives for his policy if he drops his insurance at the end of 10 yr. By subtracting this amount from the net amount paid in 10 yr. (found in ex. 1), find how much the 10 years' insurance has actually cost him. Also find the average cost per year of carrying this \$1000 insurance.

3. Mr. Alden carried a 20-payment life policy for \$1000 for 10 yr. His yearly premium was \$36.22. During the 10 yr. his dividends totaled \$61.69. Find his average yearly payment.

4. At the end of 10 yr. Mr. Alden (ex. 3) dropped his policy. Its cash value was then \$255.78. How much per year, on an average, had the insurance actually cost him?

5. Mr. Rose took out a \$10,000 ordinary life policy at age 30, the yearly premium being \$22.85 per \$1000. During the first 10 yr. the company paid him a total of \$687.30 in dividends. Find the average net cost per year to Mr. Rose for each \$1000 insurance that he carried.

6. During the second 10 yr. Mr. Rose (ex. 5) received \$897.40 in dividends. Find the average net cost per year for each \$1000 insurance for these 10 yr.; the average for the 20 yr.

NOTE. Instead of using the dividends each year to decrease the cost of their insurance, some policyholders leave them with the company, where they earn compound interest like savings deposits. If the policyholder dies, his beneficiary receives the face of the policy plus the accumulated dividends. There is also a plan by which each year's dividends may be used gradually to increase the face of the policy.

TAXES

Why We Are Taxed

Mr. Livingston is the principal of an elementary school in a small city. He finds that it costs the city $102.83 a year to educate each pupil in his school.

Every town needs large sums of money yearly to educate its girls and boys, to build and maintain roads, streets, and public buildings, to pay its firemen, policemen, and governing officials, and for many other purposes. The raising of this money is called **taxation.** The amount which each inhabitant is *taxed* depends on the value of his possessions, because the man with much property is better able to pay a large tax, and also has more need for the town's protection.

How We Are Taxed by Our Town or City

Most towns have tax officers, called **assessors,** who fix a value each year for the property to be taxed, including *real estate,* like houses and lots, and *personal property,* like automobiles. This value is called the **assessed valuation.**

Suppose the total assessed valuation of the property in a town is $1,250,000 for a certain year. If the town needs $35,500 that year, the **tax rate** will be $35,500 ÷ $1,250,000, or .0284. Then every owner of property will have to pay .0284, or 2.84%, of the assessed value of his property in taxes.

To find the tax rate on property, find what per cent the amount to be raised is of the total assessed valuation.

The tax rate may be expressed in several different ways. A tax rate of 2.84% may also be given as a rate of 28.4 mills on $1, $2.84 on $100, or $28.40 on $1000.

City and town governments get most of their money by taxes on real or personal property. State and national governments usually raise money by other forms of taxation.

FINDING THE TAX RATE

1. A town whose total assessed valuation is \$2,388,000 needs \$70,446 for its expenses. Find the tax rate.

2. Express the tax rate in ex. 1 in three other ways.

3. A town with property having a total assessed valuation of \$685,000 needs to raise \$26,441 by taxation. Find the tax rate. Then express this rate in three other ways.

Find the tax rate per \$100 *for the following. Then express each rate in three other ways:*

	Assessed Valuation	Amount Needed		Assessed Valuation	Amount Needed
4.	\$38,000,000	\$836,000	**9.**	\$34,656,000	\$918,384
5.	\$20,850,000	\$781,875	**10.**	\$78,924,000	\$1,764,290
6.	\$51,464,800	\$1,981,364	**11.**	\$60,371,000	\$1,626,510
7.	\$85,000,000	\$3,110,000	**12.**	\$105,930,500	\$4,180,900
8.	\$93,700,000	\$4,046,000	**13.**	\$450,380,000	\$13,870,600

14. The actual value of the property in a town is \$4,562,500. The assessed valuation is \$3,650,000. If \$154,000 is to be raised, what is the tax rate? If the property were assessed at its actual value, what would the rate be?

15. What per cent of the actual value of the property is the assessed valuation in ex. 14?

16. The assessed value of property is often only from 50% to 80% of the real value. How does this affect the tax rate?

17. Find the tax rate in your town for last year and express it in four different ways. Also find what per cent the assessed valuation of some property was of the real value.

18. Hospitals, schools, public buildings, charitable institutions, and churches are not taxed. Can you suggest why? List some buildings in your town that are not taxed.

A TOWN BUDGET

To find the amount to be raised by taxation, many towns list their probable expenses for the coming year in a **budget.** Here are the 1927 and 1928 budgets of Ocean City:

	A 1928 Amount Needed	*B* Tax Rate per $100	*C* 1927 Amount Needed	*D* Tax Rate per $100
Municipal Requirements	$350,957.86	$1.177	$351,036.78	$1.349
Local School Tax	418,346.54	1.403	382,263.18	....
County Tax	153,860.88	.516	139,998.36	....
State School Tax	71,861.38	.241	69,478.74	....
State Road Tax	29,818.00	.100	26,022.00	....
State Institutional Tax	14,909.00	.050	13,011.00	....
Bridge and Tunnel Tax	10,734.48	.036	9,888.36	....
Soldiers' Bonus Tax	5,069.06	.017	4,944.18	....
Total Amount Needed	$1,055,557.20		$996,642.60	
Total Assessed Valuation	$29,818,000.00		$26,022,000.00	
Total Tax Rate	$3.54 per $100		.....	

Exercises

1. What was the total amount of money needed by this town for 1928? What was the total assessed valuation? Show how the total rate was found, using the rule on page 430.

2. Column *B* shows the rate of taxation for each item of expense. Thus, for the support of local schools in 1928, every inhabitant had to pay $1.403 for every $100 of the assessed valuation of his property. By dividing the amount needed for local schools by the assessed valuation, show how this rate was found.

3. Show how the rate was found for each item in Column *B*.

4. Add all the rates in Column *B*. How does this sum compare with the total rate found in ex. 1?

5. Find the tax rate for each item in Column *D*. Then find the total rate per $100 for 1927 in two different ways.

6. Name five purposes for which your town needs money.

FINDING THE AMOUNT OF TAXES YOU MUST PAY

1. Suppose you lived in the town whose budget is given on page 432. If you owned property assessed in 1928 at $14,500, how much would you be taxed on it?

The rate for 1928 in this town was $3.54 per $100, which equals 3.54%. Hence your total tax would be 3.54% of $14,500, or .0354×$14,500.

2. If you owned property assessed at $8000 in the above town in 1928, how much would your local school tax be? your state school tax? your state road tax?

The state school tax helps schools in poorer districts.

3. Mr. Lane owns a lot 75 ft. wide, assessed at $35 a front foot. His house is assessed at $8500, his automobile at $950, and his other personal property at $1500. Find the total amount he is assessed. If the tax rate is $3.94 on $100, what is the total tax on his property?

Find the amount of taxes paid in a given year by persons whose property is assessed as follows:

	Assessed Value	Rate per $100		Assessed Value	Rate per $1000
4.	$20,000	$4.02	**8.**	$203,500	$34.50
5.	$97,540	$2.04	**9.**	$610,400	$28.75
6.	$51,230	$3.27	**10.**	$115,000	$41.60
7.	$74,000	$5.12	**11.**	$108,500	$49.50

12. In Hatton the assessed valuation of property is 45% of the real value and the tax rate is $5.83 on each $100 of assessed value. In Lakewood the property is assessed at 85% of its real value and the rate is $3.24 on each $100 of assessed value. Find the tax in each town on property actually worth $10,000; on a house actually worth $8400.

A tax rate means little unless the per cent which the assessed valuation is of the real value is also considered. A city with a high tax rate and a low valuation may be taxing its citizens less than one with a low rate and a high valuation.

USING A TAX TABLE

To save time in computing taxes, most cities use a tax table. Here is a portion of such a table, used by Ocean City:

Tax Table for Rate of $3.54 per $100

	$00	$1000	$2000	$3000	$4000	$5000	$6000	$7000
$ 00		35.40	70.80	106.20	141.60	177.00	212.40	247.80
50	1.77	37.17	72.57	107.97	143.37	178.77	214.17	249.57
100	3.54	38.94	74.34	109.74	145.14	180.54	215.94	251.34
150	5.31	40.71	76.11	111.51	146.91	182.31	217.71	
200	7.08	42.48	77.88	113.28	148.68	184.08	219.48	
250	8.85	44.25	79.65	115.05	150.45	185.85	221.25	
300	10.62	46.02	81.42	116.82	152.22	187.62	223.02	
350	12.39	47.79	83.19	118.59	153.99	189.39	224.79	
400	14.16	49.56	84.96	120.36	155.76	191.16		
450	15.93	51.33	86.73	122.13	157.53	192.93		
500	17.70	53.10	88.50	123.90	159.30	194.70		

To find the tax on property assessed at $6350, place a ruler horizontally under the number 350 in the column at the left. Then, in the column headed $6000, find the number just above the ruler, which is 224.79. Hence the tax on $6350 is $224.79.

Exercises

1. Using the table, find the tax on $6200; on $6150; on $6050; $6000; $5000; $5150; $4350; $3300; $2400; $7100.

2. Find the tax on $11,250 by adding the taxes on $6000 and $5250. Check by multiplying $11,250 by .0354.

3. Find the tax on $12,250 and on $9350, as in ex. 2.

4. How much greater is the tax on $6250 than on $6200? on $6300 than on $6250? on $6350 than on $6300? Explain how the table is made. Then fill in the amounts that have been omitted in the columns headed $6000 and $7000.

5. By the method used in ex. 4, continue the column headed $1000, giving the tax on $1550, $1600, etc., to $1950.

THE PENALTY FOR PAYING TAXES LATE

1. Mr. Lake's taxes for 1927 were \$522 and were due on Dec. 1, 1927, but he did not pay them until Oct. 13, 1928. As a penalty for paying the taxes late, he was charged 7% interest on the \$522 for the time it was overdue. Find the exact number of days the taxes were overdue. Then find, by the 6% method, the total amount he paid on Oct. 13, 1928.

Remember that 1928 was a leap year.

2. Mr. Carter received a tax bill for \$274.50, due Dec. 1, 1925. He paid the bill on Jan. 3, 1928. If he was charged simple interest at 7% for the time the taxes were overdue, how much extra was he charged for paying late?

From Dec. 1, 1925 to Dec. 1, 1927 is 2 years. From Dec. 1, 1927 to Jan. 3, 1928 is 33 da. Hence the exact time is 2 full years and 33 da.

3. On May 2, Mr. Reed received a bill for \$570 for his 1927 taxes. In Mr. Reed's city taxes are payable in two installments. One half of the \$570 was due on June 1, 1927 and the other half on Dec. 1, 1927. Mr. Reed was unable to pay either half until May 10, 1928. Then he paid both installments with interest at 8%. What amount did he pay in all?

Find the amount of interest paid on these overdue taxes:

	Taxes	Date Due	Date Paid	Interest
4.	\$984.20	Dec. 1, 1927	Aug. 8, 1928	9%
5.	\$360.40	June 1, 1927	Sept. 5, 1927	8%
6.	\$670.16	Jan. 10, 1928	Apr. 16, 1928	8%
7.	\$1255.00	Dec. 1, 1924	Sept. 1, 1928	7%

8. In Georgetown the interest on unpaid taxes is 9%. What is the interest there on a tax of \$540, due June 1, 1927 and paid Feb. 14, 1928?

9. If the taxpayer in ex. 8 had borrowed \$540 at 6% in order to pay his taxes on time, how much would he have saved?

HELPING TO SUPPORT THE NATIONAL GOVERNMENT

When John and his mother were in Europe, they purchased many articles much cheaper than they could buy them in this country. But when they returned here, their baggage was examined and they had to pay taxes, known as **duties** (also called *customs* or *tariff*), on many of these purchases. The duty on an article may be an *ad valorem* (a.v.) *duty*, which is a certain per cent of the price for which the article was bought, or a *specific* (s.) *duty*, which is a fixed amount for each unit of measure. A duty of 50% of the cost is an **ad valorem duty**, while a duty of 25¢ a pound is a **specific duty.** This duty is collected by officials of the United States government and is used for Federal expenses.

John's mother was allowed to bring in $100 worth of taxable material free. Certain things, like foreign books and magazines, were not taxed. On the other purchases, she paid the duties given in the exercises below.

Exercises

Find what duty John's mother paid on the items in ex. 1 *to* 11:

	Cost	Duty		Cost	Duty
1. Bag,	$13.70	35% a.v.	**5.** Dresses,	$89.23	65% a.v.
2. Linens,	$80.00	55% a.v.	**6.** Jewelry,	$51.25	80% a.v.
3. Games,	$22.00	70% a.v.	**7.** Perfume,	$104.80	75% a.v.
4. Lace,	$65.50	90% a.v.	**8.** Fur coat,	$275.50	50% a.v.

9. 5 lace handkerchiefs, cost $2.90 each; a specific duty of 4¢ each and a 40% a.v. duty.

10. One Oriental rug 4 ft. 9 in. by 8 ft., costing $146.15. Duty 50¢ a square foot, but *not less* than 45% (a.v.).

11. 1 boys' suit and overcoat, weight 8 lb., cost $61.70. One duty of $.50 a pound (s.), and another of 50% (a.v.).

12. Find the total duties paid by John's mother.

13. In New York John's mother bought a French hat for $25. The merchant who imported it paid $10 for it in France and paid a 50% ad valorem duty on it. In fixing the price he allowed $10 to cover selling expenses and profits. Who really paid the government duty, the merchant or John's mother?

A tax like this, which is paid by the final purchaser as part of the price of an article, is an *indirect tax*.

14. There is a tax of 10% on theater tickets that cost over $3.00. How much must you pay for a theater ticket whose price without the tax is $3.50? whose price is $4.00? $5.00?

15. If John's father pays $15.40 (including tax) for 4 theater tickets, how much tax is he paying in all? What would be the price of each ticket without the tax?

16. John's father bought an automobile whose factory price was $1750 when there was a tax of 3% on the factory price. How much tax did he pay on the automobile?

17. John's uncle bought a car for $1884. This amount included the factory price, a 3% tax on it, and a freight charge of $30. Find the factory price.

18. John's grandfather died and left $586,000. The Federal *estate tax* on this amount was as follows: no tax on the first $100,000; 1% on the next $50,000; 2% on the next $50,000; 3% on the next $100,000; 4% on the next $200,000, and 5% on the remaining $86,000. How much did the Federal government get from this estate?

19. To settle the grandfather's estate it was necessary to sell $375,800 worth of stock on which there was a *stamp tax* of 2¢ per $100. What was the total stamp tax paid?

Taxes on luxuries made within this country and on inheritances, incomes, etc. are called **internal revenue taxes.**

The **income tax,** which is a large source of revenue, is not included here because income tax laws change so often.

EQUATIONS AND FORMULAS

A Number Game

1. A class was playing a number game. Jane said, "If you add 4 to the number I am thinking of, you get 9."

The teacher wrote this statement on the board thus:

Jane's number plus 4 *makes* 9.

Then she wrote the same statement in a kind of shorthand, letting n stand for the number Jane was thinking of, thus:

$$n + 4 = 9$$

The class saw that Jane's number was 5 because $5 + 4 = 9$.

2. Then it was Edna's turn. She said, "Take 10 from the number I'm thinking of and you get 19."

Letting n stand for Edna's number, the teacher wrote:

$$n - 10 = 19$$

The class saw that n must be 29 because $29 - 10 = 19$.

3. Next Jack said, "Six times my number is 48." This was also written on the board in the shorthand way, thus:

$$6\,n = 48$$

Here $6\,n$ means "6 times some number." Can you tell what Jack's number was? How do you know that you are right?

Using Letters for Unknown Numbers

The letter n or any other letter may be used to represent an unknown number. Often x stands for the unknown number.

In writing "6 times some number" by this short method, you omit the sign of multiplication and write $6\,n$. Likewise, $4\,x$ means 4 times x. In the same way, in your work with formulas, you wrote two letters side by side to indicate multiplication. In the formula for the area of a rectangle, $A = lw$, you know that lw means *length times width.*

UNKNOWN NUMBERS

In the game on page 438, other pupils made these statements. Write each in the short way and find the unknown number:

1. 7 times my number gives 21.
2. Add my number to 4 and you get 7.
3. Add 7 to my number and you get 20.
4. Multiply my number by 2 and you get 18.
5. If you take 4 from my number, you get 13.
6. If you find $\frac{1}{4}$ of my number, the answer is 6.
7. If you take my number from 5, you have 1 left.

Here are the short forms in which several statements were written on the board. Tell what each original statement was and what number each letter stands for:

8. $y + 2 = 7$	**12.** $8 + n = 15$	**16.** $\frac{1}{2} m = 4$
9. $d + 3 = 4$	**13.** $3 + x = 12$	**17.** $3 n = 21$
10. $9 - x = 7$	**14.** $n - 7 = 11$	**18.** $6 x = 30$
11. $11 - n = 5$	**15.** $z + 3 = 14$	**19.** $\frac{1}{2} x = 12$

20. Make up other statements like those above, writing them the short way, and let the class tell the unknown numbers.

The Equation

A statement like $x + 5 = 9$, in which one expression is shown to equal another, is called an **equation.**

You *solve* the equation $x + 5 = 9$ when you find that $x = 4$. Here 4 is the *value* of x because x stands for 4.

In these equations, find the value of x mentally:

1. $x + 1 = 7$	**4.** $15 = x + 8$	**7.** $2 x = 12$
2. $x + 7 = 9$	**5.** $26 = x + 6$	**8.** $12 = 3 x$
3. $2 - x = 1$	**6.** $13 = 15 - x$	**9.** $7 x = 42$

THE NATURE OF EQUATIONS

It is not always easy to solve an equation at a glance. In an equation like $3x + 51 = 348$, it would be difficult to find the value of x mentally. It is therefore necessary to study the nature of equations more closely and to learn some simple principles for use in solving them.

Every equation has two sides separated by the sign $=$. Thus, the equation $x + 5 = 9$ has a left side, $x + 5$, and a right side, 9, with the sign $=$ between them.

Left side		Right side
$x + 5$	$=$	9

The entire left side of an equation is exactly equal to the entire right side. Therefore an equation is like a perfectly balanced seesaw, in which the weight on one end is exactly equal to that on the other end. The sign $=$ is like the wooden horse that supports the board of the seesaw. In solving an equation, you find a value for x that will keep the balance perfect.

The sides of an equation are sometimes called its *members*. The left side is the *first member* and the right side is the *second member*. In the equation $x + 5 = 9$, the first member is $x + 5$ and the second member is 9.

Balancing Equations

If these equations were balanced on a seesaw, tell which part of each would be set on the left end of the board and which on the right end. Solve each equation mentally:

1. $x + 4 = 16$ **5.** $3n = 9$ **9.** $5x = 55$

2. $5 + x = 12$ **6.** $\frac{1}{4}x = 6$ **10.** $2x = 48$

3. $x - 7 = 13$ **7.** $4m = 8$ **11.** $x - 9 = 20$

4. $n - 2 = 17$ **8.** $\frac{1}{10}x = 8$ **12.** $3 + x = 25$

KEEPING THE BALANCE

Here John, who weighs 100 lb., and Sam, who weighs 40 lb., are on one end of the seesaw, while Mary, who weighs 100 lb., is on the other end with a 40-pound bag of sand. The seesaw balances because there is the same weight on each end of the board.

If Sam gets off, what weight is taken from his end of the seesaw? What must be taken from the other end to keep the balance? What weight is then left on each end?

This story may be briefly written in equation form, thus:

Start with $100 + 40 = 100 + 40$

Then subtract 40 from both sides, like this:

$$100 + 40 - 40 = 100 + 40 - 40$$

This leaves $100 = 100$

If you subtract the same number from both sides of an equation, the sides are still equal.

Solving an Equation by Subtraction

Now try the above plan on the following equation:

$$x + 347 = 925$$

Subtract 347 from both sides, as shown below:

$$x + 347 - 347 = 925 - 347$$

This gives $x = 578$

To make sure that 578 is the value of x, *check it* by putting 578 in place of x in the original equation, thus:

$$578 + 347 = 925$$

This gives $925 = 925$

Since both sides are still equal, 578 is the correct value of x.

SOLVING EQUATIONS BY SUBTRACTION

You have seen that if 347 is subtracted from both sides of the equation $x + 347 = 925$, the sides are still equal and the result is $x = 578$. Hence by subtracting the same number from both sides of the equation, you get x on one side and its value on the other. Thus the rule on page 441 may be used in solving certain equations.

To solve an equation by subtraction, subtract from both sides of the equation the number that will leave only x on one side and its value on the other.

Exercises

1. In the equation $x + 3 = 12$, what number do you subtract from both sides to get the value of x? What does x equal?

Find the value of x by subtraction. Check by putting the answer in place of x in the given equation:

2. $x + 7 = 13$
3. $x + 9 = 14$
4. $x + 6 = 48$
5. $x + 8 = 57$
6. $x + 5 = 20$
7. $x + 7 = 34$
8. $x + 256 = 524$
9. $x + 347 = 623$
10. $x + 562 = 730$
11. $x + 777 = 900$
12. $600 + x = 801$
13. $240 + x = 680$
14. $x + 1.15 = 2.11$
15. $x + 3.65 = 7.09$
16. $x + 2.25 = 4.15$
17. $x + 3.74 = 5.23$
18. $x + 41.1 = 89.2$
19. $2.41 + x = 4.45$

20. A certain number plus 9 equals 34. Find the number.
First write the statement as an equation, letting x stand for the number.

21. Will is thinking of a number. If he adds 16 to it, he gets 45. Write this as an equation and find the number.

22. Add 340 to the number which Anna has in mind and you obtain 657. Express this as an equation and solve it.

23. Ida lives on Oak Street. If you add 412 to the number of her house, you get 1000. Find the number of her house.

SOLVING EQUATIONS BY ADDITION

If Tom and Jane, each weighing 100 lb., are on opposite ends of the seesaw, the seesaw balances. But if Tom's dog, which weighs 20 lb., gets on with Tom, a 20-pound weight is needed on Jane's side to keep the balance.

You can write these facts in equation form, thus:

Start with $100 = 100$
Add 20 to both sides: $100 + 20 = 100 + 20$
This gives $120 = 120$

If the same number is added to both sides of an equation, the sides are still equal.

This principle is used in solving an equation like $x - 3 = 5$, in which $x - 3$ means that 3 has been taken away from x. If you add 3 to $x - 3$, you put back what was taken away and get x again. Hence $x - 3 + 3$ is the same as x.

Start with $x - 3 = 5$
Add 3 to both sides: $x - 3 + 3 = 5 + 3$
This gives $x = 8$
To check, put 8 in place of x in the original equation.
This gives $8 - 3 = 5$
Then $5 = 5$
Since the sides are still equal, the work is right.

Exercises

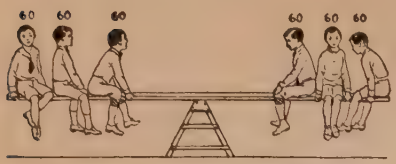
Solve the following equations. Then check your work:

1. $x - 5 = 7$ **2.** $x - 8 = 5$ **3.** $x - 7 = 15$ **4.** $x - 1 = 18$ **5.** $x - 2 = 43$

6. $p - 217 = 186$ **7.** $h - 356 = 245$ **8.** $x - 900 = 250$ **9.** $c - 255 = 846$ **10.** $n - 180 = 900$

11. $n - \frac{1}{2} = 4\frac{1}{2}$ **12.** $x - 2.5 = 2$ **13.** $c - 12\frac{1}{4} = 10\frac{1}{2}$ **14.** $x - .20 = 1.20$ **15.** $x - .30 = 3.00$

SOLVING EQUATIONS BY MULTIPLICATION



If Henry and James, each weighing 60 lb., are on opposite ends of a seesaw, the seesaw balances.

If two more boys, each weighing 60 lb., jump on Henry's side, and two other boys, each weighing 60 lb., get on with James, the seesaw still balances.

You can express the above in equation form, thus:

Start with	$60 = 60$
Multiply both sides by 3:	$3 \times 60 = 3 \times 60$
This gives	$180 = 180$

If both sides of an equation are multiplied by the same number, the sides are still equal.

This principle may be used to solve an equation like $\frac{1}{2}n = 5$, in which $\frac{1}{2}n$ means $\frac{n}{2}$. To change $\frac{1}{2}n$ to n, you multiply by 2.

Start with	$\frac{1}{2}n = 5$
Multiply both sides by 2:	$2 \times \frac{1}{2}n = 2 \times 5$
This gives	$n = 10$
Check.	$\frac{1}{2} \times 10 = 5$

Exercises

Solve the following equations. Check your results:

1. $\frac{1}{4}x = 9$

2. $\frac{1}{3}x = 8$

3. $\frac{1}{2}x = 6$

4. $\frac{b}{4} = 30$

5. $\frac{1}{4}x = 7$

6. $\frac{1}{5}x = 4$

7. $\frac{1}{7}x = 8$

8. $\frac{c}{5} = 85$

9. $\frac{1}{4}a = 3.25$

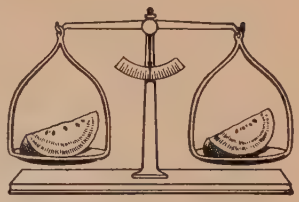
10. $\frac{1}{3}x = 0.45$

11. $\frac{1}{10}x = 10\frac{1}{2}$

12. $\frac{x}{2} = 75.5$

SOLVING EQUATIONS BY DIVISION

If you have two 18-pound watermelons and put one on each side of the scale, the scale balances.



If you cut each melon into six equal parts and leave one part on each side of the scale, the scale still balances. Tell why it balances.

You can express the above in equation form, as follows:

Start with	$18 = 18$
Divide both sides by 6:	$\frac{18}{6} = \frac{18}{6}$
This gives	$3 = 3$

If both sides of an equation are divided by the same number, the sides are still equal.

This principle is used to solve an equation like $4x = 28$, in which $4x$ means 4 times x. To change $4x$ to x, divide $4x$ by 4; also divide 28 by 4 to keep both sides equal.

Start with	$4x = 28$
Divide both sides by 4:	$\frac{4x}{4} = \frac{28}{4}$
This gives	$x = 7$
Check.	$4 \times 7 = 28$

Exercises

1. By what number do you divide $7x$ to change it to x?

Solve the following equations. Check your results:

2. $7x = 42$

3. $12b = 36$

4. $15n = 45$

5. $13n = 13$

6. $4x = 2$

7. $5x = 1$

8. $4r = 7$

9. $6x = 3$

10. $1\frac{1}{2}x = 4\frac{1}{2}$

11. $2.5x = 7.5$

12. $32s = 96.96$

13. $115x = 6.90$

USING THE RIGHT PRINCIPLE IN EQUATIONS

You have seen that the sides of an equation remain equal

(1) if you add the same number to both sides;
(2) if you subtract the same number from both sides;
(3) if you multiply both sides by the same number;
(4) if you divide both sides by the same number.

Of the above principles, use the one which leaves only x (or the letter that stands for the unknown number) on one side of the equation and only known numbers on the other side.

Exercises

1. In order to have only x left, what do you do to $x + 4$? to $x - 2$? to $\frac{1}{4}x$? to $2x$? to $6x$? to $6 + x$? to $\frac{1}{2}x$?

Solve, being careful to use the correct principle:

2. $\frac{1}{4}x = 170$
3. $5x = 405$
4. $4 + x = 10$
5. $m + 7 = 56$
6. $\frac{1}{5}t = 125$
7. $2x = 24$
8. $24 = 3x$
9. $72 = 9r$
10. $d - 8 = 6$
11. $n + \frac{1}{2} = \frac{3}{4}$
12. $x - 8 = 27$
13. $6 = m + 1$

Adding x's in Solving an Equation

1. Solve the equation $x + 3x = 28$.

You must first add the x's. Since x means $1x$, $x + 3x$ means $4x$. Hence $4x = 28$. Dividing both sides of the equation by 4, you get $x = 7$.

$$x + 3x = 28$$
$$4x = 28$$
$$x = 7$$

Check. $7 + 21 = 28$

2. How else can you write $p + 2p$? $x + x$? $2x + \frac{1}{2}x$?

Solve the following and check the work:

3. $6x - x = 5$
4. $2x + x = 15$
5. $p + 4p = 10$
6. $n + n = 30$
7. $x - \frac{1}{2}x = 10$
8. $4x - 2x = 20$
9. $b - \frac{3}{4}b = 8$
10. $\frac{1}{3}q + \frac{2}{3}q = 24$
11. $\frac{1}{2}x + \frac{1}{2}x = 30$

USING MORE THAN ONE PRINCIPLE

In solving an equation, you must sometimes use more than one of the principles you have learned, as shown here:

Given:	$5x - 7 = 38$	Given:	$\frac{3x}{2} = 15$
Add 7:	$5x = 45$	Multiply by 2:	$3x = 30$
Divide by 5:	$x = 9$	Divide by 3:	$x = 10$
Check.	$45 - 7 = 38$	*Check.*	$\frac{30}{2} = 15$

When more than one principle is used, first do the addition or subtraction, then the multiplication, if any, and then the division.

Exercises

1. In order to have only x left, what two things do you do to $2x + 1$? to $3x - 4$? to $\frac{x}{2} + 6$? to $\frac{3x}{2}$? to $4 + 3x$?

2. Solve the equation $\frac{3}{4}x = 16$. $\frac{3}{4}x$ means $\frac{3x}{4}$.

Solve the following and check your results:

3. $\frac{2x}{3} = 18$

4. $4x + 9 = 29$

5. $8x - 30 = 2$

6. $24 + 5x = 44$

7. $6x + 7x = 65$

8. $\frac{3x}{4} = 90$

9. $\frac{2}{3}x = 12$

10. $\frac{3}{4}x = 27$

11. $\frac{15}{100}x = 75$

12. $\frac{1}{5}x + 3 = 5$

13. $\frac{80x}{100} = \$16.80$

14. $2 + \frac{1}{4}x = 12$

15. $x + \frac{1}{2}x = 21$

16. $\frac{124}{100}x + 8 = 256$

17. $100x + 20x = 60$

18. Jack was asked how old he was. He said, "Multiply my age by 7, add 9, and you get 100." How old was he?

19. If 5 is subtracted from 5 times a number, the result is 5. Find the number. Check your work.

20. If you add 4 to $\frac{1}{2}$ of Elizabeth's age, the result is 10. How old is Elizabeth? Check the result.

HOW TO SOLVE PROBLEMS BY EQUATIONS

Sam needs 15 more stamps to give him a collection of 100 stamps. How many stamps has he now?

This problem can be solved by an equation, as follows:

Let x = the number of stamps Sam has now

If he gets 15 more stamps, he will have 100. Write the equation thus:

$$x + 15 = 100$$

Subtract 15: $x = 85$

Check. $85 + 15 = 100$

Since $x = 85$, you know that Sam now has 85 stamps.

In working a problem by equations, let x (or any letter) stand for the number to be found, write what the problem tells about x in the form of an equation, and solve for x.

Problems

1. If Lucy had x books and gave 17 of them to a hospital, express the number of books she had left in terms of x and 17.

2. John sold his radio set at a profit of \$20. Letting x = the cost in dollars, express the selling price in terms of 20 and x.

3. If John (ex. 2) sold his radio set for \$65, find out, by making an equation and solving it, how much it cost him.

4. Ed sold a dog at a loss of \$3. If x = the cost in dollars, what does $x - 3$ represent? If the selling price was \$20, what number does $x - 3$ equal? Solve the equation thus formed to find the cost of the dog.

Use equations to solve the following problems. Check the work:

5. A man sold a lot for \$1200 and lost \$300. Find the cost.

6. George has \$17.50 more than Sarah. If George has \$26, how much has Sarah?

7. If John earns \$6 a week, how much can he earn in x weeks? How many weeks must he work to earn \$84?

HOW MUCH IS EACH BOY'S SHARE?

Jack and his brother Bert take care of Mr. Howe's tennis court. Mr. Howe pays them \$15, of which Jack is to get twice as much as Bert. How much does each boy get?

Here you have to find two quantities, Bert's share and Jack's share. Jack's share is twice as great as Bert's.

Let	x = Bert's share, in dollars
Then	$2x$ = Jack's share, in dollars
Both together get \$15:	$x + 2x = 15$
Add the x's:	$3x = 15$
Divide by 3:	$x = 5$
Check.	$5 + 10 = 15$

Since $x = 5$, $2x = 10$. Hence Bert gets \$5 and Jack gets \$10.

When two numbers are to be found in a problem, let x represent one of them and express the other in terms of x.

Problems

1. Jane bought a tennis racket for x dollars and a sweater for twice as much. How can you express the cost of the sweater? the cost of both?

2. If the sweater and tennis racket together (ex. 1) cost \$12, form an equation and find the cost of each.

3. A book and pencil together cost \$1.40. If the book cost 13 times as much as the pencil, find the cost of each.

4. A house and lot together cost \$48,000. The house cost \$18,000 more than the lot. Find the cost of each.

5. To make cocoa, Ada uses twice as much milk as water. How many cups of each does she use for 9 cups of cocoa?

6. Alice divides 5 yd. of silk into two pieces, one of which is 4 times as long as the other. How long is each piece?

7. Together Mary and Jane have \$172. Mary has \$4 more than 3 times as much as Jane. How much has each?

HOW OLD IS JOHN?

John said to Ruth, "Add my age now to my age 4 yr. from now and you get 30." How can Ruth figure out John's age?

Let	x = John's present age, in years
Then	$x + 4$ = his age 4 yr. from now
The equation is	$x + x + 4 = 30$
Add the x's:	$2x + 4 = 30$
Subtract 4:	$2x = 26$
Divide by 2:	$x = 13$
Check.	$13 + 13 + 4 = 30$

Since $x = 13$, John's present age is 13 yr.

Age Puzzles

1. Martha is now 12 yr. old. How old will she be 5 yr. from now? x years from now? How old was she x years ago?

2. Julia is now z years old. Express her age 2 yr. ago.

3. Betty is 3 times as old as Frank, who is x years old. Express Betty's age. Express the sum of their ages. If the sum of their ages is 16, how old is Frank? How old is Betty?

4. Tom said to Ann, "If I were 7 yr. younger than my present age, I should be 8 yr. old." How old is Tom?

5. Henry's age is 4 times Susan's. The sum of their ages is 30 yr. How old is each?

6. Edna says that her age is $\frac{1}{4}$ of her mother's, and that their ages together total 40 yr. Find how old each is.

7. Harry's age is 3 yr. less than 3 times Joe's age. Their ages total 29 yr. Find how old they are.

8. Hugh is now twice as old as Lena. Two years ago the sum of their ages was 17 yr. How old is each of them now?

9. Mark and Molly are twins. If Mark were 4 times as old as he is and Molly were 4 yr. older than she is, the sum of their ages would be 59. Find how old they are.

IMPROVEMENT TEST No. 30

Test 30 A — Addition (Time 3 min.)

1.	2.	3.	4.
34975	53926	80640	53598
69855	44285	69141	68364
7480	61638	1797	71959
90096	3869	51784	37876
77823	7805	78194	5896
98767	67535	86921	9570
3849	21363	99225	52721
47692	22357	3748	58168

Test 30 B — Addition (Time 3 min.)

1.	2.	3.	4.
244	601	736	852
132	627	405	144
913	851	833	489
238	103	946	372
155	346	697	410
614	480	241	362
326	602	268	274
505	573	502	100
430	565	980	821
174	974	899	523
273	541	125	165
921	339	278	988
242	180	678	953

Test 30 C — Subtraction (Time 3 min.)

1. 95742061	2. 81695640	3. 54451431
28228346	15145860	25064163
4. 74337627	5. 33173028	6. 60068426
36432775	18875585	11372750
7. 50621488	8. 68127215	9. 59705933
15227939	45145569	17814375

THE EQUATION IN PERCENTAGE

Mary bought a tennis racket for \$3.76, which was 6% less than the regular price. What was the regular price?

The easiest way to solve this problem is by equations:

Let p = regular price, in dollars

Then $6\%\,p$, or $\frac{6}{100}\,p$ = discount, in dollars

Since Regular Price − Discount = Net Price, you have this equation:

$$p - \tfrac{6}{100}\,p = 3.76$$

Subtract the p's: $\quad \tfrac{94}{100}\,p = 3.76$

Multiply by 100: $\quad 94\,p = 376$

Divide by 94: $\quad p = 4$

The regular price of the racket is \$4. Check the work.

Exercises

Solve these problems by equations and check the work.

1. At a 15% discount sale Helen bought goods for a dress for \$9.20. What was the regular price of the material?

Let x = the regular price, in dollars.

2. Miss Vail receives 6% discount on a bill. If the net amount of the bill is \$42.30, find the original amount of the bill.

3. John made a radio set and sold it for \$75. This was 25% more than it cost him. How much did it cost him?

Let x = cost, in dollars. Then $\frac{25}{100}\,x$ = profit. Hence $x + \frac{25}{100}\,x = 75$.

4. Mary sold some candy for 60¢ a pound, thus gaining 20% of the cost. How much per pound did the candy cost Mary?

5. Tom sold books at a commission of 40%. If he received \$14 as his commission, find the amount of his sales.

Let x = number of dollars' worth sold. Then $40\%\,x$ = commission.

6. How much money must Mr. Brown invest at 6% interest in order to have a yearly income of \$1200 from it?

7. When I buy furniture for \$320, at what price do I sell it if my expenses and profit together are 36% of the selling price?

COMPARING THE EQUATION WITH OUR FORMER METHOD

A real estate dealer sold a house for \$18,400, which was 15% more than he paid for it. How much did it cost him?

	Old Method — Analysis	*New Method — Equation*
1.	100% of cost = whole cost 15% of cost = increase	Let c = cost Then $\frac{15}{100}c$ = increase
2.	100% of cost +15% of cost = \$18,400	$c + \frac{15}{100}c = 18{,}400$
3.	115% of cost = \$18,400	$\frac{115}{100}c = 18{,}400$
4.	1% of cost = $\frac{\$18{,}400}{115}$	$115c = 1{,}840{,}000$
5.	100% of cost = $100 \times \frac{\$18400}{115}$	$c = 16{,}000$
6.	Hence cost = \$16,000	Hence cost = \$16,000

Studying the above solutions, you notice these points:

1. In Steps 1 and 2 the methods are much alike. But the equation, which expresses the facts in shorthand, is quicker.

2. In Steps 3 to 6 of the old method, you must think out every step in terms of the original problem. But in using equations, you think of the problem only until the equation is formed (Step 2) and then mechanically solve for the unknown number.

3. In using equations in percentage, it is best to write each per cent as a fraction with 100 as the denominator.

Exercises

Solve these problems by both the old and the new method:

1. Mr. Low's expenses and profit are 35% of his sales. At what price should he mark a coat that cost him \$13?

2. A dealer is allowing his customers a discount of 15% on the marked price of fur coats. At what price shall he mark a coat for which he wants to get \$340?

FORMULAS AS EQUATIONS

The house that Ida lives in stands on a plot of ground 100 ft. long and 51 ft. wide. Find the area of the plot.

In this problem use the formula for finding the area (A) of a rectangle when the length (l) and the width (w) are known.

The formula is	$A = lw$
Substitute:	$A = 100 \times 51$
This gives	$A = 5100$

Hence the area of the plot of ground is 5100 sq. ft.

The expression $A = lw$, which you have known only as a formula, is also an equation. In the above example, in the equation $A = lw$, the letter A represents the *unknown number* while l and w stand for *known numbers*. To solve for A you substitute the given values for l and w.

In the equation $A = lw$, A is expressed in terms of l and w.

Exercises

In the equation $A = lw$, find the value of A:

1. If $l = 12$ and $w = 8$

2. If $l = 10$ and $w = 7$

3. If $l = 22$ and $w = 15$

4. If $l = 19$ and $w = 14$

5. If $l = 11$ mi. and $w = 2$ mi.

6. If $l = 90$ yd. and $w = 45$ yd.

7. If $l = 97$ ft. and $w = 43$ ft.

8. If $l = 100$ ft. and $w = 7$ ft.

9. Find the area (A) of a triangle whose base (b) is 8 ft. and whose height (h) is 4 ft., using the equation $A = \frac{bh}{2}$.

Using the equation $A = \frac{1}{2}bh$, find the value of A:

10. If $b = 9$ and $h = 4$

11. If $b = 7$ and $h = 5$

12. If $b = 10$ and $h = 6$

13. If $b = 12$ and $h = 5$

14. If $b = 11$ in. and $h = 5$ in.

15. If $b = 10$ ft. and $h = 5\frac{1}{2}$ ft.

16. If $b = 12\frac{1}{2}$ ft. and $h = 8$ ft.

17. If $b = 10\frac{1}{2}$ ft. and $h = 5$ ft.

HOW WIDE SHOULD EACH PLOT BE?

The Garden Club of the Palmer School was having a vegetable-growing contest. Each child was to cultivate a plot of 120 sq. ft. during the summer.

1. Harriet had a plot 20 ft. long. She found how wide it should be by dividing 120 by 20. This gave her 6 ft. To check this result, she multiplied 20 by 6, which gave 120.

2. Ann's plot was 15 ft. long. Dividing 120 by 15, she got 8 as the width of her plot. How can you check this result?

When the area and the length of a rectangle are known, the width can be found by dividing the area by the length.

You can express this as an equation, as follows:

$$w = \frac{A}{l}$$

Now you have a formula for w in terms of A and l.

A Shorter Way of Finding $w = \frac{A}{l}$

By means of equations, you can quickly find the formula $w = \frac{A}{l}$. In the equation $A = lw$, regard w as the unknown number and A and l as known numbers, and solve for w:

Start with $A = lw$

To solve for w, you must get w by itself on one side of the equation.

Divide both sides by l: $\frac{A}{l} = w$

Turn the equation around: $w = \frac{A}{l}$

To make a new formula from an old one, start with the old formula, and solve it, as an equation, for the letter whose value you wish to find, regarding the other letters as known numbers.

USING THE NEW FORMULA

1. Using the formula $w = \frac{A}{l}$, find how wide a plot 24 ft. long must be to contain 120 sq. ft. Check the work by multiplying the result by 24 to see if it gives 120.

2. Ruth's plot is 30 ft. long. How wide should it be to have an area of 120 sq. ft.? Check the work.

3. A class decided to cultivate plots of 144 sq. ft. How wide should a plot be whose length is 12 ft.? 18 ft.? 16 ft.? 24 ft.? 27 ft.? 20 ft.? 36 ft.? Check each result.

4. Starting with $A = lw$, make a formula for finding the length of a rectangle when you know the area and the width.

5. Use the formula in ex. 4 to find how long a room must be to have a floor space of 384 sq. ft. if it is 16 ft. wide.

Find the lengths of these rectangular rooms. Check the work:

6. $A = 210$ sq. ft., $w = 14$ ft.

7. $A = 425$ sq. ft., $w = 17$ ft.

8. $A = 126$ sq. ft., $w = 9$ ft.

9. $A = 108$ sq. ft., $w = 9$ ft.

Use the formula $w = \frac{A}{l}$ *or* $l = \frac{A}{w}$ *in the following, and check:*

10. $A = 180, l = 15$. Find w.

11. $A = 608, l = 38$. Find w.

12. $A = 416, l = 26$. Find w.

13. $A = 504, w = 14$. Find l.

14. $A = 960, w = 30$. Find l.

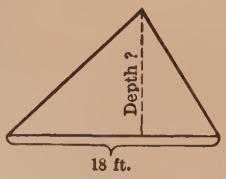
15. $A = 918, w = 27$. Find l.

16. A school 180 ft. long is to have along its entire length a playground that can accommodate 450 pupils at a time. How wide must it be to allow 30 sq. ft. per pupil?

17. In a certain class a rectangular school garden 12 ft. long and 9 ft. wide is assigned to each pupil. Jack enters the class late and has to use for his garden a strip 6 ft. wide along the fence. What length should his strip be so that his garden will have the same area as the others?

FRANK MAKES A TRIANGULAR GARDEN

Frank is making a triangular garden in back of his home. He wants the base to be 18 ft. and the area to be 108 sq. ft. What depth (or height) shall he make the garden?



Here the area (A) and the base (b) are known, and a formula for the height (h) of a triangle must be found in terms of A and b.

The formula for the area is: $A = \frac{bh}{2}$

Multiply by 2: $2A = bh$

Divide by b: $\frac{2A}{b} = h$

Turn the formula around: $h = \frac{2A}{b}$

Substitute values of A and b: $h = \frac{2 \times 108}{18}$, or 12

Then the height, or depth, of the garden should be 12 ft.

Check the work by showing that $\frac{1}{2} \times 18 \times 12 = 108$.

Exercises

1. Doris is making a triangular garden 54 sq. ft. in area. Its base is 12 ft. long. How deep should it be? Check the work.

Find the depths of these triangular plots, and check:

2. $b = 4$ ft., $A = 10$ sq. ft. **3.** $b = 15$ ft., $A = 45$ sq. ft.

4. Make a formula for finding the base of a triangle when the area and the height are known. Then find the base of a triangle whose area is 195 sq. ft. and whose height is 15 ft.

Work and check the following, using the proper formulas:

5. $b = 18$, $h = 24$. Find A. **7.** $A = 340$, $b = 8$. Find h.

6. $b = 12$, $h = 16$. Find A. **8.** $A = 380$, $h = 19$. Find b.

DETERMINING THE DIMENSIONS OF A BIN

Mr. Day is building a bin to hold his coal for the winter. It is to be 8 ft. long and 7 ft. wide and high enough to contain 280 cu. ft. How high should the bin be?

You must find a formula for the height (h) of a rectangular solid when the volume (V), the length (l), and the width (w) are known.

Start with the formula for the volume of a solid, thus:

$$V = lwh$$

Divide by lw: $$\frac{V}{lw} = h$$

Turn formula around: $$h = \frac{V}{lw}$$

Substitute: $$h = \frac{280}{8 \times 7}, \text{ or } 5$$

Hence the bin should be 5 ft. high. *Check.* $8 \times 7 \times 5 = 280$.

Exercises

1. How high should a coal bin be to hold 5 T. of coal if it is 7 ft. long and 4 ft. wide and 1 T. occupies 35 cu. ft.?

2. From the formula for the volume of a rectangular solid, make new formulas for the length and the width.

Select the proper formula; then work and check the following:

3. Find V when $l = 6$, $w = 6$, and $h = 6$.

4. Find l when $V = 114$, $w = 2$, and $h = 3$.

5. Find w when $V = 920$, $l = 23$, and $h = 8$.

6. Mr. Fox is making a tank 4 ft. long and 2 ft. high for 120 gal. of oil. If $7\frac{1}{2}$ gal. = 1 cu. ft., how wide should the tank be?

Tell what should be put in place of the dots:

7. The height of a bin equals the . . . divided by the product of the . . . and the . . . .

8. To solve for h in $V = lwh$, . . . both sides by . . . .

FINDING THE DIAMETER OF A TREE

1. When Helen was in California, she saw a very large tree. She could not cut through the tree to find its diameter; but she measured the circumference (C) with a string and found that it was 198 in. Then she derived a formula for the diameter (D) from the formula $C = \pi D$. What formula did Helen make? If $\pi = 3\frac{1}{7}$, what was the diameter of the tree? How can you check the work?

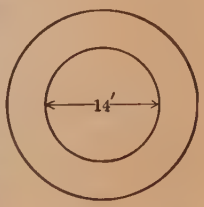
2. Helen measured another tree whose circumference was 88 in. Using Helen's formula for D, find the diameter.

3. Find the diameter of a tree whose circumference is 77 in.; 132 in.; 154 in. Check each result.

4. State in words the rule for finding the diameter of a tree when the circumference is known.

5. Find the diameter of a column measuring 11 ft. around.

6. The circular stone tower of an old castle has an outside circumference of 72 ft. Find the diameter of the tower; the radius.



7. A tunnel leads to a circular room in the tower (ex. 6). This room is 14 ft. in diameter. Find its radius. If the difference between the radius of the tower and the radius of the room shows the thickness of the wall of the tower, how thick is the wall?

8. The formula for the volume of a cylinder in terms of the radius and height is $V = \pi r^2 h$. Make a formula for h.

9. The radius of a cylindrical gas tank is 35 ft. Find the height of the tank when it contains 115,500 cu. ft. of gas; 142,450 cu. ft.; 238,700 cu. ft.; 82,775 cu. ft. Check the work.

REVIEW

If you do not remember the formulas called for in the problems below, make them from formulas which you do know.

Find the widths of these rectangles and check the results:

1. $l = 16$ ft., $A = 128$ sq. ft.
2. $l = 21$ in., $A = 147$ sq. in.
3. $l = 64$ ft., $A = 960$ sq. ft.
4. $l = 20$ ft., $A = 130$ sq. ft.

Find the bases of these triangles. Check the results:

5. $h = 24$ in., $A = 90$ sq. in.
6. $h = 45$ ft., $A = 75$ sq. ft.
7. $h = 13$ ft., $A = 32\frac{1}{2}$ sq. ft.
8. $h = 11$ in., $A = 286$ sq. in.

9. Make a formula for the radius (r) of a circle from the formula $C = 2\pi r$. Then find the radii of circles with these circumferences, and check: 40 in.; 11 in.; 264 in.; 198 in.

Find the heights of rectangular rooms having the following volumes, lengths, and widths. Check the results:

10. 504 cu. ft., 9 ft., 8 ft.
11. 378 cu. ft., 9 ft., 7 ft.
12. 960 cu. ft., 12 ft., 10 ft.
13. 935 cu. ft., 10 ft., $8\frac{1}{2}$ ft.

14. A cylindrical tank 14 ft. in radius contains 6468 cu. ft. of water. Find the depth of the water.

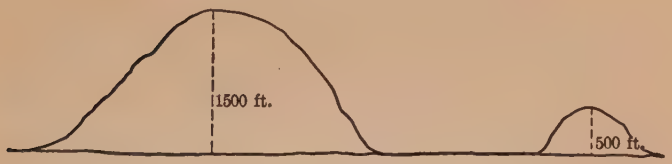
15. Which is higher, a cylindrical tank 7 ft. in radius containing 3080 cu. ft. or a rectangular bin of the same volume, 14 ft. long and 10 ft. wide? how much higher?

16. Show that 6% of \$150 = \$9. In a problem like this, 6% is the *rate*, \$150 is the *base*, and \$9 is the *percentage*. Hence *Rate* × *Base* = *Percentage*, or, more briefly, $RB = P$. From this formula derive one for R; for B. Then use these formulas below.

Tell what numbers should be put in place of the dots:

17. 315 is 7% of . . .
18. 13 is 4% of . . .
19. . . .% of \$2250 is \$612.
20. . . .% of \$1450 is \$174.

RATIO AND PROPORTION



Comparing Numbers

John was telling Alice about his trip to Blue Mountain. "The mountain," he said, "is about 1000 ft. higher than Bald Hill. I think it is 3 times as high." He was comparing the heights of the mountain and the hill in these two ways:

(1) He compared their heights by *subtraction* by showing the *difference* in the heights. In the picture above, the mountain is 1500 ft. − 500 ft., or 1000 ft., higher than the hill.

(2) He compared them by *division* by showing *how many times* one height is contained in the other. Thus, the mountain is $\frac{1500}{500}$ times, or 3 times, as high as the hill.

Similarly, if Ruth has \$5 and Sarah has \$2, you may say that Ruth has \$3 *more than* Sarah or that Ruth has $\frac{5}{2}$ times, or $2\frac{1}{2}$ times, *as much as* Sarah.

Exercises

1. 30 is how much greater than 10? 30 is how many times as large as 10? 10 is how much less than 30?

2. Compare 6 and 2 by subtraction and by division.

Compare the first number with the second in two ways:

3. 8, 2	**5.** 6, 1	**7.** 12, 4	**9.** 40, 5	**11.** 28, 14
4. 5, 2	**6.** 9, 6	**8.** 10, 3	**10.** 21, 6	**12.** 15, 15

13. Compare the numbers in ex. 3 to 12 by telling what part the second number is of the first number.

COMPARING NUMBERS BY DIVISION

Will's fish pole is 12 ft. long, while Ted's is 6 ft. long. You can compare the length of Will's pole with the length of Ted's by dividing 12 by 6, to find *how many times* 12 contains 6. Will's pole is $\frac{12}{6}$, or 2, times as long as Ted's.

You can also compare the length of Ted's pole with Will's by dividing 6 by 12 to find *what part* 6 is of 12. Ted's pole is $\frac{6}{12}$, or $\frac{1}{2}$, as long as Will's.

These comparisons are both made by division. The result is called the **ratio** of the one number to the other. Thus, 2 is the *ratio* of the length of Will's pole to Ted's, while $\frac{1}{2}$ is the *ratio* of Ted's pole to Will's.

When two numbers are compared by division, the result is called their ratio.

Ratio is just a technical name for the quotient that results when numbers are compared by division. A ratio may be a fraction, like $\frac{7}{5}$ or $\frac{5}{7}$, or a whole number, like 2, or a mixed number, like $2\frac{1}{2}$.

Exercises

1. What part is 5 of 9? What is the ratio of 5 to 9?

2. How many times 3 is 9? What is the ratio of 9 to 3?

3. Ida has \$11 and Paul has \$6. What is the ratio of Ida's savings to Paul's? of Paul's savings to Ida's?

4. Alma weighs 96 lb. and Edna, 84 lb. What is the ratio of Alma's weight to Edna's? of Edna's weight to Alma's?

Reduce to lowest terms the fraction that expresses the ratio.

5. Margaret says that she weighs twice as much as little Susan. Say this in *ratio* language.

Find the ratio of the first number to the second:

6. 3, 5	**8.** 9, 9	**10.** 10, 3	**12.** 19, 20
7. 7, 9	**9.** 25, 5	**11.** 144, 8	**13.** 25, 10

HOW TO WRITE RATIOS

In comparing two quantities, such as 7 lb. and 5 lb., you may say that they have the ratio of 7 lb. to 5 lb., or of 7 to 5. This ratio may be written as $\frac{7}{5}$. It may also be written as $7 \div 5$ or as $7:5$. Hence $\frac{7}{5}$, $7 \div 5$, and $7:5$ all show the same ratio. 7 and 5 are the **terms** of the ratio.

The ratio of 2 to 6 may be written as $\frac{2}{6}$. But $\frac{2}{6} = \frac{1}{3}$; hence the ratio of 2 to 6 is the same as the ratio of 1 to 3. This means that 2 compares with 6 in the same way that 1 compares with 3.

Similarly, by multiplying both terms of $\frac{2}{6}$ by 4, you find that the ratio $\frac{2}{6} = \frac{8}{24}$. This means that 2 compares with 6 in the same way that 8 compares with 24.

You can multiply or divide both terms of a ratio by the same number without changing its value.

Exercises

1. Write in three ways the ratio of 3 to 11; of 18 to 13.

Write each of the following ratios in two other ways:

2. $5 \div 2$ **3.** $7 \div 8$ **4.** $6 \div 7$ **5.** $9 \div 9$ **6.** $1:8$ **7.** $7:1$ **8.** $\frac{5}{12}$ **9.** $\frac{100}{19}$

Write each ratio as a fraction in its simplest form:

10. $6 \div 12$ **11.** $22 \div 4$ **12.** $36 \div 8$ **13.** 4 in. $\div$ 8 in. **14.** 6 ft. $\div$ 9 ft. **15.** 9 lb. $\div$ 6 lb. **16.** $\frac{27}{45}$ **17.** $\frac{14}{21}$ **18.** $\frac{80}{30}$ **19.** 18 yd.: 12 yd. **20.** 36 in.: 84 in. **21.** 48 oz.: 16 oz.

22. Which of these ratios equal $\frac{3}{4}$: $\frac{9}{12}$, $\frac{12}{20}$, $\frac{18}{24}$, $\frac{24}{32}$, $\frac{6}{9}$?

23. Write as fractions three ratios each equal to $\frac{14}{16}$.

24. Last week Ann earned \$5 and spent \$4. Dora earned \$15 and spent \$12. Write the ratio of each girl's expenses to her earnings. How do these ratios compare?

FINDING RATIOS

Rose wanted to know the ratio of the length of her desk to its width. The desk was 2 ft. long and 18 in. wide. Since she could not compare feet with inches, Rose first changed 2 ft. to 24 in. Then the ratio of the length to the width was $\frac{24}{18}$, or $\frac{4}{3}$.

If the ratio of two quantities is to be found, the quantities must first be expressed in the same unit of measure.

Ratios in Your Classroom

1. Measure your desk and find the ratio of its width to its length. What is the ratio of its length to its width?

2. Have a member of the class measure the teacher's desk. Then find the ratio of its length to its width.

3. Measure the blackboard in your room and compare its length with its width by finding their ratio.

4. Using the dimensions found in ex. 1 and 2, find the ratio between the area of the teacher's desk and of yours.

5. Measure the floor of your classroom. What is the ratio of its length to its width? Tell why you cannot compare its area with its length.

6. How many girls are there in your class? how many boys? Find the ratio of the number of boys to the number of girls; of the number of girls to the total number of pupils.

Express these ratios as fractions in their simplest forms:

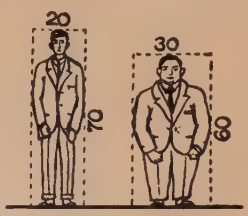
7. 2 in. : 2 ft.
8. 7 yd. : 2 ft.
9. 3 qt. : 2 bu.
10. 8 oz. : 5 lb.
11. 9 in. : $1\frac{1}{2}$ yd.
12. 5 lb. : 10 oz.
13. 3 gal. : 3 qt.
14. 15 qt. : 1 pt.
15. 14 ft. : 6 in.
16. $1\frac{1}{2}$ hr. : 1 da.
17. 15 ft. : 9 in.
18. 1320 ft. : 2 mi.

19. What is the ratio of 1 in. to 1 ft.? of 1 in. to 1 yd.?

20. What is the ratio of a nickel to \$1? of a dime to \$1?

PROPORTION

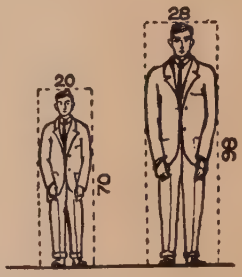
Joe is standing beside the fat man at the circus. Joe's shape is that of a normal young man, his height being 70 in. and his breadth, 20 in. The ratio of Joe's height to his breadth is $\frac{70}{20}$, or $3\frac{1}{2}$. If the fat man's height, 60 in., is compared with his breadth, 30 in., the ratio is $\frac{60}{30}$, or 2. The fat man is *out of proportion* because the ratio of his height to his breadth is only 2, while the ratio of a normal man is $3\frac{1}{2}$.



Here Joe is beside the circus giant, who is 98 in. tall and 28 in. broad. The ratio of the giant's height to his breadth is $\frac{98}{28}$, or $3\frac{1}{2}$, which is the same as Joe's. The giant is *well proportioned,* or *in proportion,* because his ratio is also normal.

The equality of Joe's ratio and the giant's is shown thus:

$$\frac{70}{20} = \frac{98}{28}$$

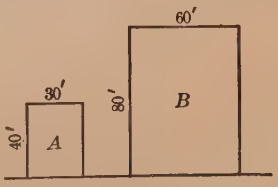


An expression like the above, which shows that two ratios are equal, is called a **proportion.**

Exercises

1. What is the ratio of the width of A to its height? of the width of B to its height? Are the ratios equal? If so, write the proportion showing it.

Do not reduce the ratios to lowest terms.



2. One picture is 2 ft. wide and 3 ft. long. Another picture is 4 ft. wide and 6 ft. long. Write the proportion showing that the ratios of their widths to their lengths are equal.

ENLARGING PICTURES

Paul's camera makes pictures 5 in. long and 4 in. wide. He wishes to enlarge one of his pictures so that it will be 10 in. wide. How long will the enlarged picture be?

The ratio of the length of the large picture to its width equals the ratio of the length of the small picture to its width. This can be expressed as a proportion.

Let x = length of large picture, in inches

Form the proportion: $\frac{x}{10} = \frac{5}{4}$

Multiply by 10: $x = \frac{5 \times 10}{4}$, or $12\frac{1}{2}$

Hence the enlarged picture will be $12\frac{1}{2}$ in. long. Check the result.

When one term of a proportion is unknown, you can easily find it by using the principles of equations.

Exercises

Find the unknown number in these proportions, and check:

1. $\frac{x}{16} = \frac{5}{8}$ **3.** $\frac{12}{24} = \frac{x}{6}$ **5.** $\frac{x}{6} = \frac{3}{9}$ **7.** $\frac{x}{14} = \frac{9}{21}$

2. $\frac{x}{15} = \frac{7}{5}$ **4.** $\frac{12}{18} = \frac{x}{3}$ **6.** $\frac{x}{3} = \frac{6}{8}$ **8.** $\frac{x}{72} = \frac{16}{9}$

9. A map is 8 in. long and 6 in. wide. How long will an enlarged map of the same proportions be if it is 45 in. wide?

10. A picture is now 1 ft. high and 2 ft. wide. If it is enlarged so as to be 9 ft. wide, how high will it be?

11. Make up three problems about enlarging photographs.

IMPROVEMENT TEST No. 31

Test 31 A — Addition (Time 3 min.)

1.	2.	3.	4.
89.406	8.1244	5318.9	338.94
23.721	.9755	8599.4	744.22
4.585	2.3727	140.3	793.58
3.746	4.6202	1596.4	37.63
91.087	.7395	6885.6	681.35
40.536	3.8656	432.3	969.69
2.728	.1908	9817.1	57.76
61.973	8.5629	3142.6	324.38

Test 31 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
40.6051	73871.4	5266.57	9.
12.3405	8426.6	3527.82	5.12346
5.	**6.**	**7.**	**8.**
562.371	89551.8	84.3786	5.
206.649	22856.5	6.4967	2.27069
9.	**10.**	**11.**	**12.**
6248.93	464.173	95719.1	997.142
1286.07	44.339	37544.2	227.824

Test 31 C — Reduction (Time 4 min. after copying)

Change to decimals. If there is a remainder, find the result to the nearest thousandth:

1. $\frac{5}{8}$	**2.** $\frac{3}{20}$	**3.** $\frac{29}{40}$	**4.** $\frac{37}{45}$
5. $\frac{13}{14}$	**6.** $\frac{17}{25}$	**7.** $\frac{15}{16}$	**8.** $\frac{61}{125}$

Test 31 D — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.	5.
9.46	972	.368	.517	.083
16.8	9.18	.023	.417	590

ANOTHER WAY TO WRITE PROPORTIONS

You know that the ratio of 7 to 5 may be written as $\frac{7}{5}$ or as 7 : 5. A proportion may also be written in several ways.

$\frac{7}{5} = \frac{14}{10}$ may be written as $7 : 5 = 14 : 10$.

This proportion is read "7 is to 5 as 14 is to 10."

When your grandparents went to school, proportions were written in the horizontal form $7 : 5 :: 14 : 10$, which is another way of writing $7 : 5 = 14 : 10$. The sign : : means *equals*. These horizontal forms are still found in many science books.

Books often print proportions in the horizontal form because it is easier to set type in that form than in the fractional form.

When a proportion containing an unknown term is given in the form $x : 36 = 2 : 9$, you may rewrite it in the fractional form and then solve for x, as shown below.

Given the proportion $x : 36 = 2 : 9$

Write in fractional form: $\frac{x}{36} = \frac{2}{9}$

Multiply by 36: $x = \frac{36 \times 2}{9}$, or 8

Check. $\frac{8}{36} = \frac{2}{9}$

Exercises

Write in fractional form and solve for x. Check the work:

1. $x : 6 = 2 : 4$ **4.** $8 : 12 = x : 54$ **7.** $x : 12 = 16 : 24$

2. $x : 9 = 2 : 6$ **5.** $10 : 7 = x : 28$ **8.** $x : 48 = 56 : 64$

3. $x : 8 = 1 : 4$ **6.** $10 : 15 = x : 27$ **9.** $x : 30 = 19 : 20$

10. Ada earns \$12 a week. For each \$4 that Ada earns, Rob earns \$5; hence Ada's earnings are to Rob's as 4 is to 5. Express this as a proportion and find how much Rob earns.

11. A-team played 27 games and won 2 out of every 3. Express this as a proportion and find how many games it won.

SIMILAR FIGURES

Similar figures are figures with the same shape. The first two triangles below are similar; the last two are not similar.

The bases AB and DE of the two similar triangles are called **corresponding sides** because they have like positions in their triangles. BC and EF are also corresponding sides. What other pair of corresponding sides have these triangles?

In the similar triangles, side AB is $4''$ and side DE is $8''$. The ratio of these two corresponding sides is $\frac{4}{8}$, or $\frac{1}{2}$.

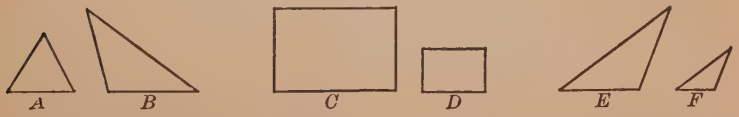
Side $BC = 3''$ and side $EF = 6''$. What is the ratio of BC to EF? What is the ratio of side AC to side DF? How do the ratios of the three pairs of corresponding sides compare?

In similar triangles, the ratio of any pair of corresponding sides equals the ratio of any other pair.

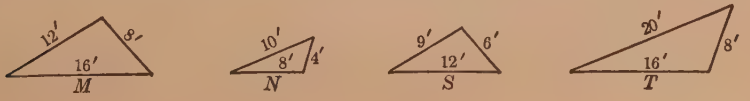
If you can not tell by the eye whether two triangles are similar, measure to see whether each pair of corresponding sides has the same ratio.

Exercises

1. Are A and B below similar? C and D? E and F?



2. Which triangle below is similar to M? to N? In each pair of triangles, do the corresponding sides have equal ratios?

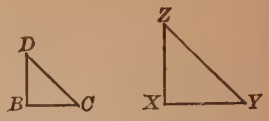


FINDING AN UNKNOWN SIDE OF A TRIANGLE

Copy and complete these proportions for the similar triangles at the right:

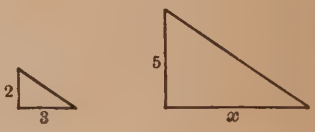
1. $\frac{BC}{XY} = \frac{BD}{\quad}$ **3.** $\frac{CD}{YZ} = \frac{\quad}{XZ}$ or $\frac{\quad}{XY}$

2. $\frac{XY}{BC} = \frac{YZ}{\quad}$ **4.** $\frac{XZ}{BD} = \frac{XY}{\quad}$ or —



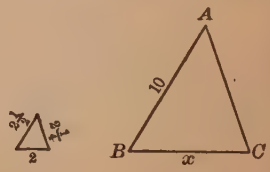
5. Using the proportion you found in ex. 2, tell the value of XY if $BC = 6$, $YZ = 15$, and $CD = 9$.

6. In a book Bertha sees an embroidery design in the form of a right triangle. The sides forming the right angle are 2 in. and 3 in. She wants to enlarge the design so that the shortest side will be 5 in. How long shall she make side x?

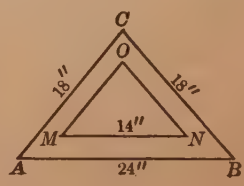


Express the problem as a proportion and solve for x.

7. Roger is making a triangular sail for his canoe similar to a pattern for a sail the sides of which measure 2 ft., $2\frac{1}{4}$ ft., and $2\frac{1}{2}$ ft. Roger wants the longest side, AB, of his sail to be 10 ft. Let x stand for the length of the shortest side, BC, of Roger's sail. Find the lengths of side BC and side AC.

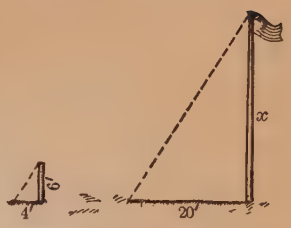


8. Mrs. Bruce wants to make a centerpiece, MNO, similar in shape to a triangular table top, ABC. The longest side of the table is 2 ft. and the other sides are both 18 in. If she wishes the longest side of the centerpiece to be 14 in., how long should each of the other sides be?



FINDING HEIGHTS BY SHADOWS

The shadow of the flagpole in front of a school is 20 ft. long when the shadow of a 6-foot post is 4 ft. long. How can you find the height of the flagpole without measuring it?



If you connect the top of the pole and the end of its shadow by a dotted line, as shown at the right, you have a triangle. By drawing a line from the top of the post to the end of its shadow, you form a smaller triangle similar to the larger one.

Let x = height of the flagpole, in feet

Since the triangles are similar, the sides representing the heights are in the same ratio as the sides representing the shadows.

Write the proportion: $\frac{x}{6} = \frac{20}{4}$

Solve: $x = 30$

Hence the flagpole is 30 ft. high. Check the result.

Exercises

1. A tree casts a shadow of 44 ft. when a post 3 ft. high casts a shadow of 2 ft. Find the height of the tree.

2. When Ben, who is 5 ft. tall, casts a shadow of 6 ft., a telegraph pole casts a shadow of 42 ft. How high is the pole?

3. A monument casts a shadow of 45 ft. when a post 8 ft. high casts a shadow of 2 ft. How high is the monument?

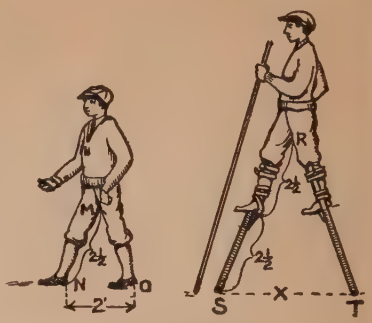
4. When a man 6 ft. tall casts a shadow of 4 ft., the Ulm Cathedral spire, which is the highest spire in Europe, casts a shadow of 352 ft. About how tall is the spire?

When a 5-foot post casts a shadow of 4 ft., tell how high trees are whose shadows have these lengths:

5. 20 ft. **6.** 12 ft. **7.** 30 ft. **8.** 23 ft. **9.** 36 ft.

HOW LONG A STEP CAN CARL TAKE ON STILTS?

Carl's legs are $2\frac{1}{2}$ ft. long and his normal step on the ground is 2 ft. He wants to know how long a step he can make on stilts that measure $2\frac{1}{2}$ ft. from his feet to the ground.



When Carl walks on the ground, his legs form triangle MNO and his normal step forms its base. When he walks on stilts, triangle RST is formed. Its base ST (or x) is Carl's enlarged step. Since the stilts are $2\frac{1}{2}$ ft. high and Carl's legs are $2\frac{1}{2}$ ft., RS and RT are each 5 ft. long. Triangles MNO and RST are similar, hence $x : 2 = RS : MN$.

Let x = length, in feet, of enlarged step

Form the proportion: $\dfrac{x}{2} = \dfrac{5}{2\frac{1}{2}}$

Solve: $x = 4$

Then Carl can make a 4-foot step on his stilts. Check the result.

Exercises

The measurements given for the stilts below are their lengths from the person's feet to the ground.

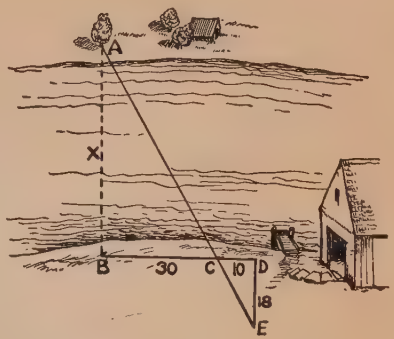
1. How long a step can a boy make on 5-foot stilts if his legs are $2\frac{1}{2}$ ft. long and his normal step is 2 ft.?

2. Helen's legs are 2 ft. long and her normal step is $1\frac{1}{2}$ ft. How long a step will she take on stilts $1\frac{2}{3}$ ft. long?

3. The tall man in the circus was a man on stilts. His legs were 3 ft. long and his normal step was $2\frac{1}{2}$ ft. The stilts were 9 ft. long. How long a step did he make on stilts? How high would his stilts have to be to take steps $12\frac{1}{2}$ ft. long?

FINDING THE WIDTH OF A RIVER

Captain Joe shows his boy scouts how to measure the width of a river without crossing it.



From point B directly across the river from tree A, he measures off $BC = 30$ ft., making BC perpendicular to BA, and drives a stake at C. He measures 10 ft. further in the same direction and sets another stake at D. Then he walks from D, at a right angle to BD, until he reaches a point E where tree A can be seen in line with stake C. He drives a stake at E. He then measures DE and finds it to be 18 ft. He is now ready to compute the width of the river.

If you think of triangle CDE as turned upside down, you see that triangles ABC and CDE are similar. Their shortest sides, BC and CD, are corresponding sides; AB and DE are also corresponding sides. Hence the scouts form the proportion $AB : DE = BC : CD$. Letting $x = AB$, since AB is the unknown side, and substituting the values of the other lines, they get $x : 18 = 30 : 10$. Solve for x and tell how wide the river is.

Exercises

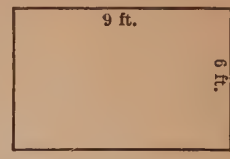
Find the width of the river if the lines above measured thus:

	BC	*CD*	*DE*		*BC*	*CD*	*DE*
1.	90′	5′	35′	**8.**	200′	15′	45′
2.	30′	4′	10′	**9.**	450′	20′	75′
3.	72′	6′	15′	**10.**	366′	12′	80′
4.	55′	3′	36′	**11.**	220′	16′	50′
5.	80′	20′	40′	**12.**	125′	$6\frac{1}{3}$′	38′
6.	40′	10′	20′	**13.**	100′	$2\frac{1}{2}$′	10′
7.	65′	10′	12′	**14.**	100′	$12\frac{1}{2}$′	25′

SCALE DRAWING

Drawing to Scale

1. Betsy made this drawing of a rug which is 9 ft. long and 6 ft. wide. She let $\frac{1}{8}$ in. stand for 1 ft.; hence she drew the rug to the scale of $\frac{1}{8}$ in. to 1 ft.



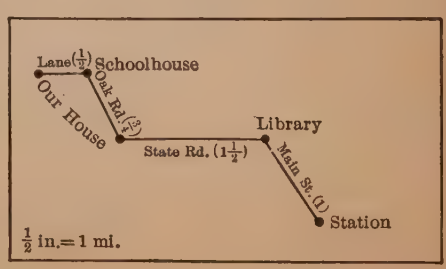
If $\frac{1}{8}$ in. stands for 1 ft., what part of an inch stands for 2 ft.? 3 ft.? 6 ft.? 9 ft.? By measuring, see if Betsy's drawing has the right length and width.

2. Kenneth is building a raft 8′ long and 7′ wide. Make a scale drawing of it, letting $\frac{1}{4}$″ stand for 1′. How many inches will stand for 4′? 5′? 7′? 8′?

3. If 1 in. stands for 10 ft., how many inches stand for 20 ft.? 15 ft.? 25 ft.? Use this scale to draw a garden 15 ft. square. Also draw a garden 20 ft. by 25 ft.

4. Ruth has a tennis court 26 yd. long and 12 yd. wide. If $\frac{1}{8}$ in. = 1 yd., how many inches equal 8 yd.? 12 yd.? 16 yd.? 24 yd.? 26 yd.? Draw the tennis court to this scale.

5. Albert was coming to visit Ben and Ben drew this map for him. In one corner he wrote $\frac{1}{2}$ in. = 1 mi. to show what scale he used. The numbers on the map show how far Albert had to go on each stretch of road. Tell how long each line should be. Then measure each line in the map to see if it is the right length.



6. To reach May's house from the station, you go 3 mi. north on Elm St. and 5 mi. east on Hill Drive. Draw a map to show this, letting 1 in. = 2 mi.

TELLING WHAT SCALE TO USE

1. Bert's house is 50 ft. long and 40 ft. wide. If he drew a diagram of the house, on a scale of $\frac{1}{2}$ in. to 1 ft., how long and how wide would his drawing be? What objection would you raise to such a drawing?

2. Tom thinks that Bert (ex. 1) should use a scale of $\frac{1}{4}$ in. = 1 ft. Anne thinks that he should let 1 in. = 10 ft. Bob suggests letting 1 in. = 5 ft. How long and how wide would the drawing be on each of these scales?

3. If Bert makes the drawing on a sheet of paper 6 in. long and 5 in. wide, which scale should he use? Why?

4. If Bert uses a scale of $\frac{1}{8}$ in. = 1 ft., can he put the drawing of his house on a sheet 6 in. long and 5 in. wide?

5. Nora's room is 10 ft. long and 8 ft. wide. She is drawing it in her diary. What dimensions will the drawing have if she lets 1 in. = 1 ft.? if she lets 4 in. = 1 ft.? $\frac{1}{8}$ in. = 2 ft.? Tell which of these scales you think it is best for Nora to use, and draw the room on that scale.

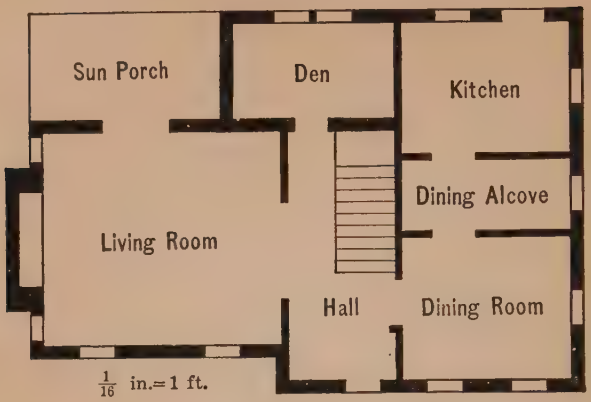
6. Our club is building a canoe shelter 30 ft. by 18 ft. Suggest a scale on which the shelter can be drawn on a sheet 9 in. by 6 in., leaving room for a small border. Then draw a plan of the shelter on the scale you have suggested.

7. Draw a bungalow 30 ft. square with a porch 30 ft. by 6 ft. on a sheet 12 in. by $10\frac{1}{2}$ in., leaving $1\frac{1}{2}$ in. border all around the drawing. What scale do you use?

8. Emil has a garden 24 ft. long and 18 ft. wide. It contains a plot of asters 24 ft. by 8 ft. and a plot of straw flowers 24 ft. by 10 ft. Draw the garden.

9. The Moose Hotel is 24 mi. south of Boston and Lydia's camp is 7 mi. west of the hotel. Should Lydia let 1 in. = 1 mi., or 8 in. = 1 mi., or $\frac{1}{8}$ in. = 1 mi. in drawing for her father the route from Boston to her camp? Draw the route.

READING THE PLAN OF A HOUSE



This plan shows the first floor of a private house.

1. As shown in the lower left corner, $\frac{1}{16}$ in. = 1 ft. Fill out and continue the table at the right to show how many feet are represented by $\frac{1}{4}$ in., $\frac{3}{8}$ in., $\frac{1}{2}$ in., $\frac{5}{8}$ in., $\frac{3}{4}$ in., $\frac{7}{8}$ in., 1 in., $1\frac{1}{8}$ in., $1\frac{1}{4}$ in., $1\frac{3}{8}$ in.

$\frac{1}{16}$ in.	= 1 ft.
$\frac{1}{4}$ in.	= . . ft.
$\frac{3}{8}$ in.	= . . ft.
$\frac{1}{2}$ in.	= . . ft.

2. Measure the length of the inside of the dining room. How many feet do $\frac{7}{8}$ in. stand for? How long is this room?

3. Measure the width of the inside of the dining room and tell how many feet wide the room is.

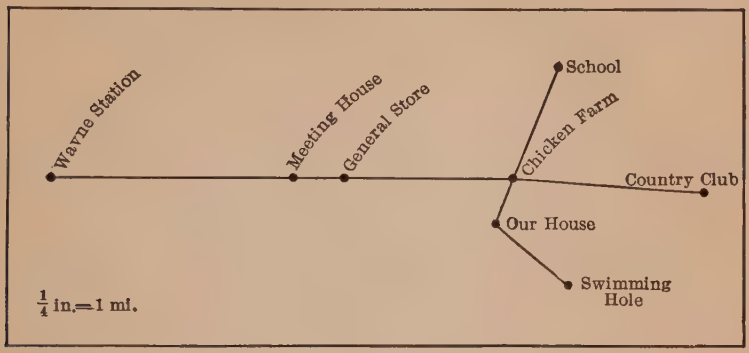
4. On the drawing measure the length and the width of the sun porch, the living room, the kitchen, the dining alcove, and the den. Then tell the dimensions of each room and of the sun porch. Use the table made in ex. 1, if necessary.

5. How long and wide is the outside of the entire house?

6. Using the dimensions found in ex. 2, 3, and 4, make a drawing of the dining room, the dining alcove, and the kitchen, letting $\frac{1}{8}$ in. = 1 ft.

PLACES NEAR DICK'S HOME

Dick lives in the country. He made this map of his home section to send to a city friend.



1. How many miles does $\frac{1}{4}''$ stand for on this map? How many miles does $\frac{1}{2}''$ stand for? $1''$? $\frac{1}{8}''$?

2. On the map, measure the line from Dick's house to the chicken farm. Tell the distance between these two points.

3. By measuring the line between them, tell how far it is from Dick's house to the swimming hole.

4. How far is it from the chicken farm to the general store? from the chicken farm to the meeting house?

5. When Dick goes to a ball game on the meeting-house grounds, he goes along the road to the chicken farm and then along the main road. How far does he travel?

6. If Dick cuts across the fields in a straight line from his house to the meeting house, how far does he have to go? How many miles can he save by not following the roads?

7. Dick walks to and from school every day. How many miles a day does this make?

8. Following the roads, how many miles is it from Dick's house to Wayne Station? to the Country Club?

DISTANCES ON A MAP

This map shows how far certain state capitals are from Washington, D. C. The scale is in the lower left-hand corner.

1. How many miles, in all, does this scale represent? How many miles does each of its small divisions represent?

2. By copying this scale, make a ruler representing 400 mi.

This ruler may be made on the edge of a sheet of paper, placing the edge along the scale to get the right length for each 10-mile space.

3. Measure the airline distance from Washington to Dover.

Using the ruler made in ex. 2, you find that the distance equals 8 of the small spaces and about 0.2 of the ninth space. Since each space is 10 mi., the distance from Washington to Dover is about 8.2×10 mi., or 82 mi.

4. Measure the airline distance from Washington to Trenton; to Annapolis; to Richmond; to Charleston; to Harrisburg.

5. Measure the airline distance from Philadelphia to Washington; from Philadelphia to each of the state capitals in ex. 4.

6. Measure the airline distance from Pittsburgh to Philadelphia; from Pittsburgh to Washington; from Pittsburgh to each of the state capitals in ex. 4.

7. Measure the distance from Baltimore to six other cities.

IMPROVEMENT TEST No. 32

Test 32 A — Comparing Numbers (Time 4 min. after copying)

Find results to the nearest tenth of 1%:

1. 18,200,000 is what per cent of 76,500,000?
2. 147,260,000 is what per cent of 125,000,000?

Test 32 B — Comparing Numbers (Time 4 min. after copying)

Find results to the nearest tenth of 1%:

1. What per cent of \$125,000,000 is \$45,650,000?
2. What per cent of \$369,500,000 is \$486,500,000?

Test 32 C — Finding the Base (Time 4 min. after copying)

What numbers should be put in place of the dots?

1. 36 is 8% of . . .
2. 68% of . . . is 3366
3. 96 is 15% of . . .
4. $66\frac{2}{3}$% of . . . is 274
5. 235.05 is 15% of . . .

Test 32 D — Trade Discount (Time 4 min. after copying)

Find, to the nearest cent, the net price when the list price and the discount are as follows:

1. \$8; $33\frac{1}{3}$%
2. \$485; $16\frac{2}{3}$%
3. \$750; 20%, 15%, 2%
4. \$8.97; 10%
5. \$752; $12\frac{1}{2}$%
6. \$2780; 25%, 20%, 3%

Test 32 E — Bank Discount (Time 3 min. after copying)

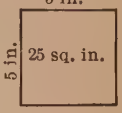
Find, to the nearest cent, the bank discount on the following, using the 6% *method:*

1. \$495 for 96 da. at 6%
2. \$870 for 150 da. at 5%
3. \$298 for 54 da. at 7%

SQUARE ROOT

Squares and Square Roots

If a square measures 5 in. on a side, its area is 5×5 sq. in., or 25 sq. in. 25 is called the **square** of 5.



A short way of writing 5×5 is 5^2, which is read "5 square." You see that $5^2 = 25$.

If you know that the area of a square is 25 sq. in. and wish to find the length of its side, you must find the **square root** of 25. This means that you must find a number which, when multiplied by itself, gives 25. Since $5 \times 5 = 25$, 5 is the *square root* of 25. Hence 5 in. is the length of the side of the square. The sign for square root is $\sqrt{\ }$. Thus, $\sqrt{25}$ is read the *square root of* 25 and $\sqrt{25} = 5$.

Exercises

1. Since $7 \times 7 = 49$, the square of 7 is 49. What is the square root of 49? What does $\sqrt{49}$ equal?

2. How many are 6×6? How many are 6^2? What is the square of 6? What is the square root of 36? What does $\sqrt{36}$ equal?

3. Give the squares of all the numbers from 1 to 12.

Find the following, remembering that 14^2 *means* 14×14:

4. 14^2	**6.** 15^2	**8.** 25^2	**10.** 30^2	**12.** 100^2
5. 20^2	**7.** 18^2	**9.** 16^2	**11.** 90^2	**13.** 200^2

Give the square roots of the following:

14. 1	**16.** 9	**18.** 16	**20.** 49	**22.** 144
15. 4	**17.** 64	**19.** 81	**21.** 100	**23.** 121

Read, and tell what numbers should be put in place of the dots:

24. $\sqrt{81} = \ldots$	**26.** $\sqrt{400} = \ldots$	**28.** $\sqrt{225} = \ldots$
25. $\sqrt{100} = \ldots$	**27.** $\sqrt{900} = \ldots$	**29.** $\sqrt{1600} = \ldots$

USING A TABLE OF SQUARES

This table gives the squares of numbers from 1 to 100:

Number	Square	Number	Square	Number	Square	Number	Square
1	1	26	676	51	2601	76	5776
2	4	27	729	52	2704	77	5929
3	9	28	784	53	2809	78	6084
4	16	29	841	54	2916	79	6241
5	25	30	900	55	3025	80	6400
6	36	31	961	56	3136	81	6561
7	49	32	1024	57	3249	82	6724
8	64	33	1089	58	3364	83	6889
9	81	34	1156	59	3481	84	7056
10	100	35	1225	60	3600	85	7225
11	121	36	1296	61	3721	86	7396
12	144	37	1369	62	3844	87	7569
13	169	38	1444	63	3969	88	7744
14	196	39	1521	64	4096	89	7921
15	225	40	1600	65	4225	90	8100
16	256	41	1681	66	4356	91	8281
17	289	42	1764	67	4489	92	8464
18	324	43	1849	68	4624	93	8649
19	361	44	1936	69	4761	94	8836
20	400	45	2025	70	4900	95	9025
21	441	46	2116	71	5041	96	9216
22	484	47	2209	72	5184	97	9409
23	529	48	2304	73	5329	98	9604
24	576	49	2401	74	5476	99	9801
25	625	50	2500	75	5625	100	10000

1. Find the square of 19; of 23; 37; 51; 65; 89.

To find the square of 19 in the table, find 19 in the left-hand column headed "Number." Then, in the column headed "Square," just to the right of 19, you will find 361. 361 is the square of 19.

2. Find the square root of 2401.

Find 2401 in one of the columns headed "Square." Then find the number just to the left of 2401, which is 49. 49 is the square root of 2401.

3. Find the square root of 81; of 256; 625; 841; 1444; 1681; 2209; 3025; 4096; 5184; 7569; 8464; 9216; 10,000.

A number like 81, 256, or 10,000, whose square root is a whole number, is called a **perfect square.**

FINDING THE APPROXIMATE SQUARE ROOT

The table on page 481 also helps you to find the square root of a number that is not a perfect square.

1. Find, to the nearest tenth, the square root of 149.

Although 149 is not in the table of squares, you find that it lies between 144 (the square of 12) and 169 (the square of 13). Hence the square root of 149 is between 12 and 13.

Since 149 is much closer to 144 than to 169, the square root of 149 is a little over 12. Find 12.1×12.1, 12.2×12.2, and 12.3×12.3, as shown at the right. Which of these squares is closest to 149? Then 12.2 is, to the nearest tenth, the square root of 149.

$12.1^2 = 146.41$
$12.2^2 = 148.84$
$12.3^2 = 151.29$

2. Find, to the nearest tenth, the square root of 145.

Since 145 is a little more than 144 (the square of 12), you may try 12.1 as the square root. $12.1^2 = 146.41$. But 144 is closer to 145 than 146.41 is; hence the approximate square root of 145 is 12.

If you cannot tell by inspection whether 144 or 146.41 is closer to 145, find the difference between 145 and each square by subtracting as shown at the right. This shows that 144 is only 1 less than 145, while 146.41 is 1.41 greater than 145.

145	146.41
144	145
1	1.41

Exercises

1. Anna is finding the square root of 968. She says that 968 lies between 961 (the square of 31) and 1024 (the square of 32). Is she right? To which of these squares is 968 closer?

2. Since 968 is a little more than the square of 31, Anna tries both 31.1 and 31.2 as the square root of 968. Find 31.1^2 and 31.2^2 and see which result is closer to 968. Show that 31.1 is the approximate square root of 968.

3. Show that the square root of 401 is a little over 20. Is 20.0 or 20.1 the approximate square root of 401?

Find, to the nearest tenth, the square root of:

4. 27 **5.** 52 **6.** 87 **7.** 789 **8.** 914 **9.** 2331

FINDING APPROXIMATE SQUARE ROOTS

1. Find, to the nearest tenth, the square root of 164.

In the table of squares on page 481, you find that 164 lies between 144 (the square of 12) and 169 (the square of 13); but it is much nearer to 169 than to 144. Hence the square root of 164 is closer to 13 than to 12. Square 12.7, 12.8, and 12.9 to find which square is closest to 164. Show that, to the nearest tenth, 12.8 is the square root of 164.

$12.7^2 = 161.29$
$12.8^2 = 163.84$
$12.9^2 = 166.41$

2. Find, to the nearest tenth, the square root of 156.

Since 156 lies about half way between 144 and 169, its square root is about half way between 12 and 13. Try 12.4, 12.5, and 12.6, squaring each as shown at the right. You find that 12.5 is the approximate square root of 156.

$12.4^2 = 153.76$
$12.5^2 = 156.25$
$12.6^2 = 158.76$

Exercises

In the following exercises, use the table on page 481.

1. Mary Lane wishes to make a square garden that will contain 130 sq. ft. of land. Find, to the nearest tenth of a foot, how long each side of the garden should be.

2. Find, to the nearest tenth of a foot, the length of the side of a square garden whose area is 750 sq. ft.; of a square garden whose area is 650 sq. ft.; 600 sq. ft.

Find, to the nearest tenth, the square root of:

3. 46	**7.** 50	**11.** 174	**15.** 449	**19.** 1450
4. 42	**8.** 56	**12.** 185	**16.** 458	**20.** 2024
5. 90	**9.** 60	**13.** 190	**17.** 471	**21.** 3230
6. 96	**10.** 24	**14.** 194	**18.** 480	**22.** 8875

23. A factory is going to make 2-quart cans which will be $5\frac{1}{2}$ in. high and square on the bottom. Find, to the nearest tenth of an inch, the length of one side of the bottom of the can.

HINT. 1 gal. = 231 cu. in.; hence 2 qt. = $115\frac{1}{2}$ cu. in. $115\frac{1}{2} \div 5\frac{1}{2} = 21$, the area of the bottom in square inches.

FINDING SQUARE ROOTS TO THE NEAREST HUNDREDTH

The method on pages 482 and 483 will also enable you to find the square root of a number correct to any number of decimal places. For example, to find the square root of 164 to the nearest hundredth, you proceed as follows:

First find, as on page 483, the square root of 164 to the nearest tenth, which gives 12.8. Since $12.8^2 = 163.84$, and since 164 is a little more than 163.8, the square root of 164 is a little over 12.8 (which equals 12.80). Trying 12.81, you find that $12.81^2 = 164.0961$.

Since 164.0961 (the square of 12.81) is a little more than 164 and since 163.84 (the square of 12.80) is a little less than 164, you must decide which is closer to 164. Subtracting, you find that $164.0961 - 164 = .0961$ and that $164 - 163.84 = .16$, or .1600. Since .0961 is less than .1600, 164.0961 is closer to 164. Therefore 12.81 is the square root of 164, to the nearest hundredth.

Exercises

1. Dorothy found that, to the nearest tenth, the square root of 85 is between 9.2 and 9.3, but nearer to 9.2. Hence to get the root to the nearest hundredth she tried 9.21, 9.22, and 9.23. Square each of these numbers and show that 9.22 is the square root of 85.

2. Show that the square root of 48 is closer to 6.9 than to 7.0. Then find the square root of 48 to the nearest hundredth by squaring 6.91, 6.92, and 6.93 to see which result is closest to 48.

Find these square roots correct to the nearest hundredth:

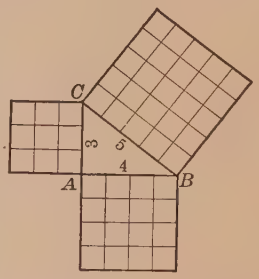
3. $\sqrt{17}$	**8.** $\sqrt{200}$	**13.** $\sqrt{217}$	**18.** $\sqrt{1085}$
4. $\sqrt{44}$	**9.** $\sqrt{428}$	**14.** $\sqrt{342}$	**19.** $\sqrt{2018}$
5. $\sqrt{61}$	**10.** $\sqrt{128}$	**15.** $\sqrt{530}$	**20.** $\sqrt{3230}$
6. $\sqrt{74}$	**11.** $\sqrt{139}$	**16.** $\sqrt{593}$	**21.** $\sqrt{4635}$
7. $\sqrt{65}$	**12.** $\sqrt{140}$	**17.** $\sqrt{945}$	**22.** $\sqrt{6815}$

USING SQUARE ROOT IN RIGHT TRIANGLES

Square root is useful in finding the length of one side of a right triangle when the other sides are known.

In any right triangle, there is one right angle. The side opposite the right angle is called the **hypotenuse;** the other sides are called the **legs.**

Draw a right triangle ABC whose legs CA and AB measure 3 in. and 4 in. Measuring the hypotenuse, BC, you find that it is exactly 5 in. long.



Now construct three squares having CA, AB, and BC as sides. Divide these squares into square inches.

The square on leg CA contains 9 sq. in. The square on leg AB contains 16 sq. in. The square on the hypotenuse BC contains 25 sq. in., which is the sum of 9 sq. in. and 16 sq. in. Hence the square on BC equals the sum of the squares on the legs.

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

This rule is called the *Pythagorean Theorem* and was discovered by Pythagoras, a Greek philosopher, about 540 B.C.

Exercises

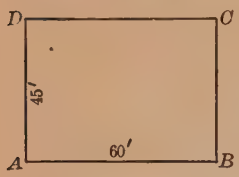
1. Draw a right triangle whose legs measure 6 in. and 8 in. Measure the length of its hypotenuse. Find the area of a 6-inch square; of an 8-inch square; of a 10-inch square. Then find 6^2, 8^2, and 10^2. Does the square of the hypotenuse here equal the sum of the squares of the legs?

2. The legs of a right triangle are 9 in. and 12 in. Find the length of the hypotenuse without drawing a triangle.

The sum of the squares of 9 and 12 is $81 + 144$, or 225. Hence the length of the hypotenuse is the square root of 225 sq. in.

FINDING DISTANCES BY SQUARE ROOT

In ex. 1 *to* 4, *find answers to the nearest tenth.*



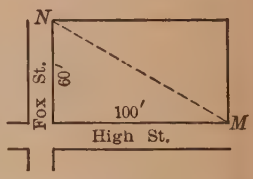
1. Dan can swim from one corner, B, of the swimming pool to the opposite corner, D. The pool measures 60 ft. by 45 ft. How far can Dan swim?

BD is the hypotenuse of triangle DAB.

2. A rectangular playground measures 45 ft. wide and 85 ft. long. Draw a diagram and find how far Agnes runs when she runs from one corner of the playground to the opposite corner.

3. Wilson Green is 70 ft. square. A path is to be made across it from one corner to the opposite corner. How long will the path be?

4. An empty corner lot 100 ft. long and 60 ft. wide has a diagonal path MN leading across it. In walking from M to N, how many feet shorter is it to take the path than to follow the sidewalks on High Street and Fox Street?



The number 13,600 does not appear in the table on page 481. Show that it lies between 116^2 and 117^2; then find the square root of 13,600 to the nearest tenth.

5. At a May party, the Maypole is to be 24 ft. high and the children holding the streamers are to stand 7 ft. from the pole. How long must the streamers be?

Do not count the height of the children.

Find, to the nearest hundredth, the hypotenuses of right triangles whose legs are as follows:

6. 6 ft., 9 ft.

7. 12 ft., 18 ft.

8. 31 ft., 43 ft.

9. 9 mi., 10 mi.

10. 60 ft., 35 ft.

11. 35 ft., 15 ft.

THE SQUARE ROOT OF A MIXED DECIMAL

The legs of a right triangle measure 2.1 in. and 5.3 in. Find, to the nearest tenth of an inch, the length of the hypotenuse.

$2.1^2 = 4.41$, and $5.3^2 = 28.09$. Hence the square of the hypotenuse, which is the sum of the squares of the legs, is $4.41 + 28.09$, or 32.50. You must find the square root of 32.50.

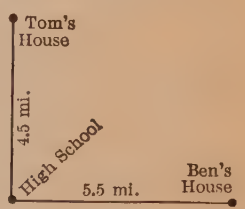
Since 32.50 lies between the perfect squares 25 and 36, the square root of 32.50 is between 5 and 6. 32.50 is nearer to 36 than to 25; hence try 5.6, 5.7, and 5.8, as shown at the right. Since 32.49 (5.7^2) is the closest to 32.50, the square root of 32.50 is 5.7. The hypotenuse is 5.7 in. long.

$5.6^2 = 31.36$
$5.7^2 = 32.49$
$5.8^2 = 33.64$

Problems

In ex. 1 *and* 2, *find answers to the nearest tenth of a mile.*

1. Tom Allen lives 4.5 mi. north of the new high school. Ben Lewis lives 5.5 mi. east of it. How many miles, in a straight line, does Ben live from Tom?



2. Oldstown is 15.1 mi. south of Smokum. Mine City is 12.2 mi. west of Smokum. Draw a diagram showing the route of an airplane flying in a straight line between Oldstown and Mine City. How long is this route?

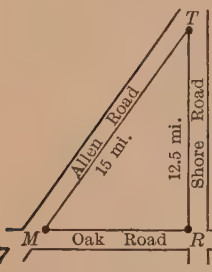
3. In a skating race (see page 302), no boy was allowed to use a sail containing more than 18.5 sq. ft. Albert wanted to make a square sail. Find, to the nearest hundredth of a foot, the greatest length which a side of the square sail could have.

Find, to the nearest tenth of a mile, the hypotenuses of right triangles whose legs are:

4. 2.1 mi., 3.7 mi.
5. 3.4 mi., 6.2 mi.
6. 2.8 mi., 3.5 mi.
7. 8.6 mi., 8.6 mi.
8. 5.3 mi., 7.5 mi.
9. 1.5 mi., 8.5 mi.
10. 9 mi., 9.9 mi.
11. 3 mi., 4.8 mi.
12. 5 mi., 6.2 mi.

PROBLEMS THAT USE SQUARE ROOT

1. It is 15 mi. along Allen Road from the Town Hall (T) to the mill (M). It is 12.5 mi. along Shore Road from the Town Hall (T) to Reed Farm (R). How far is it along Oak Road from Reed Farm to the mill?

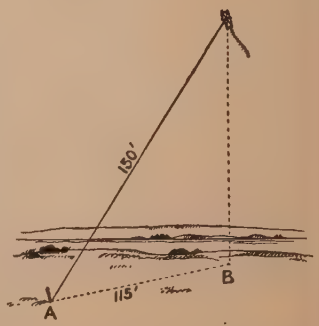


Here you know the length of the hypotenuse and one leg of the triangle and you must find the length of the other leg. Let x stand for MR. Then $x^2 + 12.5^2 = 15^2$, or $x^2 + 156.25 = 225$. Show that $x^2 = 68.75$. Then show that $x = \sqrt{68.75}$.

2. An acre contains 43,560 sq. ft. If a square field contains an acre, find, to the nearest hundredth of a foot, the length of one side of the field.

The table on page 481 does not contain squares as large as 43,560. First show by trial that 43,560 lies between 208^2 and 209^2. Then find the result to the nearest hundredth.

3. George is flying his kite. When he has let out 150 ft. of string, he fastens the end to a stick (A) in the ground and walks until he is directly under the kite at B. He then measures AB and finds that it is 115 ft. long. How high, to the nearest tenth of a foot, is George's kite?



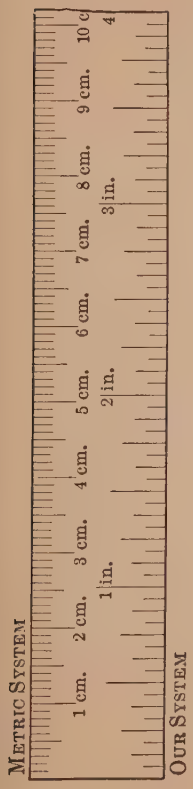
4. The formula for the area (A) of a circle is $A = \pi r^2$. From this formula show how to make the following formula for the radius: $r = \sqrt{\frac{A}{\pi}}$.

5. Ruth has a garden 11 ft. by 8 ft. Dan wants to make a circular garden with the same area as Ruth's. Find the radius of Dan's garden, to the nearest tenth of a foot. (See ex. 4.)

METRIC SYSTEM

Metric Units of Length

Instead of measuring things by the foot, the pound, the quart, etc., most countries of Europe, Asia, and South America use a system of units called the **metric system.**



The basic unit in the metric system is the **meter,** which equals 39.37 in. To find the other units of length, join to the word *meter*

milli = $\frac{1}{1000}$	deka = 10
centi = $\frac{1}{100}$	hekto = 100
deci = $\frac{1}{10}$	kilo = 1000

the prefixes shown above. This gives the following table of *metric length:*

1 millimeter (mm.)	= $\frac{1}{1000}$ meter (m.)
1 centimeter (cm.)	= $\frac{1}{100}$ meter
1 decimeter (dm.)	= $\frac{1}{10}$ meter
1 dekameter	= 10 meters
1 hektometer (hm.)	= 100 meters
1 kilometer (km.)	= 1000 meters

The illustration at the left shows the actual length of 1 decimeter. Each of the larger divisions, like the division from the number 1 to the number 2, is 1 centimeter. Each of the very small divisions is 1 millimeter.

Exercises

1. A centimeter is what part of a meter? A cent is what part of a dollar? Does *cent* mean $\frac{1}{100}$ in both cases?

2. A millimeter is what part of a meter? A mill is what part of a dollar? What does *mill* mean in both cases?

MEASURING AND COMPARING DISTANCES

1. In the decimeter shown on page 489, how many centimeters are there? how many millimeters? How many millimeters are there in 1 centimeter? in 1 meter?

2. Write the table of metric length another way, beginning: 10 millimeters = 1 centimeter, 10 centimeters = 1 decimeter.

3. Make a ruler 15 cm. long out of cardboard and mark it off into centimeters and millimeters, as on page 489.

4. With the ruler made in ex. 3, measure in centimeters and millimeters the width and the length of this book; the length of your middle finger; the length of the nail on your middle finger.

The middle finger of the average adult is about 1 dm. long; the nail of the middle finger is about 1 cm. long and .3 mm. thick.

5. At the Olympic Games, the best record for throwing the discus was 46 m. 16 cm. Express this distance in meters.

Since 1 centimeter = $\frac{1}{100}$ meter, divide by 100 to change centimeters to meters. To change 16 cm. to meters, move the decimal point two places to the left, thus: 16 cm. = .16 m. Hence 46 m. 16 cm. = 46.16 m.

6. The winner of the hammer throw at the Olympic Games made a distance of 53 m. 29.5 cm. The second best distance was 50 m. 84 cm. Change each distance to meters and tell how much longer the first throw was than the second.

7. In a United States intercollegiate meet, the winning distance for the hammer throw was 156 ft. $\frac{1}{8}$ in.; the second best was 152 ft. $9\frac{7}{8}$ in. How much farther, in feet and inches, was the best throw than the second best?

8. Which is easier, comparing distances by metric units, as in ex. 6, or by feet and inches, as in ex. 7?

9. Among the events at the Olympic Games are the 1500-meter run, the 5000-meter run, and the 800-meter swimming relay. By dividing by 1000, change each distance to kilometers. Is this easier than changing 440 yd. to miles?

EQUIVALENTS TO METRIC UNITS OF LENGTH

You have seen that it is easier to compare distances by metric units than by our units (foot, inch, etc.). Now, since the United States has not yet adopted the metric system, you must learn the approximate equivalents to the metric units of length:

1 meter (m.) = 39.37 in. or 1.1 yd.
1 kilometer (km.) = .6 mi., or $\frac{3}{5}$ mi.

Problems

Find answers to the nearest tenth of a unit.

1. The waves sent out by a certain radio broadcasting station are 422 m. in length. How many yards long are they?

2. The world's walking champion covered 12.975 km. in 1 hr. How many miles an hour did he walk?

3. The swimming events at the Olympic Games include the 100-meter back stroke, the 200-meter breast stroke, and the 1500-meter free style. Change each of these distances to yards.

4. Lucy's mother bought 20 m. of silk in Paris. How many yards of silk did she buy?

5. Mr. and Mrs. Deane made a trip of 245 km. on a train in Italy. How many miles long was the trip?

6. In a recent year an aviator from Argentina flew 15,000 km. in a seaplane in 114 hr. How many miles an hour did he fly?

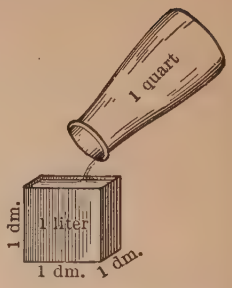
7. A French aviator reached an altitude of 12,442 m. in an airplane. An American aviator later reached an altitude of 38,559 ft. Which altitude was higher? how many feet higher?

NOTE. There are also metric tables of square measure and of cubic measure which may easily be constructed from the metric table of length. These tables are omitted here, since the pupil does not have sufficient use for them at this time.

METRIC MEASURES OF CAPACITY

The metric unit of capacity, used chiefly in measuring liquids, is the **liter.** A cube measuring 1 dm. (or 10 cm.) on each side contains 1 liter. The liter is closely related to the meter, since a liter equals a cubic decimeter.

A complete table of metric capacity can be made by joining the prefixes *milli, centi, deci,* etc., whose meanings are given on page 489, to the word *liter.* Here are the most common measures in *metric capacity:*



1 Cubic Decimeter or 1 Liter

1 milliliter (ml.) = $\frac{1}{1000}$ liter (l.)
1 hektoliter (hl.) = 100 liters

A liter is equivalent to about 1 quart in our measure. Our quart, however, is not closely related to the yard, or to the foot, as the liter is related to the meter.

Exercises

1. Write a table of capacity, thus: 1 milliliter = $\frac{1}{1000}$ liter, 1 centiliter = $\frac{1}{100}$ liter, etc.

2. Mr. Tilman is traveling in Europe in his car. He needs about 12 gal. of gasoline. How many liters shall he buy?

3. An aquarium is 5 dm. long, 4 dm. wide, and 3 dm. deep. How many cubic decimeters does it contain? How many liters of water will it hold?

4. An aquarium is 20 in. $\times$ 16 in. $\times$ 12 in. How many cubic inches does it contain? How many quarts of water will it hold?

5. Is the work shorter and easier with the metric system, as in ex. 3, or with our system, as in ex. 4?

METRIC MEASURES OF WEIGHT

The metric unit of weight is the **gram,** though the unit most commonly used is 1000 grams, or the **kilogram.** A kilogram is equal to the weight of 1 liter (1 cubic decimeter) of water. Hence a relationship exists between the liter and the kilogram.

Here are the most common measures in *metric weight:*

1 kilogram (kg.) = 1000 grams (g.)
1 metric ton (t.) = 1000 kilograms

One kilogram is approximately equal to 2.2 pounds.

Exercises

1. What part of a gram is 1 milligram? 1 centigram? 1 decigram? Write a complete table of metric weight, similar to the one given for metric length on page 489.

2. A United States dime (ten-cent piece) weighs 2 grams 5 decigrams. Write this in grams. If a half dollar weighs 5 times as much, find the weight, in grams, of a half dollar.

3. A United States nickel (five-cent piece) weighs 5 grams. One nickel weighs as much as how many dimes (see ex. 2)? How many nickels will it take to weigh 1 kg.?

4. A tank is 2 m. long, 2 m. wide, and 3 m. high. Express its dimensions in decimeters. Then find its volume in cubic decimeters. How many liters of water will this tank hold? How many kilograms will the water weigh?

5. A tank is 6 ft. long, 6 ft. wide, and 9 ft. high. How many cubic feet does it contain? If $7\frac{1}{2}$ gal. = 1 cu. ft., how many gallons of water will it hold? How many pounds will the water weigh if 1 gal. weighs 8.3 lb.?

6. Is it easier to find the weight of the water by the metric system, as in ex. 4, or by our system, as in ex. 5?

MEASURING THE ENERGY YOU GET FROM FOODS

Food is needed to build up your body, to give you energy, and to keep you warm. The energy and heat that foods give are measured by a unit called a **calorie**. A calorie is the amount of heat needed to raise the temperature of 1 kg. of water 1° centigrade or of 1 lb. of water 4° Fahrenheit. If a boy eats a large apple, it will give him 100 calories of heat and energy.

The energy in food comes from three substances called *carbohydrate*, *protein*, and *fat*. Carbohydrate is contained in sweet and starchy foods like sugar, bread, cereals, and potatoes. Much protein is found in meat, eggs, cheese, and milk, while fat is furnished by butter, olive oil, and the fat of meat. Most foods give more than one of these substances, as is shown in the table below.

The United States government publishes tables showing the per cent of carbohydrate, protein, and fat in the more common foods. In the table below, the first line shows that 9% of the weight of a piece of white bread is protein, 53% is carbohydrate, and 1% is fat; the rest is water or refuse, which explains why the sum of these three per cents is not 100%.

	Protein	*Carbohydrate*	*Fat*
White Bread	9%	53%	1%
Beefsteak	19%	—	18%
Chicken	21%	—	3%
Whole Egg	13%	—	10%
Whole Milk	3%	5%	4%
Butter	1%	—	85%
Cheese	28%	—	36%
Doughnut	7%	52%	21%
Cabbage	1%	5%	—
Carrots	1%	9%	—
Potato	2%	18%	—
Spinach	2%	3%	—
Banana	1%	22%	0.6%

FINDING THE NUMBER OF CALORIES IN FOODS

Mary has a loaf of white bread weighing 1 lb. She wishes to know how much protein, carbohydrate, and fat it contains, and how many calories of energy it will furnish.

The table on page 494 shows that 9% of white bread is protein. Since 9% of 16 oz. is 1.44 oz., the pound of bread contains 1.44 oz. of protein. Similarly, it contains 53% of 16 oz., or 8.48 oz., of carbohydrate, and 1% of 16 oz., or 0.16 oz., of fat.

To find the number of calories, use this table:

1 oz. carbohydrate	= 115 calories
1 oz. protein	= 115 calories
1 oz. fat	= 255 calories

You have found above that 1 lb. bread contains 1.44 oz. of protein, 8.48 oz. of carbohydrate, and 0.16 oz. of fat. Hence it furnishes:

$1.44 \times 115 = 166$, number of calories from protein
$8.48 \times 115 = 975$, number of calories from carbohydrate
$0.16 \times 255 = 41$, number of calories from fat
1182, number of calories in all

Hence 1 lb. bread gives 1182 calories, of which 166 come from protein, 975 from carbohydrate, and 41 from fat.

Exercises

From the table on page 494, *find how many ounces of protein, of carbohydrate, and of fat each of the following contains. Then find the total calories in each:*

1. 1 lb. spinach
2. 1 lb. cabbage
3. 1 lb. carrots
4. 1 lb. bananas
5. 3 lb. milk
6. $\frac{1}{2}$ lb. eggs
7. $\frac{1}{4}$ lb. cheese
8. 1 lb. butter
9. $\frac{1}{2}$ lb. chicken
10. 2 lb. potatoes
11. 1 lb. beefsteak
12. $\frac{1}{4}$ lb. doughnuts
13. A quart of whole milk, which weighs 34 oz.

A DAY'S MENU FOR A BOY

A sensible meal contains the right number of calories and also the right proportion of carbohydrate, protein, and fat.

Here is a menu for a day suitable for a boy 11 yr. old:

	Total Calories	Kinds of Calories		
Breakfast		*Protein*	*Fat*	*Carbohydrate*
Banana	100	5	6	89
Shredded Wheat Biscuit	100	13	5	82
Glass of Milk	160	31	83	46
Bread	100	14	6	80
Butter	100	1	99	—
Luncheon				
Cup of Bean Soup	200	30	90	80
French Rolls	200	24	16	160
Butter	100	1	99	—
Cup Custard	300	51	117	132
Dinner				
Beef Loaf	200	80	120	—
Baked White Potato	100	11	1	88
Canned Peas	134	35	4	95
Lettuce, French Dressing	100	1	95	4
Bread	100	14	6	80
Butter	100	1	99	—
Baked Apple, Whipped Cream	200	2	62	136
Gingerbread	100	12	1	87

See the footnote about 100-calorie portions on page 497.

Exercises

1. Find the number of calories in each meal and the total calories for the day on the above menu.

2. How many calories in each meal are protein calories? How many are fat? How many are carbohydrate?

3. Find the number of each kind of calorie in the total day's menu and what per cent each is of the total calories for the day.

EATING THE RIGHT AMOUNT OF PROTEIN

In a well-balanced menu, from 10% to 15% of the total calories for the day are protein. Hence a day's menu of 2200 calories should contain from 220 to 330 protein calories.

1. In the menu on page 496, do the total calories for the day include the right per cent of protein? (See ex. 3, page 496.)

2. Suppose these were added to the menu on page 496:

		Total Calories	*Protein Calories*
BREAKFAST:	2 Eggs	150	54
LUNCHEON:	2 Lamb Chops	200	80
DINNER:	3 Crackers	50	5
	Cheese	100	25

What would then be the total number of calories consumed during the day? How many of these would be protein? Is the per cent of protein more or less than it should be? by how much?

Selecting School Lunches

Below is a list of foods served in a school lunchroom. Select five different luncheons, each having at least one article from each group and a total of from 750 to 900 calories. Let each luncheon have the right per cent of protein.

GROUP 1	CALORIES *Total*	*Protein*	GROUP 2	CALORIES *Total*	*Protein*
Cream of Celery Soup	200	22	Cocoa . . .	250	40
Split Pea Soup . .	167	44	Milk . . .	160	31
Macaroni and Cheese	200	34	GROUP 3		
Baked Beans . . .	300	63	Orange . . .	100	7
Lentil Meat Loaf . .	200	56	Rice pudding .	200	32
Scalloped Potatoes .	160	14	Brown Betty .	200	6
Egg Salad	250	35	Chocolate Bar	250	18
Bread and Butter . .	200	15	Ice Cream .	200	12

Note. In making menus, tables are often used giving 100-calorie portions of each food, thus making it easy to select portions of 100, 200, or 300 calories each, as in the items above and on page 496.

HOW MANY CALORIES DO YOU NEED?

The amount of food you need each day depends on your weight and age and on how you spend your time. An active child 12 yr. of age needs 32 calories a day for every pound of his weight; hence a child 12 yr. old and weighing 78 lb. needs 32×78 calories, or 2496 calories a day.

The table at the right gives the average number of calories which children of different ages need per day for every pound of their weight. To find how many calories a day a child needs, multiply his weight by the average number of calories for a child of his age.

Age in Years	Calories per Pound of Weight
1 to 2	43
3 to 5	38
6 to 9	34
10 to 13	32
14 to 17	27

Exercises

1. What is your weight and age? Then how many calories a day do you need?

2. Billie is 6 yr. old. How many calories a day should he have if he weighs 40 lb.? if he weighs 44 lb.? 48 lb.? 50 lb.?

Tell how many calories a day these children need:

3. Doris, who is 13 yr. old and weighs 89 lb.

4. Joseph, who is 7 yr. old and weighs 54 lb.

5. George, who is 14 yr. old and weighs 94 lb.

6. Evelyn, who is 16 yr. old and weighs 104 lb.

7. Ann is 12 yr. old and weighs 84 lb. She has 450 calories for breakfast, 700 calories for lunch, and 900 calories for dinner. How many calories less than she needs has she?

8. Has the menu on page 496 enough calories for you?

9. How many calories (see pages 494 and 495) are there in a pound of spinach? in a pound of bread? in a pound of butter? Why is it impossible for a person to live on spinach alone? Why do people eat so much bread and butter?

QUESTIONS FOR REVIEW

1. A man in Galveston who has no checking account wishes to send \$75 to some one in Detroit. Name two forms in which he can send the money safely.

2. How is a commercial bank paid for the trouble and work of letting its depositors have checking accounts?

3. What two things must you do to both sides of the equation $3x - 7 = 11$ to solve for x in this equation? If you start with the formula $C = 2\pi r$, what principle do you use in deriving a new formula for the radius of a circle?

4. Why is a large factory or a railroad usually owned by a corporation rather than by an individual?

5. Mrs. Reed has \$2240 in the savings bank at $4\frac{1}{2}\%$ compound interest. A friend advises her to use the money to buy stocks with a par value of \$100 which are selling at $38\frac{3}{4}$ with few prospects ahead. What advice would you give her? Why?

6. If you buy a 4% bond at 94 and keep it until it matures, will your investment earn more or less, in all, than 4% a year? Give reasons for your answer.

7. Mr. Gregg is 36 yr. old and married and earns \$40 a week. What kind of life insurance policy should he purchase?

8. The fire insurance rate on the Red Brook Mountain House, a frame structure 4 mi. from the village of Red Brook, is \$2.29 per \$100. The rate on Mr. Gray's brick dwelling in St. Louis is \$.28 per \$100. Explain the difference in rates.

9. The actual values of the property in East Milton and in West Milton are about the same, but the assessed valuation of East Milton is only 75% of that of West Milton. If both towns must raise about the same amount for their expenses, which will have the higher tax rate?

10. In what ways is the metric system easier to compute with than our system of measures?

REVIEW OF MEASUREMENTS

1. An outdoor swimming pool is 80 ft. long and 50 ft. wide. The water is 4 ft. deep at one end and 8 ft. deep at the other end. How much does it cost to fill this pool if the water company charges 30¢ per 100 cu. ft. for water?

2. Mr. Knox is building a cylindrical tank with a diameter of 30 in. He wishes it to hold 75 gal. when filled to within 1 in. of the top. How many inches high shall he make the tank?

3. A marble monument consists of a ball 4 ft. in diameter on top of a cylindrical column $3\frac{1}{2}$ ft. in diameter and 21 ft. high. The column stands on a base which is a rectangular solid 12 ft. square and 3 ft. deep. Draw to scale a side view of the monument. If 1 cu. ft. of marble weighs 168 lb., what is the weight of the monument in pounds? in tons?

Find the curved surfaces and the volumes of cylinders whose heights (h) and radii (r) are as follows:

4. $h = 14$ in., $r = 6$ in.

5. $h = 12$ in., $r = 7$ in.

6. $h = 22$ in., $r = 5$ in.

7. $h = 5\frac{1}{4}$ ft., $r = 1\frac{1}{2}$ ft.

8. $h = 6\frac{2}{3}$ ft., $r = 3\frac{1}{2}$ ft.

9. $h = 5.6$ ft., $r = 1.4$ ft.

Find the volumes of the following pyramids:

	Area of Base	Height		Area of Base	Height
10.	9 sq. in.	5 in.	**13.**	$23\frac{1}{4}$ sq. ft.	8 ft.
11.	13 sq. in.	7 in.	**14.**	16.5 sq. ft.	3.5 ft.
12.	18 sq. in.	$5\frac{1}{4}$ in.	**15.**	29.3 sq. ft.	8.1 ft.

Find the volumes of the following cones:

	Radius of Base	Height		Radius of Base	Height
16.	4 ft.	14 ft.	**18.**	$4\frac{3}{8}$ in.	16 in.
17.	0.7 ft.	2.9 ft.	**19.**	9.1 yd.	18.5 yd.

Find the areas and the volumes of spheres having these radii:

20. 21 ft. **21.** $3\frac{1}{2}$ in. **22.** $10\frac{1}{2}$ yd. **23.** 7.7 ft. **24.** 3.5 ft.

TABLES FOR REFERENCE

Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards = 1 rod (rd.)
5280 feet or 320 rods = 1 mile (mi.)

Square Measure

144 square (sq.) inches = 1 square foot
9 square feet = 1 square yard
$30\frac{1}{4}$ square yards = 1 square rod
43,560 square feet or 160 square rods = 1 acre (A.)
640 acres = 1 square mile

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) or 365 days = 1 year (yr.)
366 days = 1 leap year
100 years = 1 century

Calendar

Thirty days have September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
To which we twenty-eight assign
Till leap year gives it twenty-nine.

Weight

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference

Counting Articles

12 units = 1 dozen (doz.)
12 doz. or 144 units = 1 gross
20 units = 1 score

Counting Paper

24 folded sheets or 25 unfolded sheets = 1 quire
20 quires = 1 ream

Useful Equivalents

1 bushel = about 2150 cu. in. or about 1¼ cu. ft.
1 cubic foot = about 0.8 bu.
1 gallon = 231 cu. in.
1 cubic foot = about 7½ gal.
1 barrel = 31½ gal.
1 barrel flour = 196 lb.

1 ton hard coal = about 35 cu. ft.
1 ton soft coal = about 42 cu. ft.
1 long ton = 2240 lb.
1 cord of wood = 128 cu. ft.
1 cu. ft. water = about 62½ lb.
1 cu. ft. ice = about 57½ lb.
π = 3.1416, or $3\frac{1}{7}$

Weight per Bushel

The legal weights per bushel are not the same throughout the country. The following weights per bushel, however, are accepted in many states.

Apples . . . 48 lb.	Clover seed . . 60 lb.	Peas . . . . 60 lb.
Barley . . . 48 lb.	Corn (shelled) . 56 lb.	Potatoes (Irish) 60 lb.
Beans . . . 60 lb.	Corn (on cob) . 70 lb.	Rye . . . . 56 lb.
Buckwheat . . 48 lb.	Oats . . . . 32 lb.	Wheat . . . 60 lb.

Metric Equivalents

1 meter = 39.37 in. or 1.1 yd.
1 kilometer = 0.6 mi.
1 liter = 1.06 liquid qt. or 0.91 dry qt.
1 kilogram = 2.2 lb.

1 yard = 0.914 meter
1 mile = 1.609 kilometer
1 dry quart = 1.10 liter
1 liquid quart = 0.95 liter
1 pound = 0.454 kilogram

For metric tables of length, capacity, and weight, see pages 489, 492, and 493.

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ANSWERS TO STRAYER–UPTON ARITHMETICS SECOND BOOK

Page 3. — *See page* xxxvii.

Page 8. — **1.** $24.67. **2.** $75.08. **3.** $49.97. **4.** $18.30. **5.** 7755 mi. **6.** $456,699,254. **7.** 1,748,000,000 people.

Page 9. — **1.** 1221 girls. **2.** $44.19. **3.** $3475. **4.** $151.50. **5.** 1,084,024 inhabitants. **6.** 21,484,460 inhabitants less. **7.** 14,360,146 telephones. **8.** 15,443,420 cars. **9.** Decrease; 15,350 farms.

Page 10. — **1.** $392.45. **2.** $335.25. **3.** $607.05. **4.** $2419.20. **5.** $459.34. **6.** $49.00; $68.25; $73.50; $91.00. **7.** $956.25. **8.** 1080 mi. farther. **9.** 689,520 cars.

Page 11. — **1.** $.84. **2.** $.35. **3.** $.08. **4.** $4.94. **5.** 18 mo. **6.** 106 mi. **7.** $.24. **8.** $30.40. **9.** $87.50.

Page 13. — **1.** $\frac{1}{36}$ yd.; $\frac{5}{36}$ yd.; $\frac{1}{4}$ yd. **2.** Yes; 9. **3.** $\frac{3}{4}$ yd. **4.** $\frac{1}{8}$ mi. **13.** $\frac{1}{16}$. **14.** $\frac{6}{12}$. **15.** $\frac{8}{36}$. **16.** $\frac{9}{24}$. **17.** $\frac{15}{60}$. **18.** $\frac{10}{100}$. **19.** $\frac{90}{100}$. **20.** $\frac{60}{100}$. **21.** $\frac{70}{100}$. **22.** $\frac{2}{9}$. **23.** $\frac{3}{4}$. **24.** $\frac{16}{25}$. **25.** $\frac{1}{2}$. **26.** $\frac{1}{2}$. **27.** $\frac{4}{5}$. **28.** $\frac{4}{5}$. **29.** $\frac{13}{20}$. **30.** $\frac{3}{4}$. **31.** $\frac{1}{3}$. **32.** $\frac{3}{5}$. **33.** $\frac{1}{2}$. **34.** $\frac{2}{3}$. **35.** $\frac{5}{6}$. **36.** $\frac{3}{8}$. **37.** $\frac{24}{25}$. **38.** $\frac{5}{6}$. **39.** $\frac{9}{10}$. **40.** $\frac{1}{3}$. **41.** $\frac{2}{13}$. **42.** $\frac{1}{4}$. **43.** $\frac{3}{5}$. **44.** $\frac{49}{50}$. **45.** $\frac{1}{4}$.

Page 14. — **2.** $1\frac{7}{8}$. **3.** $3\frac{1}{3}$. **4.** 12. **5.** $7\frac{2}{5}$. **6.** $1\frac{7}{12}$. **7.** $2\frac{1}{2}$. **8.** $8\frac{1}{3}$. **9.** $4\frac{1}{2}$. **10.** $2\frac{1}{4}$. **11.** $2\frac{1}{12}$. **12.** $7\frac{4}{5}$. **13.** $3\frac{3}{4}$. **14.** 3. **15.** $1\frac{4}{5}$. **16.** $1\frac{5}{8}$. **17.** $3\frac{2}{3}$. **18.** $2\frac{1}{2}$. **19.** $3\frac{1}{4}$. **20.** $8\frac{1}{2}$. **21.** $6\frac{1}{2}$. **22.** $1\frac{3}{16}$. **23.** $1\frac{5}{8}$. **24.** $4\frac{1}{3}$. **25.** $3\frac{1}{4}$.

Page 15. — **2.** $16\frac{7}{8}$ mi.

1. $17\frac{3}{8}$. **2.** $22\frac{1}{8}$. **3.** 18. **4.** $18\frac{3}{8}$. **5.** $19\frac{1}{8}$. **6.** 23. **7.** $24\frac{1}{2}$. **8.** $9\frac{7}{12}$. **9.** $19\frac{5}{8}$. **10.** $16\frac{3}{5}$. **11.** $16\frac{11}{12}$. **12.** $12\frac{9}{16}$. **13.** $26\frac{7}{16}$. **14.** $20\frac{5}{6}$. **15.** $21\frac{4}{5}$. **16.** $18\frac{1}{6}$. **17.** $16\frac{1}{16}$. **18.** $13\frac{11}{16}$.

Page 16. — **2.** $3\frac{5}{12}$ doz. **3.** $4\frac{7}{12}$ doz.

1. $9\frac{19}{24}$. **2.** $17\frac{1}{4}$. **3.** $21\frac{23}{30}$. **4.** $10\frac{7}{24}$. **5.** $20\frac{11}{12}$. **6.** $19\frac{1}{24}$. **7.** $16\frac{7}{12}$. **8.** $16\frac{1}{24}$. **9.** $17\frac{19}{24}$. **10.** $18\frac{13}{18}$. **11.** $12\frac{1}{16}$. **12.** $13\frac{1}{4}$. **13.** 21. **14.** $26\frac{1}{12}$. **15.** $14\frac{3}{4}$. **16.** $22\frac{3}{4}$. **17.** $15\frac{1}{8}$. **18.** $31\frac{1}{8}$.

Page 17. — **1.** $\frac{13}{2}$. **2.** $\frac{14}{3}$. **3.** $\frac{26}{5}$. **4.** $\frac{31}{6}$. **5.** $\frac{59}{6}$. **6.** $\frac{52}{7}$. **7.** $\frac{23}{8}$. **8.** $\frac{43}{5}$. **9.** $\frac{29}{6}$. **10.** $\frac{67}{8}$. **11.** $\frac{20}{3}$. **12.** $\frac{19}{5}$. **13.** $\frac{17}{3}$. **14.** $\frac{113}{12}$. **15.** $\frac{73}{10}$. **16.** $\frac{133}{16}$. **17.** $\frac{27}{10}$. **18.** $\frac{225}{100}$. **19.** 11 roses.

1. 4. **2.** 5. **3.** 6. **4.** 15. **5.** 14. **6.** 2. **7.** 4. **8.** 8. **9.** 2. **10.** 7. **11.** 6. **12.** 6. **13.** 12. **14.** 12. **15.** 14. **16.** 20. **17.** 30.

Page 18. — **2.** $\frac{4}{5}$ sec. **3.** $\frac{1}{2}$ sec.

1. $2\frac{1}{2}$. **2.** $1\frac{3}{5}$. **3.** $3\frac{3}{4}$. **4.** $1\frac{1}{4}$. **5.** $4\frac{1}{8}$. **6.** $3\frac{11}{16}$. **7.** $2\frac{1}{2}$. **8.** $5\frac{1}{2}$. **9.** $1\frac{2}{3}$. **10.** $1\frac{2}{5}$. **11.** $2\frac{1}{3}$. **12.** $2\frac{3}{4}$. **13.** $\frac{3}{8}$. **14.** $4\frac{1}{2}$. **15.** $2\frac{3}{16}$. **16.** $2\frac{1}{2}$. **17.** $5\frac{1}{12}$. **18.** $5\frac{5}{16}$. **19.** $4\frac{7}{8}$. **20.** $3\frac{9}{16}$. **21.** $2\frac{2}{3}$. **22.** $1\frac{7}{12}$. **23.** $5\frac{1}{2}$. **24.** $\frac{4}{5}$. **25.** $1\frac{1}{6}$. **26.** $4\frac{5}{12}$. **27.** $1\frac{1}{15}$. **28.** $6\frac{11}{12}$. **29.** $2\frac{1}{24}$. **30.** $3\frac{1}{12}$.

Page 19. — *See page* xxxvii.

Page 20. — **1.** $6\frac{3}{4}$ mi. **2.** $7\frac{3}{4}$ mi. **3.** $11\frac{7}{8}$ mi. **4.** $68\frac{13}{16}$ lb. **5.** $120\frac{9}{10}$ mi. **6.** $1\frac{7}{12}$ ft. **7.** $9\frac{1}{8}$ doz.

Page 22. — **1.** \$2.09. **2.** 99¢. **3.** \$5.60. **4.** \$1.24. **5.** \$5.76. **6.** \$2.89.

10. \$.08. **11.** \$.54. **12.** \$.27. **13.** \$.32. **14.** \$.18. **15.** \$.23. **16.** \$.74. **17.** \$1.44. **18.** \$.72. **19.** \$2.56. **20.** \$3.10. **21.** \$4.31. **22.** \$2.03. **23.** \$2.53. **24.** \$.80. **25.** \$3.03. **26.** \$4.45. **27.** \$1.84.

Page 23. — **1.** $3\frac{3}{4}$ mi. **14.** $2\frac{1}{2}$. **15.** 6. **16.** 28. **17.** $22\frac{1}{2}$. **18.** $3\frac{1}{2}$. **19.** $2\frac{1}{2}$. **20.** 20. **21.** $31\frac{1}{2}$. **22.** 78. **23.** 200. **24.** 250. **25.** $562\frac{1}{2}$. **26.** $115\frac{1}{2}$. **27.** $569\frac{1}{3}$.

Page 24. — **1.** $\frac{21}{32}$ in. **2.** $3\frac{1}{3}$ cups.

Page 25. — **1.** $\frac{3}{16}$. **2.** $3\frac{1}{9}$. **3.** $8\frac{3}{4}$. **4.** $\frac{9}{32}$. **5.** $10\frac{5}{6}$. **6.** $2\frac{14}{15}$. **7.** $1\frac{1}{5}$. **8.** $7\frac{1}{2}$. **9.** $10\frac{5}{8}$. **10.** $2\frac{4}{5}$. **11.** $32\frac{1}{5}$. **12.** $6\frac{2}{3}$. **13.** $\frac{5}{12}$. **14.** $3\frac{5}{12}$. **15.** $1\frac{9}{10}$. **16.** $\frac{3}{16}$. **17.** $\frac{7}{10}$. **18.** $3\frac{1}{4}$. **19.** $4\frac{1}{6}$. **20.** $2\frac{1}{4}$. **21.** $2\frac{1}{10}$. **22.** $\frac{27}{32}$. **23.** $32\frac{1}{4}$. **24.** $7\frac{14}{15}$. **25.** $\frac{4}{9}$. **26.** $1\frac{2}{3}$. **27.** $\frac{4}{5}$. **28.** $\frac{2}{15}$. **29.** $\frac{2}{5}$. **30.** $\frac{3}{5}$. **31.** $\frac{1}{6}$. **32.** $\frac{1}{6}$. **33.** $\frac{1}{10}$. **34.** $\frac{1}{9}$. **35.** $\frac{1}{6}$. **36.** $\frac{1}{5}$. **37.** 8. **38.** 20. **39.** 9. **40.** 15. **41.** 60. **42.** 77. **43.** $\frac{3}{8}$. **44.** $2\frac{1}{2}$. **45.** $3\frac{1}{2}$. **46.** $\frac{1}{3}$. **47.** $\frac{1}{2}$. **48.** $\frac{1}{3}$. **49.** 18. **50.** 2. **51.** 2. **52.** $8\frac{1}{4}$. **53.** $13\frac{3}{4}$. **54.** $4\frac{1}{2}$.

Page 26. — **4.** $4\frac{1}{2}$ gal. **5.** 4 curtains.

Page 27. — **1.** $1\frac{1}{2}$. **2.** $1\frac{1}{5}$. **3.** $\frac{2}{9}$. **4.** $\frac{3}{16}$. **5.** $\frac{4}{15}$. **6.** $\frac{15}{32}$. **7.** $\frac{3}{8}$. **8.** $\frac{5}{12}$. **9.** $2\frac{1}{10}$. **10.** $4\frac{2}{3}$. **11.** $1\frac{1}{3}$. **12.** $4\frac{1}{5}$. **13.** $\frac{5}{6}$. **14.** $1\frac{1}{4}$. **15.** $1\frac{1}{10}$. **16.** $\frac{2}{15}$. **17.** $\frac{15}{32}$. **18.** $\frac{3}{10}$. **19.** $\frac{2}{15}$. **20.** $2\frac{1}{4}$. **21.** $\frac{2}{15}$. **22.** $3\frac{9}{10}$. **23.** $\frac{3}{16}$. **24.** $\frac{7}{10}$. **25.** $\frac{5}{64}$. **26.** $\frac{3}{20}$. **27.** $\frac{9}{16}$. **28.** $1\frac{1}{2}$. **29.** $2\frac{1}{2}$. **30.** $\frac{3}{16}$. **31.** $\frac{4}{9}$. **32.** $1\frac{2}{3}$. **33.** $\frac{2}{3}$. **34.** $\frac{3}{10}$. **35.** $\frac{3}{5}$. **36.** $1\frac{1}{5}$. **37.** $\frac{1}{8}$. **38.** $\frac{1}{15}$. **39.** $\frac{1}{16}$. **40.** $\frac{1}{12}$. **41.** $\frac{1}{12}$. **42.** $\frac{1}{10}$. **43.** $\frac{1}{12}$. **44.** $\frac{1}{8}$. **45.** $\frac{1}{10}$. **46.** 14. **47.** 6. **48.** 72. **49.** $\frac{2}{3}$. **50.** $1\frac{1}{3}$. **51.** $\frac{2}{3}$. **52.** $\frac{1}{2}$. **53.** $\frac{1}{2}$. **54.** $\frac{1}{5}$. **55.** 4. **56.** 3. **57.** 20. **58.** $\frac{3}{8}$. **59.** $1\frac{5}{7}$. **60.** $\frac{9}{10}$.

Page 32. — **1.** 87.5. **2.** \$2.00. **3.** Correct. **4.** 8.3. **5.** 4.9. **6.** \$.25. **7.** Correct. **8.** \$.48. **9.** 21.6. **10.** \$18.75, \$1.39. **11.** 58.5. **12.** 23. **13.** 9.3. **14.** \$1235. **15.** 92. **16.** \$.25. **17.** 48. **18.** \$1.00. **19.** \$.65. **20.** \$.85.

Page 33. — **7.** .210 and .209. **8.** .850 and .868. **9.** .81 and .90. **10.** .640 and .037. **11.** .020 and .018. **12.** .0500 and .3185.

Page 34. — **1.** .4. **2.** Both have the same value. **3.** .82. **4.** .1. **5.** .900. **6.** .45. **7.** .273. **8.** .02. **9.** .28. **10.** .030. **11.** .003. **12.** .201. **13.** James. **14.** More. **15.** .820, .800, .791, .780, .700, .610, .295.

Page 36. — **1.** 21.22. **2.** 22.926. **3.** 20.0705. **4.** 12.67926. **5.** 31.27. **6.** 33.213. **7.** 9.7067. **8.** 16.87649. **9.** 19.76. **10.** 26.028. **11.** 21.116. **12.** 21.50423. **13.** 187.823. **14.** 52.776. **15.** 87.4. **16.** 50.018. **17.** 25.4005. **18.** 57.7355. **19.** 105.5855. **20.** 177.2588.

Page 37. — **1.** 67.9 mi. **2.** 349.71 ft. **3.** 340.3 lb. **4.** 62.147 ft. **5.** 15.2 sec. **6.** .0498 in.

Page 38. — **2.** 1.4 sec. **3.** 189.9 mi. **4.** $.1335. **5.** 1.677 ft. **6.** 2.55 in. **7.** 49.9 mi.; 8.3 mi.; 12.8 mi.; 15.5 mi.; 5.7 mi.

Page 39. — **1.** .583. **2.** 1.785. **3.** .236. **4.** .6994. **5.** 765.5. **6.** 1.039. **7.** .125. **8.** 3.245. **9.** .4985. **10.** .1840. **11.** 1.005. **12.** 2.693. **13.** 2.859. **14.** 4.834. **15.** .2544. **16.** .0760. **17.** 2.008. **18.** 7.786. **19.** 146.9. **20.** .1422. **21.** 5.495. **22.** 1.6902. **23.** 1.2771. **24.** 8.4369. **25.** 8.7338. **26.** .00048. **27.** 8.166. **28.** .57736. **29.** 1.73. **30.** 4.58. **31.** 7.6025. **32.** 3.0124. **33.** 1.349. **34.** .833. **35.** .7595. **36.** .3857. **37.** 37.572. **38.** 34.938. **39.** 49.0146. **40.** 89.6725. **41.** 62.738.

Page 41. — *See page* xxxvii.

Page 44. — **1.** 597.24 lb.; $32.85. **2.** 729.68 lb.; $73.70. **3.** $52.54. **4.** $40.85; $19.69.

1. $1.33. **2.** $1.59. **3.** 227.3 in.

Page 45. — **1.** 104.88. **2.** 248.14. **3.** 6.976. **4.** 1.596. **5.** 70.704. **6.** 3.927. **7.** 48.762. **8.** .464. **9.** 4.914. **10.** .696. **11.** .7. **12.** .3444. **13.** .119. **14.** .237. **15.** .23. **16.** .342. **17.** .0768. **18.** .0024. **19.** .0144. **20.** .0071. **21.** .0006. **22.** .00728. **23.** .05463. **24.** .01833. **25.** .000042. **26.** .04256. **27.** .000045. **28.** .05922. **29.** 9. **30.** 432. **31.** 348. **32.** 29,000. **33.** .261. **34.** .2118. **35.** .0567. **36.** .2135.

Page 46. — **1.** 64.17 ft. **2.** 203.21 ft. **3.** 115.86 ft. **4.** 591.79 ft.; 730.83 ft.; 707.65 ft. **5.** 620.31 ft.; 283.48 ft.; 415.21 ft.; 123.40 ft. **6.** 958.8 ft.

Page 47. — **1.** 8.4. **2.** .63. **3.** .246. **4.** 38.86. **5.** .966. **6.** .2109. **7.** .1778. **8.** .0600. **9.** 2.94. **10.** 1.533. **11.** 42.03. **12.** 4.336. **13.** .32. **14.** .63. **15.** .115. **16.** .528. **17.** .08. **18.** .0054. **19.** .0862. **20.** .00147. **21.** .012. **22.** .0650. **23.** .0704. **24.** .0999. **25.** 54.00. **26.** 42.000. **27.** 22.800. **28.** 196.00. **29.** 24.030. **30.** .01040. **31.** .56400. **32.** 5.06700.

Page 48. — **6.** $6.50. **7.** $12.50. **8.** $15.

Page 49. — **6.** $2.70. **7.** $1.25.

Page 50. — **2.** $5.13. **3.** $2.33. **4.** $1.74. **5.** 2.16 cwt.; 8.69 cwt.; 42.6 cwt.; 10.96 cwt.; .86 cwt. **6.** $58.86. **7.** $60.31. **8.** $16.32.

Page 51. — *See page* xxxvii.

Page 53. — **1.** 3.26. **2.** 35.4. **3.** 1.79. **4.** 4.57. **5.** 52.6. **6.** 3.69. **7.** 3.59. **8.** 51.7. **9.** 7.3. **10.** 7.75. **11.** 8.25. **12.** 6.25. **13.** 4.75. **14.** 3.72. **15.** .625. **16.** .875. **17.** .875. **18.** .375. **19.** .875. **20.** .625. **21.** 95.5. **22.** 72.5. **23.** 92.5. **24.** 74.5. **25.** 9.58. **26.** 9.55. **27.** 2.55. **28.** 9.7. **29.** 8.3. **30.** 9.4. **31.** 6.1. **32.** 9.2. **33.** 7.3. **34.** 4.8. **35.** 4.2. **36.** 4.9. **37.** 8.4. **38.** 9.9. **39.** 3.9. **40.** 4.9. **41.** 2.9. **42.** 9.1. **43.** 3.4. **44.** 6.4. **45.** 6.0. **46.** 5.41. **47.** 1.91. **48.** 2.33. **49.** 3.59. **50.** 7.86. **51.** 4.84. **52.** 7.08. **53.** 5.39. **54.** 2.41. **55.** 3.70. **56.** 4.19. **57.** 4.78.

Page 54. — **1.** 11.5 lb. **2.** 18.9 lb. **3.** $.27. **4.** $.23. **5.** $1.25. **6.** $1.36. **7.** .7 mi. **8.** 6.1 mi. **9.** 6.4 mi. **10.** $.25. **11.** 5.4 hr.

Page 55. — **1.** .25; .2; .35; .4; .34; .15; .32. **2.** .625; .75; .92; .6; .12; .95; .98. **3.** .36; .125; .344; .875; .452; .1875; .3125. **4.** 7.75, or $7\frac{3}{4}$. **5.** 8.875, or $8\frac{7}{8}$. **6.** 5.9, or $5\frac{9}{10}$. **7.** 39.625, or $39\frac{5}{8}$. **8.** 16.375, or $16\frac{3}{8}$.

Page 56. — **1.** .67; .17; .44; .90; .45; .78; .15. **2.** .57; .64; .14; .82; .11; .86; .12. **3.** .42; .58; .43; .08; .87; .56; .05. **4.** .714; .533; .644; .139; .391; .944; .426. **5.** .746; .824; .512; .411; .753; .373; .162. **6.** .513; .920; .519; .333; .156; .459; .012.

Page 57. — **1.** .17 mi. **2.** .15 mi. **3.** .17 yr. **4.** .28 mi. **5.** .81 mi. **6.** .11 T. **7.** .54 bu. **8.** .29.

Page 58. — **1.** .075. **2.** .0925. **3.** .2875. **4.** 2.25. **5.** 3.775. **6.** 9.0675. **7.** 1.8333$\frac{1}{3}$, or 1.8333. **8.** 2.5025. **9.** 7.365. **10.** 9.2166$\frac{2}{3}$, or 9.2167. **11.** 4.0025. **12.** 19.425. **13.** 2.5833$\frac{1}{3}$, or 2.5833. **14.** 1.2633$\frac{1}{3}$, or 1.2633. **15.** 9.0175. **16.** .11625. **17.** .1055. **18.** 1.005. **19.** 4.6666$\frac{2}{3}$, or 4.6667. **20.** .10666$\frac{2}{3}$, or .10667. **21.** $5.69.

Page 59. — **2.** 11. **3.** 45. **4.** 13. **5.** 15. **6.** 83. **7.** 15. **8.** 34. **9.** 161.

Page 60. — **1.** 86.4. **2.** 75.1. **3.** 62.8. **4.** 35.3. **5.** 185. **6.** 467. **7.** 378. **8.** 467.

Page 61. — **1.** 5.5 mi. **2.** $2.50. **3.** .18 in. **4.** 17 wk. **5.** 16 gal. **6.** 3.5 mi. **7.** 34.1. **8.** 3.27. **9.** 80.9. **10.** 70.2. **11.** 5.41. **12.** 56.9. **13.** 5.67. **14.** 52.7. **15.** 43.8. **16.** 40.9. **17.** 83.7. **18.** 74.7. **19.** 4.66. **20.** 72.9. **21.** 48.6. **22.** 57.3. **23.** 43.6. **24.** 56.9. **25.** 91.4. **26.** 50.2. **27.** 82.3. **28.** 61.7. **29.** 9.04. **30.** 3.21.

Page 62. — **1.** 9.025 ft. **2.** 4.4 in.; 3.6 ft. long, 2.6 ft. wide. **3.** 6.3 ft.; 4.8 ft. long, 3.4 ft. wide.

Page 63. — **4.** 9.2 in. **5.** No; 15.2 ft. **6.** The flags 6 ft. × 11.4 ft. and 10 ft. × 19 ft. **7.** The flag 5 ft. × 9.5 ft. **8.** 37.1 ft. long, 19.5 ft. wide. **9.** (*a*) 114 ft.; (*b*) 45.6 ft. long, 32.3 ft. wide; (*c*) 4.6 ft. **11.** 41.8 in.; 1.7 in.; 16.7 in. long, 11.8 in. wide.

Page 64. — **1.** 13 da. **2.** 43 da. **3.** 30 da. **4.** 324 gal. **5.** $786. **6.** $2.39½. **7.** $3.25. **8.** $.85½.

Page 65. — **1.** 94.2 mi. **2.** 121.1 mi. **3.** 154.5 mi.

Page 66. — **1.** 1.2. **2.** 11. **3.** .6. **4.** 5.9. **5.** .057. **6.** .083. **7.** 90.7. **8.** 84. **9.** .037. **10.** 690. **11.** 670. **12.** 490. **13.** 35.9. **14.** 12.4. **15.** 357. **16.** 42.8. **17.** 8.26. **18.** 461. **19.** 528. **20.** 368. **21.** 258. **22.** .0591. **23.** .0812. **24.** .0254. **25.** .0841. **26.** .0543. **27.** .0513. **28.** 78,600. **29.** 31,900. **30.** 4380. **31.** 6720. **32.** 6140. **33.** 3870. **34.** 87.67. **35.** 5.32. **36.** 2.35. **37.** 5.87. **38.** 4.93. **39.** 6.58. **40.** 7.61. **41.** 89.04. **42.** 51.82. **43.** 842.31. **44.** 324.57. **45.** 6.83.

Page 67. — **1.** 1.2. **2.** .3. **3.** 3.3. **4.** 2.6. **5.** .21. **6.** .023. **7.** 8760. **8.** 37. **9.** 93. **10.** 100. **11.** 54. **12.** 6400. **13.** 23. **14.** .041. **15.** 537. **16.** .0063. **17.** .058. **18.** 12,700. **19.** 2.45. **20.** 245. **21.** .245. **22.** .0245. **23.** 2450. **24.** .00245. **25.** 24,500. **26.** 245. **27.** 2450.

Page 68. — **1.** Wrong; correct answer is .1. **2.** Right. **3.** Wrong; correct answer is 2. **4.** Wrong; correct answer is 3.7. **5.** Wrong; correct answer is .0253. **6.** Wrong; correct answer is .0736. **7.** Wrong; correct answer is 283. **8.** Wrong; correct answer is 45.3. **9.** Wrong; correct answer is 9310. **10.** Right. **11.** Wrong; correct answer is 523. **12.** Wrong; correct answer is 243.

Page 69. — *See page* xxxviii.

Page 70. — **1.** New York to Albany, 151.7 mi.; Albany to Utica, 93.2 mi.; Utica to Syracuse, 51.1 mi.; Syracuse to Geneva, 44.4 mi.; Geneva to Buffalo, 111.6 mi.; Buffalo to Niagara Falls, 26.8 mi. **2.** The distance from Niagara Falls to each of the following cities is: Watkins, 168.2 mi.; Owego, 226.3 mi.; Binghamton, 247.1 mi.; Monticello, 347 mi.; Ramsey, 416.3 mi.; New York, 437.2 mi.

Page 71. — **3.** Niagara Falls to Lockport, 23.0 mi.; Lockport to Middleport, 14.2 mi.; Middleport to Medina, 5.3 mi.; Medina to Albion, 10.9 mi.; Albion to Brockport, 15.1 mi.; Brockport to Rochester, 19.7 mi.; Rochester to Watkins, 80.0 mi.; Watkins to Owego, 58.1 mi.; Owego to Binghamton, 20.8 mi.; Binghamton to Monticello, 99.9 mi.; Monticello to Ramsey, 69.3 mi.; Ramsey to New York, 20.9 mi. **4.** 916 mi. **5.** 41.6 mi. **6.** 9 gal. **7.** 20.8 mi. **8.** 1029.1 mi.; 113.1 mi. **9.** 1.4¢. **10.** $14.41; $37.05; $22.64. **11.** 20.5 mi.

Page 72. — **1.** 14:9 ft. **2.** $\frac{7}{20}$. **3.** $\frac{1}{8}$. **4.** $2\frac{1}{4}$. **5.** $4\frac{5}{8}$. **6.** $\frac{9}{40}$. **7.** .25. **8.** $.83\frac{1}{3}$. **9.** $.37\frac{1}{2}$. **10.** $.87\frac{1}{2}$. **11.** $.66\frac{2}{3}$. **12.** .399. **13.** 8. **14.** 1950. **15.** 46.364. **16.** 64,080. **17.** 250. **18.** .245. **19.** 1520. **20.** 26.136. **21.** 4050. **22.** 1.692. **23.** 90. **24.** 60; 6; .6. **25.** $5; $.50; $.05. **26.** 3.8; .38; .038. **27.** .94; .094; .0094. **28.** .07; .007; .0007. **29.** 1.2. **30.** 7. **31.** .047. **32.** 38.2. **33.** 509. **34.** 953. **35.** .0324. **36.** 5920. **37.** 3850. **38.** 7.83. **39.** 2.48. **40.** 6.95. **41.** 7.18. **42.** 10.86. **43.** 53.29. **44.** 59.29. **45.** 96.5. **46.** 4.73.

Page 73. — **1.** $15\frac{1}{8}$. **2.** $14\frac{1}{2}$. **3.** $17\frac{7}{8}$. **4.** $7\frac{1}{3}$. **5.** $18\frac{1}{10}$. **6.** $20\frac{1}{6}$. **7.** $26\frac{1}{16}$. **8.** $7\frac{1}{4}$. **9.** $13\frac{2}{5}$. **10.** $35\frac{1}{5}$. **11.** $12\frac{5}{8}$. **12.** $16\frac{7}{12}$. **13.** $5\frac{3}{8}$. **14.** $\frac{1}{4}$. **15.** $3\frac{1}{4}$. **16.** $\frac{3}{8}$. **17.** $4\frac{1}{4}$. **18.** $2\frac{1}{8}$. **19.** $2\frac{5}{8}$. **20.** $1\frac{9}{10}$. **21.** $2\frac{2}{3}$. **22.** $3\frac{5}{12}$. **23.** $3\frac{4}{5}$. **24.** $6\frac{15}{16}$. **25.** $\frac{5}{16}$. **26.** $\frac{7}{20}$. **27.** $13\frac{1}{2}$. **28.** $35\frac{1}{4}$. **29.** $5\frac{1}{4}$. **30.** $1\frac{5}{8}$. **31.** 20. **32.** $208\frac{1}{3}$. **33.** $3.38. **34.** $8.70. **35.** $2.63. **36.** $9.52. **37.** $\frac{1}{4}$. **38.** $\frac{1}{8}$. **39.** $1\frac{3}{4}$. **40.** $1\frac{1}{6}$. **41.** $\frac{15}{16}$. **42.** 112. **43.** $2\frac{1}{2}$. **44.** $4\frac{17}{22}$. **45.** $\frac{3}{4}$. **46.** $\frac{3}{4}$. **47.** 3. **48.** $1\frac{1}{4}$.

Page 76. — **1.** $5.53. **2.** $15.25. **3.** $1.66. **4.** $7.31. **5.** $24.88. **6.** $26.60. **7.** $24.24. **8.** $13.18. **9.** $73.55. **10.** $3.20. **11.** $3.90. **12.** $8.11.

Page 77. — **1.** $16.85. **2.** $112.78.

Page 79. — *See page* xxxviii.

Page 80. — **1.** 34¢. **2.** 14¢.

Page 81. — **3.** $18.41. **4.** 12¢; 76¢. **5.** $1.14. **6.** 48¢. **7.** $2.05. **8.** 50¢. **9.** $1.22. **10.** $1.23. **11.** 22¢. **12.** 49¢.

Page 83. — 1. $1.20. 2. Second plan is $5 cheaper. 3. $16.05. 4. 28 da. 5. $12.90.

Page 86. — **1.** $3.00. 2. How much do they give in all; $2.35. **3.** How much did it cost in all; $2.87. **4.** Pat is how much older than Kathleen; 7 yr. **5.** How many pages has she still to read; 88 pages. **6.** How much did the use of the tent cost them; $6.45. **7.** How much change did Jim give Mr. Dean; $1.18. **8.** How long did it take her to ride 1 mile; 5 min. **9.** How many miles is it from Memphis to St. Louis; 316 mi. **10.** How much had he left; 34¢.

Page 87. — **1.** $6.21. **2.** 19 mi. **3.** $342. **4.** $5\frac{1}{2}$ da. **5.** $8.75. **6.** $5.05. **7.** 1 hr. **8.** $1.15. **9.** $96.40.

Page 88. — **1.** 2 da. **2.** 67,500 T. **3.** $37\frac{1}{2}$ yr. **4.** 1200 yr. **5.** 800 men.

Page 90. — **35.** 93. **36.** 12. **37.** 17. **38.** 9. **39.** 3. **40.** 24.

Page 91. — **1.** .85; 34 words. **2.** 13 examples. **3.** Alice, 49 words; George, 43 words; Fred, 37 words; Tony, 34 words; Marion, 46 words; Julia, 47 words. **4.** 19 games. **5.** 63 children. **6.** $150; $180. **7.** $.12. **8.** 3553 children.

Page 92. — **9.** $.25 a day. **10.** 89%. **11.** $\frac{35}{100}$; 35%. **12.** $\frac{7}{100}$; 7%. **13.** 87%.

Page 93. — **1.** 14. **2.** 28. **3.** 39. **4.** 34. **5.** 42. **6.** 17. **7.** 18. **8.** 20. **9.** 12. **10.** 9. **11.** 66. **12.** $15. **13.** $34. **14.** $18. **15.** $24.50. **16.** $14.40. **17.** $22.62. **18.** $17.82. **19.** $269.80. **20.** $277.14. **21.** $41.28. **22.** $253.30. **23.** 57. **24.** 238. **25.** 216. **26.** 595. **27.** 1122. **28.** 6355. **29.** 623. **30.** 935. **31.** 1665. **32.** 1977. **33.** 14,706. **34.** 23,782 children. **36.** $2.59. **37.** $4.35. **38.** $22.03. **39.** $118.27. **40.** $309.57. **41.** $163.09. **42.** $55.08. **43.** $217.25.

Page 94. — **1.** 8 problems. **2.** 75%. **3.** $600.

Page 95. — **1.** 40 horse power. **2.** 25%. **3.** 45 words. **5.** 16. **6.** 40. **7.** 57. **8.** 12. **9.** 30. **10.** $.41. **11.** $1.30. **12.** $6.36. **13.** $2.50. **14.** $.68. **15.** $2.70. **16.** $1.44. **17.** $3.42. **18.** $.31. **19.** $.68.

Page 96. — **3.** $\frac{1}{10}$. **4.** $3. **5.** 80%. **6.** 7. **7.** 9. **8.** 51. **9.** 18. **10.** 3. **11.** 2. **12.** 19. **13.** 168. **14.** 104. **15.** 160. **16.** 174. **17.** 24. **18.** 55. **19.** 468. **20.** $.13. **21.** $1.97. **22.** $.06. **23.** $3.40. **24.** $1.34. **25.** $.04. **26.** $.60.

Page 97. — **1.** 66⅔%. **2.** $18.50.

Page 98. — **1.** ⅓; 3,590,000 bu. **2.** $54,750,000. **3.** 75,000,000 lb. **4.** 1,833,333⅓ bales.

1. 24; 48; 12. **2.** 32; 64; 16. **3.** 28; 56; 14. **4.** 78; 156; 39. **5.** 94; 188; 47. **6.** 52; 104; 26. **7.** 148; 296; 74. **8.** 90; 180; 45. **9.** 172; 344; 86. **10.** $1.14; $2.28; $.57. **11.** $.76; $1.52; $.38. **12.** $1.66; $3.32; $.83. **13.** $.96; $1.92; $.48. **14.** $1.45; $2.90; $.73. **15.** $2.09; $4.18; $1.05. **16.** 8; 24; 40; 56. **17.** 12; 36; 60; 84. **18.** 7; 21; 35; 49. **19.** 24; 72; 120; 168. **20.** 39; 117; 195; 273. **21.** 21; 63; 105; 147. **22.** 43; 129; 215; 301. **23.** 105; 315; 525; 735. **24.** 76; 228; 380; 532. **25.** $.58; $1.74; $2.90; $4.06. **26.** $.67; $2.01; $3.35; $4.69. **27.** $.29; $.87; $1.45; $2.03. **28.** $.78; $2.34; $3.90; $5.46. **29.** $1.25; $3.75; $6.25; $8.75. **30.** $1.99; $5.97; $9.95; $13.93. **31.** $5.29. **32.** $2.84. **33.** $10.34. **34.** $192.60. **35.** $675. **36.** $1361.07.

Page 99. — *See page* xxxviii

Page 100. — **1.** 25% of $400; less. **2.** About 25 words. **3.** About 90 boys. **4.** 100; 102. **5.** 30; 28.5. **6.** 60; 61.2. **7.** 660; 651.2. **8.** 450; 445. **9.** 85; 76.5. **10.** 198. **11.** 100; 104. **12.** 100; 102.5. **13.** 75; 72. **14.** $.96; $.97. **15.** $.94; $1.03. **16.** $1.00; $1.08. **17.** $1.13; $1.17. **18.** $.30; $.29. **19.** 624 children.

Page 102. — **2.** 25%. **3.** 80%. **4.** 35%. **5.** 10%. **6.** 25%. **7.** 80%. **8.** 90%. **9.** 30%. **10.** 64%. **11.** 14%.

Page 103. — **4.** $5.10; $20.40. **5.** $17.50; $73.50; $119.

Page 104. — **1.** $14; $336. **2.** $160; $7840. **3.** $14.56. **4.** $26.70. **6.** $380; $570. **7.** $127.75; $237.25. **8.** $38; $342. **9.** $35.40; $141.60. **10.** $72.75; $412.25. **11.** $112.50; $262.50. **12.** $160; $480. **13.** $29.67; $148.33. **14.** $83.33; $166.67. **15.** $95; $380. **16.** $52.50; $297.50. **17.** $20; $140. **18.** $34; $306. **19.** $109.35; $133.65. **20.** $108.33; $541.67. **21.** $60; $420. **22.** $203; $522. **23.** $35.85; $1159.15. **24.** $146; $3504. **25.** $249; $4731. **26.** $84.60; $4145.40.

Page 105. — **1.** Bed, $11.70; desk, $17.14; bookcase, $10.50; chiffonier, $13.50; table, $3.56; stool, $1.27. **2.** Bed, $19.13; chair, $4.25; table, $2.76; bureau, $15.64; lamp, $6.80; pillow, $1.96. **3.** China closet, $23.33; table, $24.50; chair, $8.40; couch, $19.60. **4.** China closet, $21; table, $22.05; chair, $7.56; couch, $17.64.

Page 106. —**1.** $7.23. **2.** $113.75. **3.** $2.30.
5. $12.75; $15. **6.** The Ajax. **7.** $.19. **8.** $.75.
9. $3.40. **10.** $2.58. **11.** $1.88. **12.** $1.61. **13.** $1.44.
14. $3.29. **15.** $3.09. **16.** $.49. **17.** $2.89. **18.** $2.24.
19. $2.35.

Page 107. —**1.** $12. **2.** $17.50. **3.** $32. **4.** $10.50.
5. $45. **6.** $17.50. **7.** $1.40. **8.** $4. **9.** $4.50.
10. $6.30. **11.** $3.30. **12.** $10.50. **13.** $87. **14.** $130.
15. $30. **16.** $50. **17.** $1.30. **18.** $1.93. **19.** $2.50. **20.** $1.

Page 108. —**2.** $.70. **3.** $18. **4.** $4.50. **5.** $8.40.
6. $14. **7.** $28.50. **8.** $1.30. **9.** $.60. **10.** $3.
11. $3.15. **12.** $.84. **13.** $.30. **14.** $2.25. **15.** $1.25.

Page 109. —**2.** $87.50. **3.** $225. **4.** $210. **5.** $20.
6. $62.50. **7.** $93.60. **8.** $27. **9.** $252. **10.** $91.
11. $68. **12.** $235.20. **13.** $450. **14.** $577.50.
15. $1255. **16.** $82.50. **17.** $105. **18.** $17.50.
19. $264.60. **20.** $83.25. **21.** $164.15. **22.** $30.63.
23. $163.35. **24.** $98.55. **25.** $57.58. **26.** $95.87.

Page 110. —**2.** $12; $2; $10; $6; $16; $18. **4.** $60; $1560.
5. $4. **6.** $5.25. **7.** $26.25. **8.** $12.83. **9.** $29.40.
10. $30.67. **11.** $4.38. **12.** $6. **13.** $2.50. **14.** $8.90.
15. $11.25. **16.** $9.50. **17.** $16.33. **18.** $12.50.
19. $50. **20.** $22.50. **21.** $57.75.

Page 111. —*See page* xxxviii.

Page 112. —**1.** $545. **2.** $152.40. **3.** $278.70.
4. $343.38. **5.** $212. **6.** $1531.48. **7.** 7.6¢. **8.** 14.7 mi.

Page 113. —**1.** 9 ft.; 33 ft.; 1320 ft.; $8\frac{1}{3}$ ft.; $\frac{1}{2}$ ft.
2. 33 yd.; 880 yd.; 32 yd. **3.** 1760 steps. **4.** 20 times.
5. 31,680 stamps. **6.** .11 mi. **7.** 5.94 mi.

Page 114. —**1.** 12 qt.; 16 qt.; $3\frac{1}{2}$ qt.; 40 qt.; 2 qt.; $\frac{1}{2}$ qt.
2. $\frac{1}{2}$ lb.; 500 lb.; 150 lb.; $1\frac{1}{2}$ lb.; 10,000 lb.; 1000 lb. **3.** $3.20; $.70. **4.** 32 da. **5.** $13\frac{1}{5}$ T.

Page 115. —**2.** 26 qt.; 5 qt.; 3 qt.; $\frac{1}{2}$ qt.; 6 qt.; 80 qt.
3. $7\frac{1}{2}$ gal.; $2\frac{1}{4}$ gal. **4.** $40. **5.** $5\frac{1}{2}$ gal.

2. 115,200 times; 806,400 times; 41,932,800 times.

Page 116. —**1.** 1 yd. 18 in. **2.** 2 hr. 15 min. **3.** 62 in.
4. 15 gal. 2 qt. **5.** 23. **6.** 66. **7.** 19. **8.** 23. **9.** 127.
10. 5; 10. **11.** 2; 8. **12.** 2; 3. **13.** 1; 9. **14.** 2; 660.

Page 117. —**1.** 8 ft. 2 in. **2.** 17 ft. 11 in. **3.** 27 ft. 11 in.
4. 27 yd. 1 ft. **5.** 14 ft. 8 in. **6.** 32 ft. 4 in. **7.** 22 ft. 5 in.
8. 27 yd. 2 ft. **9.** 19 yd. 6 in.

Page **118.** — **1.** John, 6 hr. 10 min.; Ellen, 8 hr. 15 min.; Nancy, 7 hr. 40 min. **2.** 10 gal. 2 qt. **3.** 13 lb. 14 oz. **4.** 9 hr. 43 min. **5.** 20 hr. 30 min. **6.** 12 bu. 3 pk. **7.** 21 lb.

Page **119.** — **1.** Clyde, 2½ in.; Henry, 7 in.; Elmer, 10 in.; Will, 5 in.; Hugh, 8 in.; Dick, 11 in. **2.** 1 ft. 4 in. **3.** 63 lb. 11 oz.; 61 lb. 13 oz. **5.** 3 ft. 7 in. **6.** 1 yd. 2 ft. **7.** 2 lb. 11 oz. **8.** 1 hr. 45 min.

Page 120. — **1.** 4′ 4″; 7′; 9 sec. **2.** 8 in.; 8 in.; 8 in.; 10 in.

Page 121. — **1.** 11 yr., 1′ 9″; 12 yr., 2′ 3″; 13 yr., 2′ 10″; 14 yr., 3′. **2.** 11 yr., ⅜ sec.; 12 yr., ⅘ sec.; 13 yr., ⅘ sec.; 14 yr., ⅘ sec. **3.** Gene, 10″ above; Will, 1′ 10″ below; Bert, 1′ 3″ below; Alice, 1′ 4″ above; Fanny, 5″ below; Julia, 8″ above. **4.** Will, 5″; Bert, 4″; Alice, 9″; Fanny, 2′ 4″; Julia, 1′ 11″. **5.** ½ sec. **6.** 7″ above; 1″ below. **8.** 8′ 3″. **9.** 13′ 9″. **10.** ⅖ sec. **11.** 3′ 4″ below.

Page **122.** — **2.** 3 lb. 6 oz. **3.** 3 lb. 6 oz. **4.** 5 lb. 6 oz. **5.** 6 lb. 4 oz.

1. 12 mi. 107 rd. 1 yd. 2 ft. **2.** 29 ft. 9 in. **3.** 32 gal. 2 qt. **4.** 16 hr. 30 min. **5.** 67 yd. 2 ft. **6.** 22 lb. 2 oz. **7.** 24 ft. **8.** 30 pk. **9.** 11 yd. 4 in. **10.** 9 ft. 8 in.

Page 123. — **2.** 2 ft. 1 in. **3.** 1 ft. 4 in.

1. 1 hr. 15 min. **2.** 1 hr. 55 min. **3.** 2 lb. 5 oz. **4.** 5 da. 23 hr. **5.** 4 ft. 10 in.

Page 124. — **1.** 4 times. **2.** 21½ ft. **3.** 120 yd. **4.** 3⅓ yd. **5.** Fred, 120 ft.; Jane, 136 ft.; Will, 130 ft.; Mary, 174 ft.; Dick, 200 ft.; Dora, 150 ft.; Fred's.

Page 125. — **1.** Hilda. **2.** 9 plants; 7 plants; 11 plants; 13 plants. **3.** 13 bulbs; 19 bulbs; 25 bulbs; 10 bulbs. **4.** 20 posts. **5.** 13 plants. **6.** 9 plants. **7.** 9 plants.

Page 126. — **1.** 24 sq. ft. **2.** 21 sq. yd. **3.** 20 sq. ft. **4.** 60 sq. in. **5.** 57 sq. in. **6.** 42¼ sq. in. **7.** 68 sq. mi. **8.** 3645 sq. ft. **9.** 180 sq. yd.

Page 127. — **1.** The scarf 44 in. long and 10 in. wide. **2.** 18 stars. **3.** 24 squares. **4.** $5600. **5.** 2½ gal. **6.** $144. **7.** 66.79 times. **8.** 1 sq. ft. **9.** 21 sq. ft. **10.** 48 sq. in. **11.** ⅜ sq. yd. **12.** 14⅛ sq. ft. **13.** 11⅞ sq. ft. **14.** 14 sq. ft. **15.** 62 sq. ft. **16.** 3 sq. ft.

Page 128. — **1.** 75 sq. ft.; 8⅓ sq. yd.; $16.25. **2.** 900 sq. yd.; $90. **3.** $37. **4.** 125 lb. **5.** 800. **6.** 60½. **7.** 58.5. **8.** 216. **9.** 26,136. **10.** 4½. **11.** $\frac{1}{20}$. **12.** ⅛. **13.** 2. **14.** 40.

Page 129. — *See pages* xxxviii *and* xxxix.

Page 130. — **2.** 320 cu. in. **3.** 224 cu. ft. **4.** 135 cu. yd. **5.** 84 cu. ft. **6.** 100 cu. ft. **7.** 472½ cu. ft. **8.** 147 cu. ft. **9.** 231 cu. ft.

Page 131. — **1.** 11,040 cu. ft. **2.** $48.75. **3.** 12,600 cu. ft.; 63 pupils. **4.** 8 ft. 4 in. **5.** 6 ft. 8 in. **6.** 10 ft. 5 in.

Page 132. — **1.** 384 bu. **2.** 84 bu. **3.** 384 bu. **4.** 200 bu. **5.** 252 bu. **6.** 290 bu. **7.** 500 bu. **8.** 84 bu. **9.** 9 T. **10.** Box (*b*).

Page 133. — **1.** 16 gal. **2.** 2 gal. **3.** A little more than a pint. **4.** 7.48 gal. **5.** 28,350 gal. **6.** 22,050 gal. **7.** 240 cu. ft.; $.67.

Page 134. — **2.** $14\frac{7}{12}$ ft.; $8\frac{1}{8}$ ft. **3.** $6\frac{7}{8}$ ft.; $9\frac{3}{8}$ ft. **4.** $3\frac{1}{2}$ ft.; $9\frac{1}{2}$ ft. **5.** $5\frac{1}{4}$ in.; $8\frac{3}{4}$ in. **6.** 4 ft.; 6 ft. **7.** $9\frac{9}{16}$ ft.; $12\frac{3}{8}$ ft. **8.** 5 in.; 1 ft.

Page 135. — *See page* xxxix.

Page 136. — **1.** 108.9 mi. **2.** 8.4 mi. **3.** 26.1 gal. **4.** No.

Page 137. — **5.** 151.2 mi. **6.** 5294 lb. **7.** 12.9 gal. **9.** 900 lb. **10.** 6 passengers. **11.** 2726.4 lb.

Page 138. — **1.** 90. **2.** 14,520. **3.** $\frac{1}{5}$. **4.** $4\frac{1}{2}$. **5.** $\frac{1}{12}$. **6.** 42. **7.** $1\frac{1}{2}$. **8.** 42. **9.** $9\frac{1}{4}$. **10.** 39. **11.** $\frac{3}{4}$. **12.** 1600. **13.** 54. **14.** $506\frac{1}{4}$. **15.** $\frac{3}{4}$. **16.** $16\frac{2}{3}$. **17.** $21\frac{3}{16}$ lb. **18.** $14\frac{5}{12}$ hr. **19.** $20\frac{1}{4}$ yd. **20.** $29\frac{3}{10}$ sec. **21.** $5\frac{7}{8}$ yd. **22.** $2\frac{11}{16}$ lb. **23.** $1\frac{1}{5}$ mi. **24.** $2\frac{1}{2}$ ft. **25.** $5\frac{1}{4}$. **26.** $\frac{3}{5}$. **27.** 48. **28.** $43\frac{3}{4}$. **29.** $8\frac{1}{4}$. **30.** $91\frac{1}{4}$. **31.** $1\frac{1}{8}$. **32.** 77. **33.** $13\frac{1}{2}$. **34.** $26\frac{2}{3}$. **35.** 22. **36.** 870. **37.** $4\frac{4}{7}$. **38.** $\frac{1}{3}$. **39.** $4\frac{1}{2}$. **40.** $8\frac{1}{2}$. **41.** 40. **42.** $\frac{6}{7}$. **43.** 1. **44.** $\frac{3}{4}$. **45.** $\frac{1}{2}$. **46.** $\frac{5}{8}$. **47.** $3\frac{1}{2}$. **48.** 4.

Page 139. — **1.** $5. **2.** 35 words; 15 words. **3.** Yes. **4.** 32%. **5.** $3. **6.** $564. **7.** $180. **8.** $.30. **9.** 17,640. **10.** 75%. **11.** 100%. **12.** 60%. **13.** 200%.

Page 143. — *See page* xxxix.

Page 149. — **1.** 35,741. **2.** 39,431. **3.** 25,345. **4.** 23,127. **5.** 31,101. **6.** 45,019. **7.** 39,301. **8.** 37,114. **9.** 31,143. **10.** 41,392.

1. 3081. **2.** 6727. **3.** 1801. **4.** 697. **5.** 2593. **6.** 8627. **7.** 1087. **8.** 3956. **9.** 112. **10.** 5378.

Page 150. — **1.** 2133 stamps. **2.** $21.65. **3.** 29 bags. **4.** $172.15. **5.** $2.82. **6.** 1209 mi. **7.** 129 mi.; 256 mi.; 127 mi.; 68 mi.; 105 mi.

Page 152. — **1.** 41,174. **2.** 279,672. **3.** 171,962. **4.** 1,203,935. **5.** 2,593,716.

Page 153. — **1.** 1,200,472. **2.** 406,016. **3.** 1,621,257. **4.** 2,701,062. **5.** 12,522,672. **6.** Arthur.

Page 154. — **1.** 279,464. **2.** 246,213. **3.** 424,096. **4.** 297,774. **5.** 323,180. **6.** 204,215. **7.** Correct. **8.** Wrong; correct ans., 4770. **9.** Wrong; correct ans., 236,076. **10.** Correct.

Page 155. — **1.** 175 hr.; \$43.75. **2.** 270 plants. **3.** 2016 mi. **4.** 1512 mi. **5.** 5280 ft.; 63,360 in. **6.** \$176. **7.** \$84.60. **8.** 360 chocolate creams; \$6. **9.** \$210. **10.** 260,280 sq. mi. **11.** \$276.

Page 156. — **1.** 138. **2.** $193\frac{4}{7}$. **3.** $2461\frac{4}{5}$. **4.** $258\frac{13}{25}\frac{3}{7}$. **5.** 247. **6.** $184\frac{15}{23}$. **7.** $345\frac{3}{5}\frac{8}{3}$. **8.** $451\frac{13}{62}$. **9.** Frank.

Page 157. — *See page* xxxix.

Page 158. — **1.** \$.15. **2.** 93; 89; 87. **3.** 28 mi. **4.** $17\frac{1}{2}$ mi. **5.** 5 mo. **6.** \$30. **7.** $8\frac{1}{2}$ hr. **8.** 18 yd.; 648 in. **9.** 13 times. **10.** \$3.75. **11.** He must measure off the length of the rope 24 times. **12.** 8 times.

Page 160. — **1.** \$.54. **2.** To serve 5 persons: 2 cups water, $\frac{3}{4}$ cup sugar, 1 cup boiling water, 6 mint sprigs, $\frac{1}{2}$ cup strawberry juice, $\frac{1}{2}$ cup raspberry juice, juice of 4 lemons. To serve 15 persons: 6 cups water, $2\frac{1}{4}$ cups sugar, 3 cups boiling water, 18 mint sprigs, $1\frac{1}{2}$ cups strawberry juice, $1\frac{1}{2}$ cups raspberry juice, juice of 12 lemons. To serve 25 persons: 10 cups water, $3\frac{3}{4}$ cups sugar, 5 cups boiling water, 30 mint sprigs, $2\frac{1}{2}$ cups strawberry juice, $2\frac{1}{2}$ cups raspberry juice, juice of 20 lemons. **3.** For 36 cookies: $\frac{3}{4}$ cup butter, $\frac{1}{2}$ cup sugar, $\frac{3}{8}$ cup raisins, $1\frac{1}{8}$ cups flour, $1\frac{1}{2}$ tsp. vanilla. For 48 cookies: 1 cup butter, $\frac{2}{3}$ cup sugar, $\frac{1}{2}$ cup raisins, $1\frac{1}{2}$ cups flour, 2 tsp. vanilla. **4.** \$.38.

Page 161. — **1.** 8 yd.; \$8.80. **2.** 180 oz., or $11\frac{1}{4}$ lb. **3.** 9 boards. **4.** 144 sq. ft. **5.** 6 qt.; \$3.20. **6.** 130 ft.

Page 162. — **2.** $\frac{1}{2}$ yd. = 18 in.; $\frac{5}{8}$ yd. = $22\frac{1}{2}$ in.; $\frac{3}{4}$ yd. = 27 in.; $\frac{7}{8}$ yd. = $31\frac{1}{2}$ in.; 1 yd. = 36 in. **3.** $\frac{7}{8}$ yd. **4.** $\frac{1}{4}$ yd. **5.** $\frac{1}{4}$ yd. **6.** $\frac{1}{2}$ yd. **7.** $\frac{3}{8}$ yd. **8.** $\frac{3}{8}$ yd. **9.** $\frac{3}{4}$ yd. **10.** $\frac{5}{8}$ yd. **11.** 1 yd. **13.** $1\frac{1}{2}$ yd. **14.** $1\frac{1}{8}$ yd. **15.** $1\frac{5}{8}$ yd. **16.** $1\frac{1}{4}$ yd. **17.** 2 yd. **18.** $1\frac{3}{8}$ yd. **19.** $1\frac{1}{2}$ yd. **20.** $3\frac{1}{8}$ yd. **21.** $5\frac{3}{8}$ yd.

Page 163. — **1.** 78 in.; 3 yd. **2.** $\frac{3}{4}$ yd. **3.** $3\frac{3}{4}$ yd.; \$2.82. **4.** $3\frac{1}{8}$ yd.; \$.79. **5.** $4\frac{5}{8}$ yd.; \$2.09. **6.** $1\frac{7}{8}$ yd. **7.** $\frac{7}{8}$ yd.

Page 164. — **1.** .05463. **2.** 53.795. **3.** 70.704. **4.** 29,000. **5.** .2135. **6.** 2.13. **7.** 7.61. **8.** 38.7. **9.** 2.35. **10.** 3.69. **11.** 7.61. **12.** .43. **13.** .06. **14.** 5.20. **15.** 9.53.

Page 165. — *See page* xxxix.

Page 166. — **1.** 25 lb.; 125 lb.; 225 lb. **2.** 375 lb. **3.** 9 cu. ft.; 337.5 lb.; yes. **4.** Yes. **5.** 225 lb. **6.** Yes.

Page 167. — **2.** .143. **3.** .444. **4.** .375. **5.** .571. **6.** .938. **7.** .644. **8.** .417. **9.** .563. **10.** .625. **11.** .156. **12.** .867. **13.** .369.

2. 43.8. **3.** .0513. **4.** 5.41. **5.** 6140. **6.** 31,900. **7.** 5.87. **8.** .0812. **9.** 40.9. **10.** 5.32. **11.** 4.93. **12.** .0254. **13.** 410.

Page 168. — **1.** 430. **2.** 580. **3.** 54. **4.** 2050. **5.** 987. **6.** 5000. **7.** 340. **8.** 6620. **9.** 63. **10.** 5. **11.** 4440. **12.** 7500. **13.** 600. **14.** 2660. **15.** 5625.

2. 57. **3.** 8.89. **4.** 106.9. **5.** .258. **6.** .49. **7.** .025. **8.** .87. **9.** .0432. **10.** .008. **11.** .66. **12.** .00451. **13.** .0298.

Page 169. — **1.** 310. **2.** 435. **3.** 1800. **4.** 2400. **5.** 2600. **6.** 700. **7.** 10,200. **8.** 18,850. **9.** 79,000. **10.** 120,000. **11.** 9287½. **12.** 19,000. **13.** 12,235. **14.** 18,475. **15.** 66,200. **16.** 12,100. **17.** 45,000. **18.** 61,000.

Page 170. — **3.** 6. **4.** 52. **5.** 85.6. **6.** 56. **7.** 26. **8.** 78.4. **9.** 36.4. **10.** 24. **11.** 60.16. **12.** 121.2. **13.** 33. **14.** 204.08.

1. 330. **2.** 550. **3.** 370. **4.** 8300. **5.** 21,500. **6.** 8700. **7.** 7.64. **8.** 70,000. **9.** .039. **10.** 456.3. **11.** 32,533⅓. **12.** 480. **13.** 204. **14.** 2800. **15.** .01. **16.** .0624. **17.** 6.4. **18.** 880,000.

Page 171. — **1.** 40 watt hours; 120 watt hours; 160 watt hours; 100 watt hours. **2.** In 1 hr.: 15 watt hours; 25 watt hours; 50 watt hours; 60 watt hours. In 5 hr.: 75 watt hours; 125 watt hours; 250 watt hours; 300 watt hours. In 7½ hr.: 112½ watt hours; 187½ watt hours; 375 watt hours; 450 watt hours. In 12 hr. 30 min.: 187½ watt hours; 312½ watt hours; 625 watt hours; 750 watt hours. **3.** 1200 watt hours; 1.2 K.W.H. **4.** 1.5 K.W.H.; 1.68 K.W.H.; 1.92 K.W.H.; 4.68 K.W.H.; 1 K.W.H. **5.** .015 K.W.H., .025 K.W.H., .05 K.W.H., .06 K.W.H.; .09 K.W.H., .15 K.W.H., .3 K.W.H., .36 K.W.H.; .15 K.W.H., .25 K.W.H., .5 K.W.H., .6 K.W.H. **6.** 1.23 K.W.H.; 1.845 K.W.H.; 1.025 K.W.H.

Page 172. — **2.** $4.08. **3.** $2.34; $2.61; $3.33. **4.** 2.4 K.W.H.; 24¢. **5.** 21¢; 12.6¢; 10.5¢. **6.** 54¢. **7.** 14.9¢. **8.** 10.2¢. **9.** 8.5¢.

Page 173. — **1.** 3.9¢. **2.** 14¢; 9.7¢. **3.** Electricity is .3¢ cheaper. **4.** .7¢. **5.** 1.8¢. **6.** 1¢; .9¢. **7.** 4.6¢; 6.9¢. **8.** Flatiron, 3.9¢; toaster, 3.5¢; vacuum cleaner, 1.1¢; fan, .3¢; stove, 4.6¢.

Page 174. — **2.** 5466 K.W.H., 6673 K.W.H.; 1207 K.W.H.

Page 175. — **4.** 66 K.W.H.

Page 176. — **2.** $7.38. **3.** $111.50. **4.** $89.20; $124.80; $66.50. **5.** 2086 K.W.H.; $128.30.

Page 177. — **1.** 77,800 cu. ft., 86,500 cu. ft.; 8700 cu. ft. **2.** 62,000 cu. ft., 66,200 cu. ft.; 4200 cu. ft. **3.** 75,700 cu. ft., 79,300 cu. ft.; 3600 cu. ft.

Page 178. — **1.** $1.89. **2.** $2.94. **3.** $.95. **4.** $2.73. **5.** $5.99. **6.** $8.61. **7.** $3.05. **8.** $4.73. **9.** $10.01; $4.83; $4.14

Page 179. — *See pages* xxxix *and* xl.

Page 182. — **1.** 72. **2.** 74. **3.** 420. **4.** 15. **5.** 27. **6.** $24.50. **7.** $106.80. **8.** $107.83. **9.** 47 words. **10.** 17 examples. **11.** 230 pupils. **12.** Bedford team, 14 games; Cadets, 9 games.

Page 183. — **2.** $1.29. **3.** $.27, $.60, $1.56, $.48; $2.91. **4.** $.90, $.42, $.54, $.30; $2.16. **5.** $22.75.

1. 12. **2.** 9. **3.** 42. **4.** 6. **5.** 20. **6.** 1.05. **7.** 18. **8.** 31.5. **9.** 18. **10.** $356. **11.** $81.06. **12.** $594. **13.** $330. **14.** $2660.70. **15.** $3199.50. **16.** $1683.60. **17.** $3619.20 **18.** $1704.96. **19.** 70 boys. **20.** 49 men. **21.** 39 dogs. **22.** 231 pens. **23.** 81 hats. **24.** 735 girls. **25.** 66 days. **26.** 153 eggs. **27.** 169 rugs.

Page 184. — **2.** 16. **3.** 45. **4.** 54. **5.** 45. **6.** 723. **7.** 85. **8.** $.60. **9.** $4.20. **10.** $2.50. **11.** $6.84. **12.** $3.65. **13.** $.95. **14.** $10.32. **15.** $11.13. **16.** $2.96. **17.** $18.75. **18.** $9.96. **19.** $6.50. **20.** 9 examples.

Page 185. — **4.** 7. **5.** 59. **6.** 50. **7.** 237. **8.** 102. **9.** 16. **10.** 470. **11.** 120. **12.** 215. **13.** 230. **14.** $493.75. **15.** $89.35. **16.** $24.15. **17.** $490.70. **18.** $318.50.

Page 186. — **1.** 17.1. **2.** 151. **3.** 89.49. **4.** 42. **5.** 176. **6.** 65. **7.** $220. **8.** $208.50. **9.** $31. **10.** $468.75. **11.** $317.07. **12.** $19. **13.** $13.58. **14.** $35.55. **15.** $16.15. **16.** $24.80. **17.** $55.30. **18.** $11.17. **19.** $\frac{1}{4}$. **20.** 15%. **21.** Same. **22.** $\frac{5}{8}$. **23.** $\frac{5}{8}$. **24.** $\frac{2}{3}$. **25.** $\frac{2}{5}$. **26.** $\frac{7}{8}$. **27.** 180 girls; 108 girls.

2. $2.66. **3.** $16.99. **4.** $519.22. **5.** $3.84. **6.** $11.39. **7.** $.25. **8.** $.42. **9.** $2.16. **10.** $4.37. **11.** $2.44. **12.** $20.42. **13.** $69.94. **14.** $237.58. **15.** $1.83. **16.** $95.41.

Page 187. — **1.** $400; $392. **2.** $136; $141.44. **3.** $124; $128.96. **4.** $530; $540.60. **5.** $761; $722.95. **6.** $938.50; $957.27. **7.** $4818; $4842.09. **8.** $31,500; $31,920. **9.** $8150; $7824. **10.** $46,750; $47,124. **11.** $48.16; $52.97. **12.** $181.92; $174.64. **13.** $237.72; $233.76. **14.** $44.53; $46.76. **15.** $484.82; $487.59. **16.** Nearly right. **17.** Too small. **18.** Too large. **19.** Too small. **20.** Nearly right. **21.** Nearly right. **22.** Nearly right. **23.** Too small.

Page 189. — **1.** 100%; 15%. **2.** $66\frac{2}{3}$%. **3.** 28%. **4.** 80%. **5.** 70%; 30%. **6.** 44%. **7.** 40%.

Page 190. — **2.** 1. **3.** 18. **4.** 348. **5.** 580. **6.** 3255. **7.** 7840. **8.** 980. **9.** 100. **10.** $1500. **11.** $15,000. **12.** $19,160. **13.** $58,050. **14.** 800% of 55. **15.** 200% of 96. **16.** 700% of 32. **17.** 800% of $700. **18.** 500% of $250. **19.** 400% of $990. **20.** 100% of 18 girls. **21.** 1000% of 15 boys. **22.** 1200% of 50 pupils. **23.** 1600 people. **24.** 300% as large. **25.** $4000. **26.** $4000; $5000.

Page 191. — **1.** .89. **2.** .43. **3.** .09. **4.** .07. **5.** 1.25. **6.** 3. **7.** 1.5. **8.** 2.15. **9.** 10. **10.** 11.5. **11.** 75%. **12.** 160%. **13.** 200%. **14.** 175%. **15.** 1250%. **16.** $891.46. **17.** $684.60. **18.** $1470. **19.** $600. **20.** $4105.50. **21.** $2392.50. **22.** $74.47. **23.** $55.61. **24.** $779.24. **25.** $89. **26.** $360; $660.

Page 192. — **1.** 165; 33. **2.** 106; 233.20. **3.** 82; 369. **4.** 28; 266. **5.** 200; 1496. **6.** 280; 1075.20. **7.** 400; 1876. **8.** 3; 927. **9.** 320; 483. **10.** For 65% more than its cost. **11.** 76%; 6384.

Page 193. — **6.** $21,000. **7.** $276. **8.** $23.75. **9.** $17,500. **10.** $1,008,000,000. **11.** 22% increase: Philadelphia, 2,224,060; San Francisco, 618,174; Detroit, 1,212,314; Atlanta, 244,732. 15% decrease: Philadelphia, 1,549,550; San Francisco, 430,695; Detroit, 844,645; Atlanta, 170,510.

Page 194. — **2.** 4,188,349 automobiles. **3.** $9\frac{1}{4}$%; 444,037 automobiles. **4.** Pennsylvania, 15,400,000 T.; Ohio, 9,000,000 T.; Michigan, 3,900,000 T.; Illinois, 3,800,000 T.; New York, 3,000,000 T.; Alabama, 2,900,000 T. **5.** United States, 514,710,000 barrels; Mexico, 158.670,000 barrels; Russia, 33,540,000 barrels; Dutch East Indies, 17,415,000 barrels; Rumania, 12,900,000 barrels; India, 7,740,000 barrels. $3\frac{3}{4}$%; 29,025,000 barrels. **6.** 16.8. **7.** 25.63. **8.** 20.93. **9.** 30.68. **10.** $41.99. **11.** $33.38. **12.** $29.93. **13.** $162. **14.** $175.05. **15.** $282.92. **16.** $353.40. **17.** $2215.40.

Page 195. — **5.** $\frac{3}{10}$ hr.; $\frac{9}{20}$ hr.; $\frac{7}{12}$ hr.; $\frac{4}{5}$ hr. **6.** 5280 ft.; $\frac{1}{8}$ mi. **7.** $\frac{5}{12}$; $\frac{5}{8}$; $\frac{5}{11}$. **8.** Alice, $\frac{2}{25}$; Jenny, $\frac{2}{25}$; Marie, $\frac{5}{6}$. **9.** Bill, $\frac{1}{4}$; Lucy, $\frac{3}{8}$; Ted, $\frac{1}{5}$; May, $\frac{1}{8}$; John, $\frac{1}{20}$. **10.** Paris, $\frac{10}{381}$; Berlin, $\frac{1}{76}$; London, $\frac{2}{289}$; Leningrad, $\frac{10}{2113}$; Madrid, $\frac{10}{151}$; Warsaw, $\frac{5}{93}$.

Page 196. — **1.** 65%. **2.** 76. **3.** 80. **4.** 75. **5.** 20. **6.** 22. **7.** 92. **8.** 55. **9.** 30. **10.** .84; 84%. **11.** .28; 28%. **12.** .18; 18%. **13.** .45; 45%. **14.** .16; 16%.

Page 197. — **1.** 88⅜. **2.** 6¼. **3.** 22½. **4.** 7½. **5.** 15¾. **6.** 44⅛. **7.** 5½. **8.** 8. **9.** 31¼. **10.** 18¾. **11.** 3¾. **12.** 2¼. **13.** 2½. **14.** 73⅓. **15.** 72½. **16.** 5½. **17.** Frank, 80%; Alice, 95%; Mollie, 97½%.

Page 198. — **2.** Roger, 14 lb., 20%; Anna, 22 lb., 33⅓%; Alice, 18 lb., 25%; Will, 31 lb., 41⅓%; Grace, 5 lb., 6¼%; John, 12 lb., 18¾%; Isaac, 12 lb., 15%; Ralph, 22 lb., 22%; Mary, 15 lb., 25%. **3.** Will; Will; Grace.

1. Dover, 5%; Baywood, 15%; Lincoln, 28%; Harrison, 14⅔%. **2.** $1000; 25%. **3.** 25% inc. **4.** 16⅔% dec. **5.** 56% inc. **6.** 46⅔% dec. **7.** 12% inc. **8.** 28% inc.

Page 199. — **3.** Agnes, 75% inc.; Laura, 28⅔% dec.; Sarah, 62½% dec.; May, 133⅓% inc.; Anne, 100% inc.; Rae, 1% inc.; Leah, 16⅔% inc.; Josie, 50% inc.; Doris, 8⅓% dec.; John, 200% inc.; Louis, 400% inc.; Fred, 75% dec.; Dan, 50% inc.; Bob, 150% inc.; Alvin, 95% dec.; Bill, 200% inc.; Jacob, 500% inc.; Carl, 75% inc. **4.** $420; $483; $63; 15%. **5.** $291; 97%.

Page 200. — **1.** $2400. **2.** Mr. Curt, $2000; Mr. Steel, $300; Mr. Voss, $5000. **3.** 200. **4.** 400. **5.** 3800. **6.** 9100. **7.** 900. **8.** 500. **9.** 725. **10.** 100. **11.** 300. **12.** 600. **13.** 450. **14.** 21. **15.** $300. **16.** $800. **17.** 50 hits.

Page 201. — *See page* xl.

Page 202. — **1.** $5.40. **2.** $148.75. **3.** $18.50; $23.80; $20.20; $17.25. **4.** $16. **5.** $8.40. **6.** $24.33. **7.** $110.40. **8.** $93. **9.** $14.10. **10.** $92.20. **11.** $166.50. **12.** $100. **13.** $28. **14.** $266. **15.** $51.33. **16.** $1.49. **17.** $3.17. **18.** $14.09. **19.** $36.26. **20.** $11.70. **21.** $8.23. **22.** $17.60.

Page 203. — **1.** $240; $2760. **2.** $182, $2618; $559, $8041; $1124.50, $16,175.50; $67.60, $972.40; $1257.36, $18,086.64. **3.** $18; $282. **4.** $45.80; $870.20. **5.** $9.75; $120.25. **6.** $46.60; $186.40. **7.** $483; $322. **8.** $96.80; $145.20.

9. $110.70; $504.30. **10.** $1.20; $118.80. **11.** $4.35; $285.65. **12.** $122.50; $367.50. **13.** $203; $377. **14.** $502.70; $411.30. **15.** $255; $510. **16.** $27.13; $189.87. **17.** $1.65; $80.99. **18.** $2.54; $61.01. **19.** $3.58; $32.17. **20.** $8.59; $34.36. **21.** $18.75; $31.25. **22.** $56.74; $28.37. **23.** $6.05; $30.25. **24.** Earnings in 1923, $1623.60; in 1924, $2371.95; in 1925, $1926; in 1926, $4367.88; in 1927, $5796; in 1928, $4510.80; total for 6 yr., $20,596.23. Net proceeds in 1923, $16,416.40; in 1924, $23,983.05; in 1925, $19,474; in 1926, $44,164.12; in 1927, $58,604; in 1928, $45,609.20.

Page 204. — **2.** $1. **3.** $4.50. **5.** $.72. **6.** $4.58. **7.** $2.52. **8.** $1.40. **9.** $2.17. **10.** $7.84. **11.** $4.25. **12.** $4.50. **13.** $.80. **14.** $1.50. **15.** $7.87. **16.** $1.33.

Page 205. — **1.** 88%; 90%; 70%; 55%; 50%; 75%. **2.** $27.60. **3.** $63. **4.** $528. **5.** $595. **6.** $312. **7.** $143. **8.** $555. **9.** $407. **10.** $812.60. **11.** $716. **12.** $297. **13.** $493.50. **14.** $529.33. **15.** $387.50. **16.** $719.01. **17.** $419.50. **18.** $526.68. **19.** $682.95. **20.** $487.65. **21.** $435. **22.** $596.64. **23.** $.17. **24.** $.10. **25.** $17.50.

Page 206. — **1.** The rate of discount on the first watch is 5% greater. **2.** The rate of discount on the second coat is $5\frac{1}{2}$% greater. **3.** The rate of discount on the second car is 1% greater. **4.** The rate of discount on the second pen is $3\frac{1}{3}$% greater. **5.** The rate of discount on the first lamp is $19\frac{3}{13}$% greater. **6.** 24%. **7.** Murray's. **8.** 15%. **9.** 30%; 25%.

Page 208. — **1.** $115.50. **2.** $38.25. **3.** $38; $57. **4.** $142.80. **5.** $93.94; $5.64. **6.** 60%; $10.56. **7.** $56; 35%. **8.** $72. **9.** $33.75. **10.** $746.30. **11.** $175. **12.** $58. **13.** $407.50. **14.** $143.90. **15.** $227.90. **16.** $167.04.

Page 209. — **1.** $432. **2.** $504. **3.** $346.87. **4.** $48.10. **5.** $457.85. **6.** $102.81. **7.** $266.36. **8.** $353.78. **9.** Mr. Marr, $13.44; Mr. Nye, $12; a discount of 50%.

Page 210. — **1.** $432.25. **2.** $398.29. **3.** $182.40. **4.** $425.32. **5.** $445.50. **6.** $185. **7.** $680.17. **8.** $367.56. **9.** $124.35. **10.** $693.36. **11.** $506.73. **12.** $533.14. **13.** Wilson, $923.40; Benson, $810; National, $957.60.

Page 212. — Net amount, $131.19.

Page 213. — **1.** $80.75. **2.** $49.43. **3.** $317.57. **4.** $182.57. **5.** $267.10. **6.** $441.67. **7.** $265.48. **8.** $137.32. **9.** $972.56.

Page 214. — **1.** \$900; \$750; \$150; $16\frac{2}{3}\%$. **2.** 80%. **3.** 25%. **4.** $12\frac{1}{2}\%$ profit. **5.** $26\frac{2}{3}\%$ loss. **6.** $16\frac{2}{3}\%$ loss. **7.** 10% profit. **8.** 20% profit. **9.** 23% loss. **10.** 15% profit. **11.** $33\frac{1}{3}\%$ loss. **12.** 40% profit. **13.** 25% profit. **14.** 25% profit. **15.** 14% loss. **16.** $28\frac{1}{2}\%$ profit. **17.** $7\frac{1}{2}\%$ profit. **18.** $16\frac{1}{4}\%$ profit.

Page 215. — **2.** \$676.80. **3.** 12%; 28%.

Page 216. — **1.** \$13.50. **2.** \$26. **3.** \$75.60. **4.** \$150. **5.** \$498. **6.** \$15.70. **7.** \$50.28. **8.** \$68.77. **9.** \$434.61. **10.** \$585; 24%; 6%. **11.** \$1246; 30%; 10%. **12.** \$1185.35; 40%; 11%. **13.** \$23.40; 35%; 15%. **14.** \$111.18; $33\frac{1}{3}\%$; 12%. **15.** \$295.11; 25%; 10%.

Page 217. — **1.** \$276. **2.** \$691.20. **3.** \$2632.50. **4.** \$10,584. **5.** \$37.93. **6.** \$256.01. **7.** \$359.05. **8.** \$174.36. **9.** \$445.50. **10.** \$1.50; 10%.

Page 218. — **1.** \$.09. **2.** \$.30. **3.** \$.12. **4.** \$7.50. **5.** \$67.60. **6.** \$46.54. **7.** \$81.40. **8.** \$3.83. **9.** \$8.31. **10.** \$61.20. **11.** \$10; $13\frac{1}{3}\%$. **12.** \$16; 20%. **13.** \$29; 25%. **14.** \$30; 8%. **15.** \$150; 25%. **16.** \$159; $13\frac{1}{4}\%$. **17.** \$.50; $10\frac{5}{12}\%$. **18.** $\$.12\frac{1}{2}$.

Page 219. — **1.** 24%. **2.** 27%. **3.** $33\frac{1}{3}\%$. **4.** 30%. **5.** 53%. **6.** \$.72; \$2.10; \$3.18.

Page 220. — **1.** \$3. **2.** \$14. **3.** \$15. **4.** \$4.75. **5.** \$54. **6.** \$27. **7.** \$32.40. **8.** \$150. **9.** \$250.

Page 221. — *See page* xl.

Page 222. — **1.** \$1.68. **2.** Newtown, 14,000; Redville, 28,000. **3.** \$300. **4.** \$3.07; \$22.53. **5.** \$147.24. **6.** Cost, \$17.60; expenses, \$5.50; total sales, \$27.50; profit, \$4.40; 16%. **7.** Mr. Lyons; 35% greater.

Page 235. — **1.** Drake, 11% more. **2.** \$31,490. **3.** 70%. **4.** \$31.75. **5.** Year before last: \$1250, 10%. Last year: \$2800. 14%. **7.** Second offer is \$3 better.

Page 236. — **6.** 112.5. **7.** 92. **8.** 378. **9.** 288. **10.** \$18. **11.** \$27.90. **12.** \$55.43. **13.** \$62.85. **14.** \$6245. **15.** \$1000. **16.** \$12,755.40. **17.** \$16,125. **18.** \$1179.39. **19.** \$6694.60. **20.** \$276.79. **21.** \$2411.68. **22.** \$12,299.39. **23.** \$20,734.97. **24.** 20%. **25.** 30%. **26.** 25%. **27.** 15%. **28.** 17%. **29.** 49%. **30.** 85%. **31.** 100%. **32.** 24%. **33.** 140%. **34.** 170%. **35.** 102%. **36.** 900. **37.** 300. **38.** 200. **39.** 500. **40.** 1100. **41.** 1900. **42.** 4150. **43.** 2300.

Page 238. — **1.** 46¢, 50¢, $17, $18, $19, $20. **2.** $3.36. **3.** $1.05; $.65. **4.** 4.45 P.M. **5.** 30,184. **6.** Mr. Green's profit was $2.60 greater.

Page 241. — *See page* xl.

Page 242. — **1.** No; 2 passengers. **2.** $19.90. **3.** $.84. **4.** 8.08 A.M. **5.** 4 hr. 24 min. **6.** $121 lost.

Page 243. — **1.** $24. **2.** $35. **3.** $27. **4.** $61.60. **5.** $30. **6.** $45. **7.** $26.40. **8.** $15. **9.** $100. **10.** $160. **11.** $510. **12.** $270. **13.** $7.80. **14.** $23.70. **15.** $42.75. **16.** $26.13. **17.** $42.30. **18.** $16.80. **19.** $11.35. **20.** $21.83. **21.** $18.55. **22.** $8.13. **23.** $55. **24.** $222. **25.** $318.50. **26.** $363. **27.** $112.73.

Page 244. — **1.** $54. **2.** $22.40. **3.** $49.50. **4.** $25.60. **5.** $94.20. **6.** $64.40. **7.** $310.80. **8.** $318. **9.** $135. **10.** $63. **11.** $58.20. **12.** $56.97. **13.** $85.50. **14.** $142.50. **15.** $34.92. **16.** $588. **17.** $478.50. **18.** $1148.58. **19.** $118.20. **20.** $170. **21.** $24.21. **22.** $84.37. **23.** $300. **24.** $114.75. **25.** $590. **26.** $24.

Page 245. — **1.** $8. **2.** $6.88. **3.** $10.80. **4.** $8.75. **5.** $46.20. **6.** $33.33. **7.** $114.38. **8.** $120. **9.** $65.50. **10.** $140. **11.** $10. **12.** $60. **13.** $53.13. **14.** $56. **15.** $81.90. **16.** $96.88. **17.** $81.25. **18.** $14.58. **19.** $58. **20.** $82.43. **21.** $61.25. **22.** $11.63.

Page 246. — **2.** $9.12.

1. $.94. **2.** $3. **3.** $6.81. **4.** $10.39. **5.** $7.67. **6.** $6.85. **7.** $1.32. **8.** $5.88. **9.** $132.08. **10.** $20.08. **11.** $11.33. **12.** $19.88. **13.** $7.12. **14.** $40.56. **15.** $119.64. **16.** $155.06. **17.** $11.82. **18.** $61.94. **19.** $4.79.

Page 247. — **1.** $5761.67. **2.** $63.08. **3.** $61.22. **4.** $95.89. **5.** 67 yr. 9 mo. 22 da.

Page 248. — **1.** $798. **2.** $45. **3.** $355. **4.** $1170. **5.** $150.

Page 249. — **1.** $5.98. **2.** $3.82. **3.** $8.07. **4.** $4.62. **5.** $2.92. **6.** $9.08. **7.** $1.17. **8.** $6.62. **9.** $18.87. **10.** $13.91. **11.** $23.77. **12.** $19.06. **13.** $17.77. **14.** $19.53. **15.** $60.49; $61.33. Latter, $.84 more. **16.** $115.64; $117.25. Latter, $1.61 more. **17.** $176.05; $178.50. Latter, $2.45 more. **18.** $1215.62; $1232.50. Latter, $16.88 more. **19.** $1020.08; $1034.25. Latter, $14.17 more. **20.** $3698.63; $3750. Latter, $51.37 more.

Page 250. — **1.** $5. **2.** $72. **3.** $5.40. **4.** $6.25. **5.** $22.40. **6.** $17.60. **7.** $11.25. **8.** $22.50. **9.** $14.10. **10.** $7. **11.** $17.40. **12.** $5.43. **13.** $2.25. **14.** $1.25. **15.** $1.20. **16.** $3.33. **17.** $.75.

Page 251. — **2.** \$3.60; \$1.80; \$.90; \$.36. **3.** \$6; \$3; \$7.50; \$1.80. **4.** \$4.80; \$9.60; \$7.20; \$.96.

Page 252. — **1.** \$3.30. **2.** \$6.67. **3.** \$1.94. **4.** \$12.35. **5.** \$4.10. **6.** \$16.68. **7.** \$5.50. **8.** \$37.50. **9.** \$28.75. **10.** \$9.50. **11.** \$42.40. **12.** \$12.83. **14.** \$1.17. **15.** \$6.13. **16.** \$5.63. **17.** \$3.60. **18.** \$2.80. **19.** \$9.38. **20.** \$6.94. **21.** \$5.83. **22.** \$2.89. **23.** \$2.38. **24.** \$3.09. **25.** \$2.70. **26.** \$12.57.

Page 253. — **1.** \$474. **2.** \$511.16. **3.** \$224.74. **4.** \$110.88. **5.** \$1320.78. **6.** 8.4¢. **7.** Mr. Scott's expense is 4.8¢ per mile more. **8.** \$.13; \$8.58. **9.** \$1.43.

Page 254. — **1.** Interest, \$9.25; depreciation, \$14.80; total loss, \$24.05. **2.** Expenses, \$168.24; cost, \$192.29. **3.** \$.20. **4.** \$4. **5.** \$6.50; \$3.61.

Page 255. — **6.** \$329.50. **7.** \$321.64. **8.** \$651.14. **9.** \$.94. **10.** \$9.40. **11.** \$13.15; \$1.75. **12.** \$1.86.

Page 257. — **1.** \$40; \$60. **2.** At 4% compounded semi-annually; \$3.06 more. **3.** \$109.44. **4.** \$100; \$9.44. **5.** 10 yr.: B, \$700; C, \$42.78. 15 yr.: B, \$800; C, \$105.34. 17½ yr.: B, \$850; C, \$149.50. 20 yr.: B, \$900; C, \$203.46. 25 yr.: B, \$1000; C, \$345. 35 yr.: B, \$1200; C, \$799.78. 50 yr.: B, \$1500; C, \$2122.32.

Page 258. — **1.** \$.67. **3.** \$.33.

Page 259. — Yes.

Page 260. — **3.** \$1.48; \$177.60.

Page 261. — *See page* xl.

Page 262. — **2.** None. **3.** \$1.25.

Page 263. — **1.** Total, \$1056.53. **2.** Total, \$799.23.

Page 265. — **3.** Total, \$491.46.

Page 267. — **1.** \$366.28; \$353.78. **2.** \$353.78; \$373.78; \$128.03. **3.** \$128.03; \$128.03; \$63.03.

1. Total, \$180.58. **2.** \$920.08. **4.** \$858.98. **6.** \$836.58.

Page 269. — **2.** \$7; \$693; \$700. **3.** \$1000; \$17.50; \$982.50. **4.** John Martin; \$400; \$4.40; \$395.60. **5.** \$447.75; \$198; \$714.12.

Page 270. — **10.** (1) \$2.10, \$347.90; (2) \$10.50, \$689.50; (3) \$.80, \$479.20; (4) \$1.15, \$113.85; (5) \$2.67, \$797.33; (6) \$7.50, \$617.50; (7) \$.70, \$279.30; (8) \$18.75, \$1231.25; (9) \$42.53; \$2157.47. **11.** More economical to borrow; \$2.

12. $148.12. **13.** $322.29. **14.** $445.87. **15.** $746.87.
16. $888. **17.** $467.08. **18.** $542.21. **19.** $288.19.
20. $369.23. **21.** $235.50. **22.** $175.87. **23.** $842.21.
24. $394.17. **25.** $510.03.

Page 271. — *See pages* xl *and* xli.

Page 286. — **1.** 45°. **2.** 60°; 45°. **3.** 180°.

Page 287. — *See page* xli.

Page 291. — **1.** Autumn, 94°; winter, 140°; spring, 76°; summer, 50°. **2.** Food, 72°; rent, 90°; clothing, 60°; other expenses, 102°; savings, 36°.

Page 292. — Literary Society, $\frac{7}{24}$, 105°; Art Club, $\frac{5}{24}$, 75°; Parents' League, $\frac{4}{15}$, 96°; Tennis Club, $\frac{1}{15}$, 24°; Athletic Association, $\frac{1}{6}$, 60°.

Page 293. — **1.** North America, 160°; Europe, 100°; South America, 60°; Asia, 40°. **2.** A, 81°; B, 144°; C, 90°; D, 45°. **3.** English, 126°; arithmetic, 108°; history, 72°; geography, 54°. **4.** College, 216°; work, 108°; home, 36°. **5.** Third grade, 90°; fourth, 75°; fifth, 69°; sixth, 48°; seventh, 42°; eighth, 36°. **6.** U. S., 219.6°; Europe, 104.4°; all others, 36°. **7.** Japan, 172.8°; China, 75.6°; Italy, 32.4°; rest of world, 79.2°. **10.** Saves 10%. Board, 144°; clothing, 90°; incidentals, 54°; recreation, 36°; saves, 36°.

Page 294. — **1.** 14 sq. in. **2.** 24 sq. ft. **3.** 24 sq. ft.
4. $22\frac{1}{2}$ sq. yd. **5.** $15\frac{3}{4}$ sq. in. **6.** $14\frac{7}{16}$ sq. ft. **7.** 3.055 sq. in.
8. 79.985 sq. ft. **9.** 36.125 sq. ft.

Page 295. — **2.** $18.80. **3.** $1\frac{1}{8}$ sq. ft. **4.** $62\frac{1}{2}$ sq. ft.
5. 183 sq. ft. **6.** 162 sq. ft. **7.** $57\frac{1}{2}$ sq. ft. **8.** $12\frac{2}{3}$ sq. ft.
9. 28 sq. ft. **10.** $23\frac{3}{8}$ sq. ft.

Page 296. — **1.** $58.80. **2.** 14 ft. **3.** Yes; 2 strips each 14 ft. long; 28 ft.; 21 running feet of 12-foot width. **4.** $102.48.
5. 54 patches. **6.** $16.20. **7.** $175.50. **8.** $144. **9.** $184.50.
10. $131.25. **11.** $133.60. **12.** $232.97. **13.** 180 sq. yd.
14. $53\frac{7}{9}$ sq. yd. **15.** 128 sq. yd. **16.** 250 sq. yd. **17.** $42\frac{1}{4}$ sq. yd.
18. 85 sq. yd.

Page 297. — **2.** Rob's; 1110 sq. ft. **3.** 6750 lb., or $3\frac{3}{8}$ T.
4. $262.50. **5.** $58.50. **6.** .38 A. **7.** 144. **8.** $\frac{1}{2}$. **9.** 50.
10. $1\frac{1}{4}$. **11.** 80. **12.** 32,670. **13.** 160. **14.** $\frac{1}{10}$. **15.** $1\frac{1}{2}$ gal.

Page 299. — **1.** 160 sq. ft. **2.** Lot *A*, 16,000 sq. ft.; lot *B*, 19,000 sq. ft. **3.** 99 sq. ft. **4.** $129\frac{1}{2}$ sq. rd. **5.** $106\frac{1}{4}$ sq. ft.
6. $175\frac{7}{12}$ sq. ft. **7.** $15\frac{5}{9}$ sq. yd. **8.** $144\frac{2}{3}$ sq. ft.

Page 301. — **1.** 700 sq. ft. **2.** Area of end, 736 sq. ft.; 14 gal. **3.** $33\frac{1}{4}$ sq. rd. **4.** 42 sq. in. **5.** 60 sq. yd. **6.** 240 sq. ft. **7.** $100\frac{3}{4}$ sq. ft. **8.** 75 sq. yd. **9.** 55 sq. yd. **10.** 41 sq. ft. **11.** 69 sq. yd. **12.** $102\frac{1}{2}$ sq. ft. **13.** $87\frac{1}{2}$ sq. ft. **14.** $117\frac{1}{8}$ sq. ft.

Page 302. — **1.** Dick, $12\frac{1}{2}$ sq. ft.; John, 21 sq. ft.; Ralph, $19\frac{1}{2}$ sq. ft.; Harry, $20\frac{7}{2}$ sq. ft; Fred, 15 sq. ft.; Paul, 20 sq. ft.; Will, $17\frac{7}{8}$ sq. ft.; Earl, $21\frac{9}{16}$ sq. ft. **2.** 197 sq. ft. **3.** 38 sq. ft. **4.** $72\frac{7}{8}$ sq. ft. **5.** 91 sq. ft. **6.** $52\frac{1}{2}$ sq. ft. **7.** $36\frac{9}{16}$ sq. ft. **8.** 35 sq. ft.

Page 304. — **1.** 6600 sq. ft. **2.** George's; $1\frac{1}{4}$ sq. ft. **3.** 364 sq. ft. **4.** *A*, 7500 sq. ft.; *B*, 11,700 sq. ft.; *C*, 17,100 sq. ft. **5.** 28 sq. in. **6.** 100 sq. ft. **7.** $73\frac{1}{2}$ sq. ft. **8.** 70 sq. ft. **9.** 350 sq. ft. **10.** 108 sq. rd. **11.** 399 sq. yd. **12.** $47\frac{1}{8}$ sq. ft. **13.** $106\frac{1}{4}$ sq. ft. **14.** 4550 sq. ft. **15.** $958\frac{3}{4}$ sq. ft. **16.** 29 sq. yd. **17.** 90 sq. rd. **18.** 52 sq. yd.

Page 305. — *See page* xli.

Page 306. — **1.** 611 sq. in. **2.** 8.5 lb.; 14.8 lb. **3.** 496 sq. in. **4.** 7.4 sq. ft. **5.** 14.8 lb.

Page 309. — **1.** 44 in. **2.** 88 in. **3.** 132 ft. **4.** 220 in. **5.** $157\frac{1}{7}$ ft. **6.** $119\frac{3}{7}$ in. **7.** 44 yd. **8.** $37\frac{5}{7}$ ft. **9.** $2\frac{5}{14}$ mi. **10.** 880 yd. **11.** 21.98; 22. **12.** 43.96; 44. **13.** 307.72; 308. **14.** 131.88; 132. **15.** 65.94; 66. **16.** 263.76; 264. **17.** 351.68; 352. **18.** 30.772; 30.8.

Page 310. — **1.** Bob; $18\frac{3}{7}$ ft. **2.** 132 ft.; $565\frac{5}{7}$ ft. **3.** 660 ft.

1. $471\frac{3}{7}$ ft.; $2357\frac{1}{7}$ ft.; $11\frac{1}{5}$ times. **2.** 200 ft.; $157\frac{1}{7}$ ft. **3.** 572 ft. **4.** 264 ft. **5.** $301\frac{5}{7}$ ft. **6.** $339\frac{3}{7}$ ft. **7.** 616 ft. **8.** 528 ft. **9.** $628\frac{4}{7}$ ft. **10.** 1320 ft.

Page 311. — **1.** 3850 sq. ft. **3.** Square flower bed is $9\frac{2}{28}$ sq. ft. larger. **4.** 2464 sq. in. **5.** 5544 sq. in. **6.** 154 sq. mi. **7.** $2828\frac{4}{7}$ sq. ft. **8.** $1018\frac{3}{7}$ sq. yd. **9.** 18,634 sq. yd. **10.** 12,474 sq. in. **11.** 3850 sq. ft. **12.** 7546 sq. yd. **13.** $6084\frac{4}{7}$ sq. ft.

Page 312. — **1.** 2464 sq. ft.; $643.38. **2.** About 176 ft. **3.** $1810\frac{2}{7}$ sq. ft. **4.** 4 times as great. **5.** $9\frac{5}{8}$ sq. ft. **6.** Same. **7.** $402\frac{2}{7}$ sq. ft. **8.** 88 ft.; 616 sq. ft. **9.** $75\frac{3}{7}$ yd.; $452\frac{4}{7}$ sq. yd. **10.** $62\frac{6}{7}$ mi.; $314\frac{2}{7}$ sq. mi. **11.** 66 ft.; $346\frac{1}{2}$ sq. ft. **12.** 704 ft.; 39,424 sq. ft. **13.** 484 in.; 18,634 sq. in. **14.** 660 ft.; 34,650 sq. ft. **15.** 1100 ft.; 96,250 sq. ft. **16.** $314\frac{2}{7}$ yd.; $7857\frac{1}{7}$ sq. yd. **17.** $1634\frac{2}{7}$ mi.; $212,457\frac{1}{7}$ sq. mi.

Page 313. — **3.** Cost, 67%, 241.2°; overhead, 23%, 82.8°; profits, 10%, 36°. **4.** Discounts, \$46.20, \$9.24, \$1.66; net amount, \$81.50. **5.** Saved \$3. **6.** \$11.90; \$838.10. **7.** \$89.42.

Page 314. — **1.** $8. **2.** $6.75. **3.** $.60. **4.** $1.67. **5.** $8.25. **6.** $.83. **7.** $4.17. **8.** $75.60. **9.** $20. **10.** $90.28. **11.** $5.42. **12.** $9.44. **13.** $4.82. **14.** $5.27. **15.** $2.84. **16.** $26.77. **17.** $5.16. **18.** $6.75. **19.** $6.21. **20.** $35.33. **21.** $492. **22.** $612.56. **23.** $235.92. **24.** $417.48. **25.** $949.44. **26.** $631.68. **27.** $353.88. **28.** $273.70. **29.** 39 sq. in. **30.** $41\frac{1}{4}$ sq. ft. **31.** $6\frac{2}{3}$ sq. yd. **32.** $2\frac{1}{4}$ sq. rd. **33.** $16\frac{8}{9}$ sq. yd. **34.** $33\frac{1}{4}$ sq. rd. **35.** 13.42 sq. ft. **36.** 16.34 sq. ft. **37.** 72.2 sq. in. **38.** 63 sq. ft. **39.** 222 sq. ft. **40.** $93\frac{3}{4}$ sq. in. **41.** $50\frac{7}{8}$ sq. in. **42.** (38) $31\frac{1}{2}$ sq. ft.; (39) 111 sq. ft.; (40) $46\frac{7}{8}$ sq. in.; (41) $25\frac{7}{16}$ sq. in. **43.** 44 in.; 154 sq. in. **44.** 88 ft.; 616 sq. ft. **45.** $25\frac{1}{7}$ mi.; $50\frac{2}{7}$ sq. mi. **46.** 220 rd.; 3850 sq. rd. **47.** $75\frac{3}{7}$ yd.; $452\frac{4}{7}$ sq. yd.

Page 316. — **1.** .135; $62.10; $51.30; $67.50. **2.** .146. **3.** .361. **4.** .174. **5.** .993. **6.** .8325. **7.** .6233. **8.** .082. **9.** .035. **10.** .003. **11.** .0018. **12.** $127.50. **13.** $12.75. **14.** $131.75. **15.** $13.18. **16.** $24.82. **17.** $19.38. **18.** $54.31. **19.** $5.93. **20.** $8.70. **21.** $16.40. **22.** $914.40. **23.** $280.50. **24.** $1,271,000. **25.** 91,060,000 T.

Page 317. — **1.** 15.7%. **2.** 43.2%. **3.** 62.5%. **4.** 75.5%. **5.** 12.5%. **6.** 4.8%. **7.** 6.7%. **8.** 9.5%. **9.** 0.3%. **10.** 96.75%. **11.** 49.33%. **12.** 66.67%. **13.** 8.75%. **14.** 9.09%. **15.** 6.25%. **16.** $2\frac{1}{2}$%. **18.** 64.3%. **19.** 7.5%. **20.** 57.3%. **21.** 21.3%. **22.** 24.9%.

Page 318. — **2.** Cincinnati, 56.6%; St. Louis, 54.2%; Chicago, 52.1%; Brooklyn, 50%; New York, 48.9%; Boston, 40%; Philadelphia, 39.8%. **3.** Boston, 40%; Philadelphia, 40%.

Page 319. — **1.** New York, 65.3%; Cleveland, 56%; Philadelphia, 52.04%; Detroit, 52%; Washington, 51.1%; Chicago, 50.5%; St. Louis, 41.8%; Boston, 30.9%. **2.** New York, 65.7%; Cleveland, 56.4%; Philadelphia, 51.52%; Detroit, 51.49%; Chicago, 51%; Washington, 50.5%; St. Louis, 41.4%; Boston, 31.6%. **3.** Pittsburgh, 57.4%; St. Louis, 56.5%; Cincinnati, 56.0%; New York, 51.79%; Chicago, 51.75%; Brooklyn, 47.0%; Boston, 41.2%; Philadelphia, 38.2%. **4.** Pittsburgh, 58.04%; St. Louis, 57.98%; Cincinnati, 57.1%; Chicago, 51.7%; New York, 50%; Brooklyn, 47.1%; Boston, 40.2%; Philadelphia, 37.7%.

Page 320. — **2.** Brown, 34.2%; Donohue, 33.8%; Cuyler, 33.25%; Roush, 33.24%; Williams, 33.16%; Walker, 32.62%; Gibson, 32.61%; Goslin, 35.07%; McNulty, 35.14%; Collins, 34.8%; Welch, 33.7%; Bennett, 33.3%; Simmons, 32.20%; Miller, 32.16%.

Page 320 (*continued*). — **3.** 26.2%. **4.** Cincinnati, 29.1%; Pittsburgh, 28.9%; Philadelphia, 28.7%; Chicago, 28.5%; St. Louis, 28.37%; New York, 28.41%; Boston, 28.35%; Brooklyn, 27.0%. **5.** 77.8%. **6.** 73.3%.

Page 321. — *See page* xli.

Page 322. — **1.** 23.7%. **2.** 8.6%. **3.** 4.1%; 6.1%.

Page 323. — **1.** 51.4%. **2.** 13.4%; 86.5%. **3.** 32.4%; 67.6%. **4.** 78.4%; 21.6%. **5.** 17.3%; 82.7%. **6.** 97.5%. **7.** New Jersey, New York, Rhode Island; Mississippi. **8.** In 1920.

Page 324. — **1.** $.36 a month. **2.** $4. **3.** $1. **4.** $6.30. **5.** $.09. **6.** $4.20. **7.** $4.50. **8.** $.51. **9.** $.70. **10.** $.72. **11.** $.89. **12.** $.98. **13.** $.33. **14.** 0.4%. **15.** 50%; same.

Page 325. — **1.** 4. **2.** 3.54. **3.** 5. **4.** 1.15. **5.** 2.09. **6.** 1.62. **7.** 7.8. **8.** 4.68. **9.** 2.01. **10.** 12. **11.** 284%. **12.** 305%. **13.** 175%. **14.** 462%. **15.** 1808%. **16.** 1005%. **17.** 412%. **18.** 193%. **19.** 300%. **20.** 1500%. **21.** 494 boys. **22.** 300% as great. **23.** $1085. **24.** $1650. **25.** $1031.24. **26.** $1347.36. **27.** $633.60. **28.** $501.32. **29.** $1127. **30.** $2260.80. **31.** $10,680. **32.** $6600. **33.** $2745. **34.** $7862.50. **35.** $75. **36.** $15,750.

Page 326. — **1.** 250%; $6000. **2.** 400%; $8. **3.** 75%. **5.** 200; $500. **6.** 60; $1200. **7.** $36. **8.** 180; $36. **10.** 20%. **11.** 25% increase. **12.** 6.3% decrease. **13.** 16.1% increase. **14.** 9% decrease. **15.** 115% increase. **16.** 12.5% increase. **17.** 15.0% decrease. **18.** 18.8% increase. **19.** 11.4% decrease.

Page 327. — **1.** 105.6%. **2.** 375.5%. **3.** 495.3%. **4.** 275%. **5.** 2320 T.; $191,980.

Page 328. — **1.** *Nitrate of soda*, 300 lb. nitrogen. *Sulfate of ammonia*, 408 lb. nitrogen. *Dried blood*, 286 lb. nitrogen. *Muriate of potash*, 1040 lb. potash. *Sulfate of potash*, 960 lb. potash. *Kainit*, 260 lb. potash. *Acid phosphate*, 320 lb. phosphoric acid. *Ground bone*, 60 lb. nitrogen, 480 lb. phosphoric acid. *Tankage*, 124 lb. nitrogen, 220 lb. phosphoric acid. *Cottonseed meal*, 140 lb. nitrogen, 50 lb. phosphoric acid, 30 lb. potash. **2.** $.198; $.178; $.203. **3.** Muriate of potash.

Page 333. — **2.** $204.50. **3.** $322.70.

Page 336. — **2.** $.27; $274.77. **7.** $684.31. **10.** Sight draft; $812.69.

Page 337. — *See page* xli.

Page 338. — **1.** $2.05. **2.** $18.12. **3.** $25.15. **4.** $62.20. **5.** $80.20. **6.** $80.42. **7.** $75.20. **8.** $11.62. **9.** $1.25. **10.** $2.87. **11.** $4.82. **12.** $5.22. **13.** $20.17. **14.** $33.95. **15.** $99.72. **16.** $100.22. **17.** $150.40. **18.** $100 and $20.12, fee $.37; $100 and $35.50, fee $.37; $100, $100, and $40.35, fee $.62; $76.60, fee $.20. **19.** $1.53.

Page 339. — **1.** $2; $2.33; $1.67. **2.** $2.10; $2.45; $1.75. **3.** $12; $14; $10. **4.** $4.32; $5.04; $3.60. **5.** $12; $14; $10. **6.** $1.25; $1.46; $1.04. **7.** $1.90; $2.22; $1.58. **8.** $5.50; $6.42; $4.58. **9.** $5.92; $6.91; $4.93. **10.** $3.13; $3.65; $2.61. **11.** $5.50; $6.42; $4.58. **12.** $.50; $.58; $.42. **13.** $4.20; $4.90; $3.50. **14.** $4.32; $5.04; $3.60. **15.** $1.56; $1.82; $1.30. **16.** $9; $10.50; $7.50. **17.** $3.80; $4.43; $3.17. **18.** $37.20; $43.40; $31.

Page 340. — **1.** 180 da. **2.** 139 da. **3.** 143 da. **4.** 168 da. **5.** 146 da. **6.** 35 da. **7.** 123 da. **8.** 165 da. **9.** 73 da. **10.** 43 da. **11.** 51 da. **12.** 124 da. **13.** 60 da.; $.90. **14.** 72 da.; $12. **15.** $2.50. **16.** $7.35. **17.** $4.14. **18.** $.50. **19.** $7.15. **20.** $7.50.

Page 342. — **1.** $.67; $1; $1.33; $1.67. **2.** $2; $.1667, or $.17; $2.17. **3.** $3.65. **4.** $2; $.20; $.02; $2.22; $20. **5.** $.60. **6.** $.27. **7.** $1.05. **8.** $.25. **9.** $.53. **10.** $1.20. **11.** $3.30. **12.** $3.87. **13.** $.47. **15.** $2.92; $2.08; $3.75. **16.** $1.13; $3.10. **17.** $4.40. **18.** $5.58. **19.** $2.66. **20.** $2.59. **21.** $5.53. **22.** $14.85. **23.** $26.92. **24.** $38.04. **25.** $4.88. **26.** $6.75. **27.** $1.23. **28.** $6.63.

Page 343. — **1.** $350. **2.** $.90; $89.10; $90. **3.** $3; $597. **4.** $.64; $119.36. **5.** $2.76; $177.24. **6.** $11.28; $538.72. **7.** $14; $1386. **8.** $8; $1592. **9.** $6.25; $1243.75. **10.** $37.50; $2462.50.

Page 344. — **2.** 46 da.; $11.50. **3.** $23; $15.50.

Page 345. — **1.** June 9; $378.88. **2.** $213.41; $214.

Page 347. — **1.** $485.28. **2.** $520.87. **3.** $609.10. **4.** $862.75. **5.** $1224. **6.** $1648. **7.** $2006.67. **8.** $3771.60. **9.** 36 da.; $3.40; $481.88; $485.28. **10.** $1615.04; $1648. **11.** (3) $604.33; (4) $855.27; (5) $1211.96; (7) $2003.33; (8) $3618.22. **12.** Aug. 4. **13.** $446.62; $450. **14.** May 31; June 4; Nov. 14; July 30; Nov. 2.

Page 348. — **1.** $54; 36%. **2.** $72; 6%. **3.** $96; 12%. **4.** 10%. **5.** 6%. **6.** $16\frac{2}{3}$%. **7.** 40%. **8.** 24%. **9.** 8%. **10.** 48%. **11.** 30%. **12.** 7%. **13.** 5%.

Page 349. — *See pages* xli *and* xlii.

Page 350. — **1.** 5 mo. **2.** $4\frac{1}{2}$ mo.; $4\frac{1}{2}$ mo. **5.** $5.25. **6.** $13. **7.** $46.

Page 352. — **1.** $27; $324; 60%. **2.** 33%. **3.** 38%. **4.** 40%. **5.** 578%. **6.** $135; 27%. **7.** 23%; 40%; Star.

Page 353. — **1.** $15. **2.** $11. **3.** $22. **4.** 57%. **5.** $21.87. **6.** About 17%.

Page 355. — 1. *5th month:* $5832, $42, $29.16, $71.16; *6th month:* $5790, $42, $28.95, $70.95; *7th month:* $5748, $42, $28.74, $70.74; *8th month:* $5706, $42, $28.53, $70.53; *9th month:* $5664, $42, $28.32, $70.32; *10th month:* $5622, $42, $28.11, $70.11; *11th month:* $5580, $42, $27.90, $69.90; *12th month:* $5538, $42, $27.69, $69.69; *13th month:* $5496, $42, $27.48, $69.48; *14th month:* $5454, $42, $27.27, $69.27; *15th month:* $5412, $42, $27.06, $69.06; *16th month:* $5370, $42, $26.85, $68.85. **2.** $2016; $3984; $19.92. **3.** $204. *139th month:* $204, $42, $1.02, $43.02; *140th month:* $162, $42, $.81, $42.81; *141st month:* $120, $42, $.60, $42.60; *142d month:* $78, $42, $.39, $42.39; *143d month:* $36, $36, $.18, $36.18. 143 mo.

Page 357. — 4. *5th month:* $5758.20, $90, $28.79, $61.21; *6th month:* $5696.99, $90, $28.48, $61.52; *7th month:* $5635.47, $90, $28.18, $61.82; *8th month:* $5573.65, $90, $27.87, $62.13; *9th month:* $5511.52, $90, $27.56, $62.44; *10th month:* $5449.08, $90, $27.25, $62.75; *11th month:* $5386.33, $90, $26.93, $63.07; *12th month:* $5323.26, $90, $26.62, $63.38; *13th month:* $5259.88, $90, $26.30, $63.70; *14th month:* $5196.18, $90, $25.98, $64.02; *15th month:* $5132.16, $90, $25.66, $64.34; *16th month:* $5067.82, $90, $25.34, $64.66; *17th month:* $5003.16, $90, $25.02, $64.98; *18th month:* $4938.18, $90, $24.69, $65.31; *19th month:* $4872.87, $90, $24.36, $65.64; *20th month:* $4807.23, $90, $24.04, $65.96; *21st month:* $4741.27, $90, $23.71, $66.29; *22d month:* $4674.98, $90, $23.37, $66.63; *23d month:* $4608.35, $90, $23.04, $66.96; *24th month:* $4541.39, $90, $22.71, $67.29. **5.** $4474.10. **6.** *1st month:* $8000, $75, $40, $35; *2d month:* $7965, $75, $39.83, $35.17; *3d month:* $7929.83, $75, $39.65, $35.35; *4th month:* $7894.48, $75, $39.47, $35.53; *5th month:* $7858.95, $75, $39.29, $35.71; *6th month:* $7823.24, $75, $39.12, $35.88; *7th month:* $7787.36, $75, $38.94, $36.06; *8th month:* $7751.30, $75, $38.76, $36.24; *9th month:* $7715.06, $75, $38.58, $36.42; *10th month:* $7678.64, $75, $38.39, $36.61; *11th month:* $7642.03, $75, $38.21, $36.79; *12th month:* $7605.24, $75, $38.03, $36.97. **7.** *1st month:* $1500, $25, $7.50, $17.50; *2d month:* $1482.50, $25, $7.41, $17.59; *3d month:* $1464.91, $25, $7.32, $17.68; *4th month:* $1447.23, $25, $7.24, $17.76; *5th month:* $1429.47, $25, $7.15, $17.85; *6th month:* $1411.62, $25, $7.06, $17.94; *7th month:* $1393.68, $25, $6.97, $18.03; *8th month:* $1375.65, $25, $6.88, $18.12. *9th month:* $1357.53, $25, $6.79, $18.21; *10th month:* $1339.32, $25, $6.70, $18.30; *11th month:* $1321.02, $25, $6.61, $18.39; *12th month:* $1302.63, $25, $6.51, $18.49. **8.** *1st month:* $3000, $60, $15, $45; *2d month:* $2955, $60, $14.78, $45.22; *3d month:* $2909.78, $60, $14.55, $45.45;

4th month: $2864.33, $60, $14.32, $45.68; *5th month:* $2818.65, $60, $14.09, $45.91; *6th month:* $2772.74, $60, $13.86, $46.14. **9.** 1*st month:* $3000, $60, $15, $75; *2d month:* $2940, $60, $14.70, $74.70; *3d month:* $2880, $60, $14.40, $74.40; *4th month:* $2820, $60, $14.10, $74.10; *5th month:* $2760, $60, $13.80, $73.80; *6th month:* $2700, $60, $13.50, $73.50.

Page 359. — **1.** 176 sq. yd.; $61.60. **2.** .19 A. **3.** $37\frac{1}{2}$ running feet; 15 ft. of each. **4.** $51.25. **5.** $92.40. **6.** 48 sq. ft. **7.** 5 sq. yd. **8.** $22\frac{3}{4}$ sq. ft. **9.** $12\frac{11}{16}$ sq. yd. **10.** 13.95 sq. in. **11.** 51.84 sq. in. **12.** 42 sq. in. **13.** 20 sq. in. **14.** $16\frac{1}{2}$ sq. ft. **15.** $1\frac{7}{8}$ sq. yd. **16.** 131.4 sq. ft. **17.** 62.56 sq. ft.

Page 360. — **1.** *M*, 10,000 sq. ft.; *N*, 35,000 sq. ft.; *R*, 60,000 sq. ft. **2.** 137 sq. ft. **3.** $7\frac{1}{2}$ sq. ft. **4.** 14 sq. ft. **5.** 15 sq. ft. **6.** 13 sq. in. **7.** $6\frac{3}{8}$ sq. ft. **8.** $5\frac{3}{4}$ sq. yd. **9.** 3.875 sq. ft. **10.** 9.24 sq. ft. **11.** 7.25 sq. ft. **12.** 18 sq. yd. **13.** $41\frac{1}{8}$ sq. yd. **14.** 340 sq. ft. **15.** $1662\frac{1}{2}$ sq. ft. **16.** 3720 sq. ft. **17.** 91 sq. ft. **18.** $89\frac{1}{4}$ sq. ft. **19.** 150 sq. ft.

Page 361. — *See page* xlii.

Page 362. — **1.** $7\frac{1}{3}$ ft. **2.** 132 ft.; 40 times. **3.** 3850 sq. mi. **4.** 44 in.; 154 sq. in. **5.** 88 ft.; 616 sq. ft. **6.** 176 yd.; 2464 sq. yd. **7.** $157\frac{1}{7}$ ft.; $1964\frac{2}{7}$ sq. ft. **8.** $6\frac{2}{7}$ yd.; $3\frac{1}{7}$ sq. yd. **9.** 308 in.; 7546 sq. in. **10.** $7\frac{1}{3}$ ft.; $4\frac{5}{18}$ sq. ft. **11.** $29\frac{1}{3}$ ft.; $68\frac{4}{9}$ sq. ft. **12.** 44 in.; 154 sq. in. **13.** $31\frac{3}{7}$ mi.; $78\frac{4}{7}$ sq. mi. **14.** 264 ft.; 5544 sq. ft. **15.** $56\frac{4}{7}$ rd.; $254\frac{4}{7}$ sq. rd. **16.** 484 ft.; 18,634 sq. ft. **17.** $314\frac{2}{7}$ ft.; $7857\frac{1}{7}$ sq. ft. **18.** $26\frac{2}{5}$ rd.; $55\frac{1}{25}$ sq. rd. **19.** $17\frac{3}{5}$ mi.; $24\frac{18}{25}$ sq. mi.

Page 363. — **1.** 105 cu. in. **2.** 512 cu. in. **3.** 60 cu. ft. **4.** 288 cu. in., or $\frac{1}{6}$ cu. ft. **5.** 14 cu. ft. **6.** 50 cu. ft. **7.** 72 cans.

Page 364. — **3.** 12,600 cu. ft.; 63 pupils. **4.** 167 cu. yd.; $108.55. **5.** 21,547$\frac{1}{2}$ lb. **6.** 6120 cu. ft.; $16.52. **7.** 1512 cu. in.; $54\frac{11}{16}$ lb. **8.** 10 loads.

Page 365. — **1.** Third can. **2.** More; $9\frac{5}{8}$ cu. in. **3.** 24 gal. **4.** 141,750 gal. **5.** 105 cu. ft.; $787\frac{1}{2}$ gal.; 25 bbl. **6.** 300 gal. **7.** 315 gal. **8.** 525 gal. **9.** 1 gal. **10.** $2\frac{1}{2}$ gal. **11.** 74,250 gal.

Page 366. — **2.** $2\frac{1}{16}$ cu. in.

Page 367. — **2.** 160 bu.; 272 bu. **3.** $9\frac{1}{2}$ T.; $11\frac{2}{3}$ T. **4.** 5 ft. **5.** 408 bu. **6.** 396 bu. **7.** 186 bu. **8.** 336 bu. **9.** 342 bu. **10.** $94\frac{1}{2}$ bu. **11.** $157\frac{1}{2}$ bu. **12.** 364 bu. **13.** 117 bu. **14.** 16.8 bu. **15.** 81.12 bu. **16.** 271.32 bu.

Page 368. — **1.** 2160 cu. in.; 10 cu. in.; about 1 in 200. **2.** 2150 cu. in.; 50 cu. in.; about 1 in 40. **3.** 1 in 86; greater. **4.** More. **5.** Box (*f*). **6.** Former.

Page 370. — **1.** 2 bd. ft. **2.** 20 bd. ft. **3.** 35 bd. ft. **4.** 7 bd. ft. **5.** 28 bd. ft. **6.** 16 bd. ft. **7.** 30 bd. ft. **8.** 875 bd. ft. **9.** 6180 bd. ft.

Page 371. — **1.** $16.38. **2.** $.76. **3.** $1.69. **4.** $3.75. **5.** $3.84. **6.** $303.39. **7.** $64.83.

Page 372. — **1.** $1.45. **2.** $9.74. **3.** $2.69. **4.** $4.27. **5.** $2.30; $2.76; $4.03; $4.72; $8.28; $12.19. **6.** $15.90.

Page 373. — *See page* xlii.

Page 375. — **1.** 196 cu. in. **2.** 27 cu. ft. **3.** $10\frac{1}{2}$ cu. in. **4.** 28 cu. in. **5.** 72 cu. ft. **6.** 540 gal. **7.** 10,500 cu. ft.; 78,750 gal. **8.** 67,500 gal.; $28.80.

Page 376. — **1.** 32 cu. in. **2.** 42 cu. in. **3.** $49\frac{1}{2}$ cu. ft. **4.** $12\frac{1}{2}$ cu. ft. **5.** $75\frac{3}{7}$ cu. ft. **6.** $226\frac{2}{7}$ cu. in. **7.** $28\frac{2}{7}$ cu. in. **8.** 6930 cu. in. **9.** $28\frac{3}{7}$ cu. in. **10.** $192\frac{1}{2}$ cu. ft.; 1444 gal.

Page 377. — **1.** $247\frac{1}{2}$ cu. ft.; 34,650 lb. **2.** $553\frac{1}{7}$ cu. in.; 2.4 gal. **3.** 40 gal.; 61 gal. **4.** 712,800 cu. ft.; $891. **5.** $2227\frac{1}{2}$ gal. **6.** $125\frac{5}{7}$ cu. ft.; $942\frac{6}{7}$ gal. **7.** 3 gal.

Page 378. — **1.** 1571 cu. ft.; 1964 cu. ft.; 2357 cu. ft.; 2750 cu. ft. **2.** 2829 cu. ft.; 3394 cu. ft.; 4526 cu. ft. **3.** 4620 cu. ft.; 5390 cu. ft.; 6160 cu. ft.; 7241 cu. ft.; 7643 cu. ft.; 8448 cu. ft. **4.** 50 cows. **5.** 159 da. **6.** 94 cu. ft.

Page 379. — **1.** 39 cu. ft.; 39 cows. **2.** Yes; no. **3.** 10 ft. in diameter.

1. 6 A. **2.** 119 T. **3.** 21,120 gal. **4.** 61,097 cu. ft.; 1741 T. **5.** One is 4 times the other.

Page 380. — **1.** $37\frac{5}{7}$ sq. in. **2.** $39.60; $60.34. **3.** 90 sq. in. **4.** 24 sq. ft. **5.** 2600 sq. in. **6.** 4350 sq. in. **7.** 880 sq. in. **8.** 660 sq. in. **9.** $942\frac{6}{7}$ sq. ft. **10.** 220 sq. in.

Page 381. — **1.** 10 cu. in. **2.** $13\frac{1}{3}$ cu. ft. **3.** 21 cu. ft. **4.** 7 cu. in. **5.** 30 cu. ft. **6.** $43\frac{1}{3}$ cu. ft. **7.** 85,517,958$\frac{2}{3}$ cu. ft. **8.** $26\frac{2}{3}$ bu.

Page 382. — **1.** 192 sq. in. **2.** 3 gal. **3.** 6 gal. **4.** 884,756 sq. ft.; 1106 da.

Page 383. — **1.** $6\frac{2}{7}$ cu. yd. **2.** $3\frac{3}{8}$ cu. in.; 63 cones. **3.** $184\frac{4}{5}$ bu. **4.** 19,800 cu. ft.; 566 T. **5.** $565\frac{5}{7}$ cu. in. **6.** 13,860 cu. in. **7.** 36,960 cu. ft. **8.** 770,000 cu. in.

Page 384. — **1.** 616 sq. in. **2.** 154 sq. in. **3.** $314\frac{2}{7}$ sq. in. **4.** $1018\frac{2}{7}$ sq. ft. **5.** 5544 sq. in. **6.** 2464 sq. in. **7.** 15,400 sq. ft. **8.** 22,176 sq. ft. **9.** 169,161$\frac{1}{7}$ sq. in. **10.** 1,540,000 sq. in. **11.** 201,142,857$\frac{1}{7}$ sq. mi. **12.** 693 sq. ft.; $519.75.

Page 385. — **2.** 11,498$\frac{2}{3}$ cu. in. **3.** 91,989$\frac{1}{3}$ cu. in. **4.** 113,142$\frac{8}{7}$ cu. in. **5.** 38,808 cu. in. **6.** 4190$\frac{1}{2}\frac{0}{1}$ cu. in., or 4190.5 cu. in. **7.** 7241$\frac{1}{7}$ cu. in. **8.** 179$\frac{2}{3}$ cu. ft., or 179.7 cu. ft. **9.** 310.5 cu. ft. **10.** 92.0 cu. ft. **11.** 4$\frac{4}{2}\frac{4}{1}$ cu. ft., or 4.2 cu. ft. **12.** 1.8 cu. in. **13.** 2.8 cu. in. **14.** 179$\frac{3}{3}$ cu. in., or 179.7 cu. in. **15.** 606$\frac{3}{3}$ cu. in., or 606.4 cu. in. **16.** 350.9 cu. in. **17.** 33$\frac{1}{2}\frac{1}{1}$ cu. in., or 33.5 cu. in.; 1767$\frac{8}{7}$ cu. in., or 1767.9 cu. in.

Page 386. — **1.** 704 lb. **2.** 5632 lb.; 8 times. **3.** 15$\frac{5}{7}$ gal. **4.** 1527$\frac{3}{7}$ cu. ft.; about 3 T. **5.** 8.18 cu. in.; 17.98 cu. in.; about 2$\frac{1}{5}$ times. **7.** 19.8 gal.; 13.2 gal.

Page 387. — *See page* xlii.

Page 388. — **1.** 404 sq. ft. **2.** 17 qt.; $10.20. **3.** 220 sq. ft.; 9 qt.; $5.40. **4.** $.60 loss. **5.** 6 quart cans; 2 gallon cans. **6.** $.40 loss.

Page 389. — **1.** $102.87. **2.** $102.05. **3.** $212.93. **7.** $538.21.

Page 390. — **1.** 7.7%. **2.** 3.6%. **3.** 21.0%. **4.** 0.8%. **5.** 306.5%. **6.** 99.8%. **7.** $4.80. **8.** $5.85. **9.** $2.70. **10.** $55. **11.** $11.13. **12.** $112.50. **13.** $240.24. **14.** $3527.63. **15.** $8895.14. **16.** 18%. **17.** 30%. **18.** 33$\frac{1}{3}$%. **19.** 7$\frac{1}{2}$%. **20.** 8$\frac{1}{4}$%. **21.** 18%. **22.** 3960 cu. ft.; 1320 sq. ft. **23.** 8045$\frac{5}{7}$ cu. ft.; 2011$\frac{3}{7}$ sq. ft. **24.** 498.96 sq. in.; 1047.816 cu. in. **25.** 1386 sq. in.; 4851 cu. in. **26.** 38$\frac{1}{2}$ sq. ft.; 22$\frac{1}{2}\frac{1}{4}$ cu. ft.

Page 391. — **1.** $8.32; $8.48. **2.** 3%. *1st half year:* $24, $824; *2d half year:* $24.72, $848.72; *3d half year:* $25.44, $874.16; *4th half year:* $26.22, $900.38; *5th half year:* $27, $927.38; *6th half year:* $27.81, $955.19; *7th half year:* $28.65, $983.84; *8th half year:* $29.49, $1013.33. **3.** 1%; $202; $208.12.

Page 392. — **3.** $65.80. **4.** $185.03; $601.66.

Page 393. — **1.** 6 mo.; 6 periods; 2%; $56.31. **2.** $450.20. **3.** $358.22. **4.** $965.42. **5.** $823.40. **6.** $574.78. **7.** $784.17. **8.** $369.58. **9.** $1444.24. **10.** $267.28. **11.** $895.54. **12.** $695.35. **13.** $548.85. **14.** $1098.21. **15.** $1985.92. **16.** $1271.86. **17.** *2 yr.:* $519.57, $39.57; *4 yr.:* $562.40, $82.40; *6 yr.:* $608.76, $128.76; *8 yr.:* $658.94, $178.94; *9 yr.:* $685.56, $205.56; *10 yr.:* $713.26, $233.26. **18.** 3 mo.; 20 periods; 1%; $122.02. **19.** $964.99.

Page 394. — **2.** $592,096,000; $192,096,000. **3.** $199.99. **4.** $1199.93; $899.95. **5.** $197.99. **6.** $203.28. **7.** $199.99, or about $200; $400. **8.** $30.42.

Page 395. — **1.** $6400; $12,800; $25,600; $51,200; $102,400. **3.** $36.50; $73; $146. **4.** $60. **5.** $4; $16; $1024. **6.** $20; $80; $5120. **7.** $80; $320; $20,480. **8.** $[illegible]0; $560; $35,840. **9.** $800; $3200; $204,800. **10.** $1000; $4000; $256,000.

Page 396. — **1.** $.60; $1.20; $1.80; $3. **2.** $.44. **3.** $3.13. **4.** $10.33; $103.30. **5.** $1957.52; $1882.52. **6.** $1957.52. **8.** $61.94.

Page 397. — *See page* xlii.

Page 399. — **1.** $140.83; $774.23; $1458.21. **2.** $3276. **3.** $113.14. **4.** $344.97. **5.** $512.95. **6.** $1157.79. **7.** $20,950.43. **8.** $45,113.12.

1. $20.60. **2.** 4 periods; $42.04; $64.34; $87.55; $111.69; $247.83; $413.79; $616.10; $862.71. **3.** $136.14; $165.96; $202.31; $246.61; 20th to 25th year.

Page 400. — **1.** $366.45; $446.69; $544.53. 3 times; $3\frac{1}{2}$ times; 4 times. **2.** $172.25. **3.** $150; $22.25. **4.** $164.25; $26,078.60. **5.** $18; $471.30. **6.** $183.18. **7.** $976.10. **8.** $2896.56. **9.** $5392.82. **10.** $11,070.60. **11.** $33,885.50.

Page 403. — **1.** 8000 shares. **2.** 7%. **3.** $7; $840. **4.** $525,000; $26,250. **5.** $4\frac{1}{2}$%. **6.** 4%; $200. **7.** 7%; $350. **8.** $5\frac{1}{2}$%; $275. **9.** 5%; $250. **10.** 6%; $300. **11.** 4%; $200. **12.** 6%; $300. **13.** $7\frac{1}{2}$%; $375.

Page 406. — **3.** $6492.50. **4.** $2521.25. **5.** $1993.13. **6.** $23,025. **7.** $1939. **8.** $2930. **9.** $8070. **10.** $7905. **11.** $8242.50. **12.** $101,550.

Page 407. — **1.** $857.50. **2.** $525; $1382.50. **3.** $1377.50.

Page 408. — **1.** $210, loss. **2.** $2775, profit. **3.** $7000, loss. **4.** $2220, profit. **5.** $912.50, loss.

2. 5%. **3.** $6\frac{1}{4}$%. **4.** $7\frac{1}{2}$%. **5.** $8\frac{3}{4}$%. **6.** 5%. **7.** 10%. **8.** $6\frac{1}{4}$%. **9.** 4%. **10.** 5%. **11.** 8%. **12.** 8%. **13.** 6%. **14.** 5.8%. **15.** 6.3%. **16.** 8.5%. **17.** 4.2%. **18.** 5.6%. **19.** 6.0%. **20.** 8.2%. **21.** 6.5%.

Page 410. — **1.** $17.50. **2.** $25. **3.** $35. **4.** $30. **5.** $32.50. **6.** $20. **8.** $200. **9.** $6,000,000; 1946. **10.** $5000.

Page 412. — **1.** $857; $4285; $12,855. **2.** $987; $50. **3.** $1032; $50. **4.** $1057; $70. **5.** $1027; $55. **6.** $5360; $212.50. **7.** $4585; $300. **8.** $19,140; $800. **9.** $1896.50; $80. **11.** $22.50; $1034.50. **12.** $8.67; $895.67.

Page 413. — **1.** $890; $50; 5.6%. **2.** 4.6%. **3.** 5.2%. **4.** 4.4%. **5.** 4.9%. **6.** 5.8%. **7.** 6.8%. **8.** 5.9%. **9.** 4.6%. **10.** Former; 0.2% higher.

Page 414. — **2.** 7.8%. **3.** 6¼%. **4.** 5.6%. **5.** 4.5%. **6.** 5.1%. **7.** 5.6%. **8.** 6.3%.

Page 415. — **1.** $10; $120; $600; $1200; $1390. **2.** $2000; $1390; $610. **3.** $250. **4.** $15. **5.** $1200; $300. **6.** $16.

Page 416. — **2.** $38.50; 33 shares. **3.** $71.50; $9938.50; $3338.50. **5.** 24 shares; $26.

Page 417. — **1.** $21. **2.** $63; $5040. **3.** $840; $840. **4.** $1680; $840. **6.** $142.50. **7.** $2400; $2400.

Page 418. — **1.** $22. **2.** $18. **3.** $12.04. **4.** $45. **5.** $148.50. **6.** $31.50. **7.** $272. **8.** $134.40. **9.** $284.40.

Page 419. — **3.** $5.25. **4.** $40.60. **5.** $9.60. **6.** $18.04. **7.** $36.25. **8.** $3315. **9.** $140. **10.** $559.15.

Page 420. — **1.** $21. **2.** $42. **3.** $537.20; $644.64. **4.** $114. **5.** $3. **6.** $91.35. **7.** $56.25. **8.** $378.40. **9.** $396.68. **10.** $116.25. **11.** $177.48. **12.** $3675. **13.** $4797.76. **14.** $4819.50. **15.** $5175; $3175, gain.

Page 421. — *See page* xliii.

Page 423. — **1.** $20.14. **2.** $80.56. **3.** $92.82. **4.** $227.25. **5.** $159.95. **6.** $296.64. **7.** $237.15. **8.** $90.90. **9.** $228.50. **10.** $444.96. **11.** $658.75. **12.** $1299.48.

Page 425. — **4.** $225.30. **5.** $98.61. **6.** $503.50. **7.** $249.25.

Page 426. — **1.** $9060; $6000; $4000. **3.** $26,350.

1. 51.7%. **2.** $1409.68. **3.** 223.7%. **4.** $197,625; $118,575.

Page 428. — **1.** $730.05. **2.** $43.50. **3.** $4253.25. **4.** $1022.40.

Page 429. — **1.** $21.09; $210.90. **2.** $146.01; $64.89; $6.49. **3.** $30.05. **4.** $4.47. **5.** $15.98. **6.** $13.88; $14.93.

Page 431. — **1.** 2.95%. **2.** 29.5 mills on $1; $2.95 on $100; $29.50 on $1000. **3.** 3.86%; 38.6 mills on $1; $3.86 on $100; $38.60 on $1000. **4.** $2.20 on $100; 2.2%; 22 mills on $1; $22 on $1000. **5.** $3.75, etc. **6.** $3.85, etc. **7.** $3.66, etc. **8.** $4.32, etc. **9.** $2.65, etc. **10.** $2.24, etc. **11.** $2.70, etc. **12.** $3.95, etc. **13.** $3.08, etc. **14.** $4.22; $3.38. **15.** 80%.

Page 432. — **4.** $3.54. **5.** $1.469, $.538, $.267, $.100, $.050, $.038, $.019; $3.83.

Page 433. — 1. \$513.30. **2.** \$112.24; \$19.28; \$8. **3.** \$13,575; \$534.86. **4.** \$804. **5.** \$1989.82. **6.** \$1675.22. **7.** \$3788.80. **8.** \$7020.75. **9.** \$17,549. **10.** \$4784. **11.** \$5370.75. **12.** Hatton: \$262.35, \$220.38; Lakewood: \$275.40, \$231.34.

Page 434. — 2. \$398.25. **3.** \$433.65; \$330.99. **4.** \$1.77; \$1.77; \$1.77; *\$6000 column:* \$226.56, \$228.33, \$230.10; *\$7000 column:* \$253.11, \$254.88, \$256.65, \$258.42, \$260.19, \$261.96, \$263.73, \$265.50. **5.** \$54.87; \$56.64; \$58.41; \$60.18; \$61.95; \$63.72; \$65.49; \$67.26; \$69.03.

Page 435. — 1. 317 da.; \$554.18. **2.** \$40.19. **3.** \$601.98. **4.** \$61.76. **5.** \$7.69. **6.** \$14.45. **7.** \$330.66. **8.** \$34.83. **9.** \$11.61.

Page 436. — 1. \$4.80. **2.** \$44. **3.** \$15.40. **4.** \$58.95. **5.** \$58. **6.** \$41. **7.** \$78.60. **8.** \$137.75. **9.** \$6. **10.** \$65.77. **11.** \$34.85. **12.** \$545.12.

Page 437. — 14. \$3.85; \$4.40; \$5.50. **15.** \$1.40; \$3.50. **16.** \$52.50. **17.** \$1800. **18.** \$16,800. **19.** \$75.16.

Page 442. — 8. 268. **9.** 276. **10.** 168. **11.** 123. **12.** 201. **13.** 440. **14.** .96. **15.** 3.44. **16.** 1.90. **17.** 1.49. **18.** 48.1. **19.** 2.04. **20.** 25. **21.** 29. **22.** 317. **23.** 588.

Page 443. — 1. 12. **2.** 13. **3.** 22. **4.** 19. **5.** 45. **6.** 403. **7.** 601. **8.** 1150. **9.** 1101. **10.** 1080. **11.** 5. **12.** 4.5. **13.** $22\frac{3}{4}$. **14.** 1.40. **15.** 3.30.

Page 444. — 1. 36. **2.** 24. **3.** 12. **4.** 120. **5.** 28. **6.** 20. **7.** 56. **8.** 425. **9.** 13. **10.** 1.35. **11.** 105. **12.** 151.

Page 445. — 1. 7. **2.** 6. **3.** 3. **4.** 3. **5.** 1. **6.** $\frac{1}{2}$. **7.** $\frac{1}{5}$. **8.** $1\frac{3}{4}$. **9.** $\frac{1}{2}$. **10.** 3. **11.** 3. **12.** 3.03. **13.** .06.

Page 446. — 2. 680. **3.** 81. **4.** 6. **5.** 49. **6.** 625. **7.** 12. **8.** 8. **9.** 8. **10.** 14. **11.** $\frac{1}{4}$. **12.** 35. **13.** 5.

3. 1. **4.** 5. **5.** 2. **6.** 15. **7.** 20. **8.** 10. **9.** 32. **10.** 24. **11.** 30.

Page 447. — 2. $21\frac{1}{3}$. **3.** 27. **4.** 5. **5.** 4. **6.** 4. **7.** 5. **8.** 120. **9.** 18. **10.** 36. **11.** 500. **12.** 10. **13.** \$21. **14.** 40. **15.** 14. **16.** 200. **17.** $\frac{1}{2}$. **18.** 13 yr. **19.** 2. **20.** 12 yr.

Page 448. — 3. \$45. **4.** \$23. **5.** \$1500. **6.** \$8.50. **7.** $6x$ dollars; 14 wk.

Page 449. — 2. Sweater, \$8; racket, \$4. **3.** Book, \$1.30; pencil, \$.10. **4.** House, \$33,000; lot, \$15,000. **5.** 6 cups milk. 3 cups water. **6.** 1 yd.; 4 yd. **7.** Jane, \$42; Mary, \$130.

Page 450. — **3.** Frank, 4 yr.; Betty, 12 yr. **4.** 15 yr. **5.** Susan, 6 yr.; Henry, 24 yr. **6.** Mother, 32 yr.; Edna, 8 yr. **7.** Joe, 8 yr.; Harry, 21 yr. **8.** Lena, 7 yr.; Hugh, 14 yr. **9.** 11 yr.

Page 451. — *See page* xliii.

Page 452. — **1.** \$10.82. **2.** \$45. **3.** \$60. **4.** 50¢. **5.** \$35. **6.** \$20,000. **7.** \$500.

Page 453. — **1.** \$20. **2.** \$400.

Page 454. — **1.** 96. **2.** 70. **3.** 330. **4.** 266. **5.** 22 sq. mi. **6.** 4050 sq. yd. **7.** 4171 sq. ft. **8.** 700 sq. ft. **9.** 16 sq. ft. **10.** 18. **11.** $17\frac{1}{2}$. **12.** 30. **13.** 30. **14.** $27\frac{1}{2}$ sq. in. **15.** $27\frac{1}{2}$ sq. ft. **16.** 50 sq. ft. **17.** $26\frac{1}{4}$ sq. ft.

Page 456. — **1.** 5 ft. **2.** 4 ft. **3.** 12 ft.; 8 ft.; 9 ft.; 6 ft.; $5\frac{1}{3}$ ft.; $7\frac{1}{5}$ ft.; 4 ft. **5.** 24 ft. **6.** 15 ft. **7.** 25 ft. **8.** 14 ft. **9.** 12 ft. **10.** 12. **11.** 16. **12.** 16. **13.** 36. **14.** 32. **15.** 34. **16.** 75 ft. **17.** 18 ft.

Page 457. — **1.** 9 ft. **2.** 5 ft. **3.** 6 ft. **4.** $b = \frac{2A}{h}$; 26 ft. **5.** 216. **6.** 96. **7.** 85. **8.** 40.

Page 458. — **1.** $6\frac{1}{4}$ ft. **2.** $l = \frac{V}{wh}$; $w = \frac{V}{lh}$. **3.** 216. **4.** 19. **5.** 5. **6.** 2 ft.

Page 459. — **1.** 63 in. **2.** 28 in. **3.** $24\frac{1}{2}$ in.; 42 in.; 49 in. **5.** $3\frac{1}{2}$ ft. **6.** $22\frac{10}{11}$ ft.; $11\frac{5}{11}$ ft. **7.** 7 ft.; $4\frac{5}{11}$ ft. **8.** $h = \frac{V}{\pi r^2}$. **9.** 30 ft.; 37 ft.; 62 ft.; 21.5 ft.

Page 460. — **1.** 8 ft. **2.** 7 in. **3.** 15 ft. **4.** $6\frac{1}{2}$ ft. **5.** $7\frac{1}{2}$ in. **6.** $3\frac{1}{3}$ ft. **7.** 5 ft. **8.** 52 in. **9.** $r = \frac{C}{2\pi}$; $6\frac{4}{11}$ in.; $1\frac{3}{4}$ in.; 42 in.; $31\frac{1}{2}$ in. **10.** 7 ft. **11.** 6 ft. **12.** 8 ft. **13.** 11 ft. **14.** $10\frac{1}{2}$ ft. **15.** Bin; 2 ft. **16.** $R = \frac{P}{B}$; $B = \frac{P}{R}$. **17.** 4500. **18.** 325. **19.** 27.2. **20.** 12.

Page 464. — **7.** $\frac{1}{12}$. **8.** $\frac{21}{2}$. **9.** $\frac{3}{64}$. **10.** $\frac{1}{10}$. **11.** $\frac{1}{8}$. **12.** $\frac{8}{1}$. **13.** $\frac{4}{1}$. **14.** $\frac{30}{1}$. **15.** $\frac{28}{1}$. **16.** $\frac{1}{16}$. **17.** $\frac{20}{1}$. **18.** $\frac{1}{8}$. **19.** $\frac{1}{12}$; $\frac{1}{36}$. **20.** $\frac{1}{20}$; $\frac{1}{10}$.

Page 465. — **1.** $\frac{3}{4}$; $\frac{6}{8}$; yes; $\frac{3}{4} = \frac{6}{8}$. **2.** $\frac{2}{3} = \frac{4}{6}$.

Page 466. — **1.** 10. **2.** 21. **3.** 3. **4.** 2. **5.** 2. **6.** $2\frac{1}{4}$. **7.** 6. **8.** 128. **9.** 60 in. **10.** $4\frac{1}{2}$ ft.

Page 467. — *See page* xliii.

Page 468. — **1.** 3. **2.** 3. **3.** 2. **4.** 36. **5.** 40. **6.** 18. **7.** 8. **8.** 42. **9.** $28\frac{1}{2}$. **10.** $15. **11.** 18 games.

Page 470. — **5.** 10. **6.** $7\frac{1}{2}$ in. **7.** $BC = 8$ ft.; $AC = 9$ ft. **8.** $MO = 10\frac{1}{2}$ in.; $NO = 10\frac{1}{2}$ in.

Page 471. — **1.** 66 ft. **2.** 35 ft. **3.** 180 ft. **4.** 528 ft. **5.** 25 ft. **6.** 15 ft. **7.** $37\frac{1}{2}$ ft. **8.** $28\frac{3}{4}$ ft. **9.** 45 ft.

Page 472. — **1.** 6 ft. **2.** $2\frac{3}{4}$ ft. **3.** 10 ft.; 12 ft.

Page 473. — **1.** 630 ft. **2.** 75 ft. **3.** 180 ft. **4.** 660 ft. **5.** 160 ft. **6.** 80 ft. **7.** 78 ft. **8.** 600 ft. **9.** $1687\frac{1}{2}$ ft. **10.** 2440 ft. **11.** $687\frac{1}{2}$ ft. **12.** 750 ft. **13.** 400 ft. **14.** 200 ft.

Page 474. — **1.** $\frac{1}{4}$ in.; $\frac{3}{8}$ in.; $\frac{3}{4}$ in.; $1\frac{1}{8}$ in. **2.** 1 in.; $1\frac{1}{4}$ in.; $1\frac{3}{4}$ in.; 2 in. **3.** 2 in.; $1\frac{1}{2}$ in.; $2\frac{1}{2}$ in. **4.** 1 in.; $1\frac{1}{2}$ in.; 2 in.; 3 in.; $3\frac{1}{4}$ in. **5.** Lane, $\frac{1}{4}$ in.; Oak Rd., $\frac{3}{8}$ in.; State Rd., $\frac{3}{4}$ in.; Main St., $\frac{1}{2}$ in.

Page 475. — **1.** 25 in. long; 20 in. wide. **2.** Tom's scale, $12\frac{1}{2}$ in. by 10 in.; Anne's scale, 5 in. by 4 in.; Bob's scale, 10 in. by 8 in. **3.** 1 in. = 10 ft. **4.** No; $\frac{1}{4}$ in. too short. **5.** 10 in. by 8 in.; 40 in. by 32 in.; $\frac{5}{8}$ in. by $\frac{1}{2}$ in.; $\frac{1}{8}$ in. = 2 ft. **7.** 1 in. = 4 ft. **9.** $\frac{1}{8}$ in. = 1 mi.

Page 476. — **1.** 4 ft., 6 ft., 8 ft., 10 ft., 12 ft., 14 ft., 16 ft., 18 ft., 20 ft., 22 ft. **2.** 14 ft. **3.** 12 ft. **4.** Sun porch, 16 ft. by 9 ft.; living room, 20 ft. by 18 ft.; kitchen, 14 ft. by 11 ft.; dining alcove, 14 ft. by 6 ft.; den, $13\frac{1}{2}$ ft. by 8 ft. **5.** 48 ft. by 32 ft.

Page 477. — **1.** 1 mi.; 2 mi.; 4 mi.; $\frac{1}{2}$ mi. **2.** 1 mi. **3.** 2 mi. **4.** $3\frac{1}{2}$ mi.; $4\frac{1}{2}$ mi. **5.** $5\frac{1}{2}$ mi. **6.** $4\frac{1}{4}$ mi.; $1\frac{1}{4}$ mi. **7.** 7 mi. **8.** $10\frac{1}{2}$ mi.; 5 mi.

Page 479. — *See page* xliii.

Page 480. — **4.** 196. **5.** 400. **6.** 225. **7.** 324. **8.** 625. **9.** 256. **10.** 900. **11.** 8100. **12.** 10,000. **13.** 40,000. **14.** 1. **15.** 2. **16.** 3. **17.** 8. **18.** 4. **19.** 9. **20.** 7. **21.** 10. **22.** 12. **23.** 11. **24.** 9. **25.** 10. **26.** 20. **27.** 30. **28.** 15. **29.** 40.

Page 482. — **4.** 5.2. **5.** 7.2. **6.** 9.3. **7.** 28.1. **8.** 30.2. **9.** 48.3.

Page 483. — **1.** 11.4 ft. **2.** 27.4 ft.; 25.5 ft.; 24.5 ft. **3.** 6.8. **4.** 6.5. **5.** 9.5. **6.** 9.8. **7.** 7.1. **8.** 7.5. **9.** 7.7. **10.** 4.9. **11.** 13.2. **12.** 13.6. **13.** 13.8. **14.** 13.9. **15.** 21.2. **16.** 21.4. **17.** 21.7. **18.** 21.9. **19.** 38.1. **20.** 45.0. **21.** 56.8. **22.** 94.2. **23.** 4.6 in.

Page 484. — **2.** 6.93. **3.** 4.12. **4.** 6.63. **5.** 7.81. **6.** 8.60. **7.** 8.06. **8.** 14.14. **9.** 20.69. **10.** 11.31. **11.** 11.79. **12.** 11.83. **13.** 14.73. **14.** 18.49. **15.** 23.02. **16.** 24.35. **17.** 30.74. **18.** 32.94. **19.** 44.92. **20.** 56.83. **21.** 68.08. **22.** 82.55.

Page 486. — **1.** 75 ft. **2.** 96.2 ft. **3.** 99 ft. **4.** 43.4 ft. **5.** 25 ft. **6.** 10.82 ft. **7.** 21.63 ft. **8.** 53.01 ft. **9.** 13.45 mi. **10.** 69.46 ft. **11.** 38.08 ft.

Page 487. — **1.** 7.1 mi. **2.** 19.4 mi. **3.** 4.30 ft. **4.** 4.3 mi. **5.** 7.1 mi. **6.** 4.5 mi. **7.** 12.2 mi. **8.** 9.2 mi. **9.** 8.6 mi **10.** 13.4 mi. **11.** 5.7 mi. **12.** 8.0 mi.

Page 488. — **1.** 8.29 mi. **2.** 208.71 ft. **3.** 96.3 ft. **5.** 5.3 ft.

Page 490. — **6.** 2.455 m. **7.** 3 ft. $2\frac{1}{4}$ in. **9.** 1.5 km.; 5 km.; .8 km.

Page 491. — **1.** 464.2 yd. **2.** 7.8 mi. **3.** 110 yd.; 220 yd.; 1650 yd. **4.** 22 yd. **5.** 147 mi. **6.** 78.9 mi. **7.** Former; 2499.6 ft.

Page 492. — **2.** 48 l. **3.** 60 cu. dm.; 60 l. **4.** 3840 cu. in.; 66.5 qt.

Page 493. — **2.** 2.5 g.; 12.5 g. **3.** 2 dimes; 200 nickels. **4.** 12,000 cu. dm.; 12,000 l.; 12,000 kg. **5.** 324 cu. ft.; 2430 gal.; 20,169 lb.

Page 495. — **1.** Protein, .32 oz.; carbohydrate, .48 oz.; 92 calories. **2.** Protein, .16 oz.; carbohydrate, .8 oz.; 110 calories. **3.** Protein, .16 oz.; carbohydrate, 1.44 oz.; 184 calories. **4.** Protein, .16 oz.; carbohydrate, 3.52 oz.; fat, .096 oz.; 447 calories. **5.** Protein, 1.44 oz.; carbohydrate, 2.4 oz.; fat, 1.92 oz.; 932 calories. **6.** Protein, 1.04 oz.; fat, .8 oz.; 324 calories. **7.** Protein, 1.12 oz.; fat, 1.44 oz.; 496 calories. **8.** Protein, .16 oz.; fat, 13.6 oz.; 3486 calories. **9.** Protein, 1.68 oz.; fat, .24 oz.; 254 calories. **10.** Protein, .64 oz.; carbohydrate, 5.76 oz.; 736 calories. **11.** Protein, 3.04 oz.; fat, 2.88 oz.; 1084 calories. **12.** Protein, .28 oz.; carbohydrate, 2.08 oz.; fat, .84 oz.; 485 calories. **13.** Protein, 1.02 oz.; carbohydrate, 1.7 oz.; fat, 1.36 oz.; 660 calories.

Page 496. — **1.** Breakfast, 560 calories; luncheon, 800 calories; dinner, 1034 calories; total, 2394 calories. **2.** Protein calories, 64, 106, 156; fat calories, 199, 322, 388; carbohydrate calories, 297, 372, 490. **3.** 326 protein calories, 13.6%; 909 fat calories, 38%; 1159 carbohydrate calories, 48.4%.

Page 497. — **1.** Yes. **2.** 2894 calories; 490 protein calories; 1.9% more.

Page 498. — **2.** 1360 calories; 1496 calories; 1632 calories; 1700 calories. **3.** 2848 calories. **4.** 1836 calories. **5.** 2538 calories. **6.** 2808 calories. **7.** 638 calories. **9.** Spinach, 92 calories; bread, 1182 calories; butter, 3486 calories.

Page 500. — **1.** \$72. **2.** 25.5 in. **3.** 112,165 lb.; 56.08 T. **4.** 528 sq. in., 1584 cu. in. **5.** 528 sq. in.; 1848 cu. in. **6.** $691\frac{3}{7}$ sq. in.; $1728\frac{4}{7}$ cu. in. **7.** $49\frac{1}{2}$ sq. ft.; $37\frac{1}{8}$ cu. ft. **8.** $146\frac{2}{3}$ sq. ft.; $256\frac{2}{3}$ cu. ft. **9.** 49.28 sq. ft.; 34.496 cu. ft. **10.** 15 cu. in. **11.** $30\frac{1}{3}$ cu. in. **12.** $31\frac{1}{2}$ cu. in. **13.** 62 cu. ft. **14.** 19.25 cu. ft. **15.** 79.11 cu. ft. **16.** $234\frac{2}{3}$ cu. ft. **17.** 1.49 cu. ft. **18.** $320\frac{5}{6}$ cu. in. **19.** 1604.94 cu. yd. **20.** 5544 sq. ft.; 38,808 cu. ft. **21.** 154 sq. in.; $179\frac{2}{3}$ cu. in. **22.** 1386 sq. yd.; 4851 cu. yd. **23.** 745.36 sq. ft.; 1913.09 cu. ft. **24.** 154 sq. ft.; $179\frac{2}{3}$ cu. ft.

ANSWERS FOR IMPROVEMENT TESTS

Page 3. **Test 1A.** — **1.** 34,020. **2.** 34,016. **3.** 31,339. **4.** 30,802. **5.** 39,048.

Test 1B. — **1.** 3570. **2.** 3890. **3.** 4869. **4.** 1903. **5.** 2096. **6.** 4598. **7.** 4713. **8.** 3547. **9.** 2537. **10.** 2880. **11.** 1161. **12.** 2473. **13.** 5241. **14.** 270. **15.** 6162.

Test 1C. — **1.** 784,875. **2.** 241,466. **3.** 139,620. **4.** 430,056. **5.** 543,244.

Test 1D. — **1.** 744. **2.** 839 *R* 25, or 839 $\frac{25}{48}$. **3.** 494. **4.** 806. **5.** 580 *R* 11, or 580$\frac{1}{7}$.

Page 19. **Test 2A.** — **1.** 37,896. **2.** 43,989. **3.** 38,998. **4.** 36,691. **5.** 30,759.

Test 2B. — **1.** 32,309. **2.** 7160. **3.** 28,888. **4.** 18,732. **5.** 55,761. **6.** 28,970. **7.** 23,255. **8.** 20,553. **9.** 75,577. **10.** 39,904. **11.** 19,289. **12.** 36,369.

Test 2C. — **1.** 376,352. **2.** 4,397,211. **3.** 368,784. **4.** 648,396. **5.** 71,372.

Test 2D. — **1.** 537 *R* 13, or 537$\frac{13}{29}$. **2.** 517 *R* 10, or 517$\frac{5}{31}$. **3.** 895. **4.** 616. **5.** 2078.

Page 41. **Test 3A.** — **1.** $24\frac{7}{16}$. **2.** $19\frac{5}{8}$. **3.** $23\frac{1}{9}$. **4.** $23\frac{11}{12}$. **5.** $20\frac{2}{3}$.

Test 3B. — **1.** $3\frac{1}{2}$. **2.** $2\frac{3}{8}$. **3.** $2\frac{5}{8}$. **4.** $4\frac{9}{10}$. **5.** $7\frac{5}{8}$. **6.** $2\frac{1}{8}$. **7.** $1\frac{3}{8}$. **8.** $1\frac{1}{2}$. **9.** $1\frac{1}{3}$. **10.** $3\frac{7}{12}$. **11.** $6\frac{3}{20}$. **12.** $2\frac{1}{10}$.

Test 3C. — **1.** 27. **2.** $\frac{1}{16}$. **3.** $1\frac{9}{10}$. **4.** $6\frac{2}{3}$. **5.** $1\frac{2}{5}$. **6.** 42. **7.** $11\frac{3}{4}$. **8.** $4\frac{1}{8}$. **9.** 12.

Test 3D. — **1.** $\frac{1}{4}$. **2.** 48. **3.** $\frac{3}{16}$. **4.** $\frac{2}{9}$. **5.** $\frac{9}{10}$. **6.** $6\frac{3}{4}$. **7.** $1\frac{1}{2}$. **8.** $2\frac{2}{3}$. **9.** $\frac{1}{2}$.

Page 51. **Test 4A.** — **1.** 27,502. **2.** 28,271. **3.** 28,564. **4.** 23,819. **5.** 26,374. **6.** 17,929. **7.** 14,652. **8.** 22,812. **9.** 30,035. **10.** 28,384.

Test 4B. — **1.** 1447. **2.** 4509. **3.** 1883. **4.** 6645. **5.** 1293. **6.** 4980. **7.** 1774. **8.** 659. **9.** 6805. **10.** 2528. **11.** 3968. **12.** 4679. **13.** 2757. **14.** 2709. **15.** 7696.

Test 4C. — **1.** 41,175. **2.** 38,938. **3.** 35,546. **4.** 33,447. **5.** 42,820.

Page 69. Test 5A. — **1.** $10\frac{5}{8}$. **2.** 14. **3.** $14\frac{1}{8}$. **4.** $12\frac{1}{10}$. **5.** $15\frac{3}{4}$. **6.** $10\frac{2}{5}$. **7.** $15\frac{19}{24}$. **8.** $8\frac{1}{6}$. **9.** $5\frac{11}{15}$. **10.** $8\frac{1}{16}$. **11.** $10\frac{5}{12}$. **12.** $10\frac{1}{12}$.

Test 5B. — **1.** \$4.34. **2.** \$3.78. **3.** $155\frac{1}{4}$. **4.** $\frac{1}{8}$. **5.** 10. **6.** 6. **7.** $\frac{5}{12}$. **8.** 15. **9.** 21. **10.** $\frac{3}{4}$. **11.** $22\frac{1}{2}$. **12.** $2\frac{1}{4}$.

Test 5C. — **1.** 10. **2.** $1\frac{1}{8}$. **3.** 1. **4.** 4. **5.** $1\frac{1}{4}$. **6.** $\frac{1}{10}$. **7.** $3\frac{1}{3}$. **8.** 6. **9.** 2. **10.** $\frac{3}{5}$. **11.** $\frac{3}{4}$. **12.** $3\frac{1}{8}$.

Page 79. Test 6A. — **1.** $\frac{6}{25}$. **2.** $\frac{7}{20}$. **3.** $\frac{3}{8}$. **4.** $\frac{1}{20}$. **5.** $\frac{9}{250}$. **6.** $\frac{3}{5}$. **7.** $\frac{3}{500}$. **8.** $\frac{1}{8}$. **9.** $\frac{3}{40}$. **10.** $\frac{7}{8}$. **11.** .45. **12.** .375. **13.** .8. **14.** .84. **15.** .26.

Test 6B. — **1.** .57. **2.** .42. **3.** .58. **4.** .56. **5.** .89. **6.** .16. **7.** .27. **8.** .36.

Test 6C. — **1.** 7.68. **2.** 34.44. **3.** 9. **4.** 1205.4. **5.** 76.44. **6.** 2.772. **7.** .5877. **8.** 19.24. **9.** .01296. **10.** 2.514. **11.** .000045. **12.** 86.4.

Test 6D. — **1.** .0341. **2.** 41,800. **3.** 5.32.

Page 99. Test 7A. — **1.** $10\frac{5}{16}$. **2.** $18\frac{1}{8}$. **3.** $25\frac{5}{12}$. **4.** $11\frac{13}{10}$. **5.** $18\frac{5}{8}$.

Test 7B. — **1.** $1\frac{1}{8}$. **2.** $1\frac{3}{8}$. **3.** $5\frac{1}{20}$. **4.** $2\frac{9}{16}$. **5.** $8\frac{5}{12}$. **6.** $\frac{11}{12}$. **7.** $2\frac{3}{8}$. **8.** $1\frac{5}{12}$. **9.** $2\frac{3}{10}$. **10.** $2\frac{3}{10}$. **11.** $1\frac{5}{12}$. **12.** $5\frac{5}{16}$.

Test 7C. — **1.** $\frac{3}{16}$. **2.** $46\frac{1}{2}$. **3.** $437\frac{1}{2}$. **4.** $\frac{5}{16}$. **5.** 195. **6.** 414. **7.** $16\frac{1}{2}$. **8.** $115\frac{1}{2}$. **9.** 18.

Test 7D. — **1.** $\frac{1}{2}$. **2.** $6\frac{2}{3}$. **3.** 3. **4.** $\frac{5}{9}$. **5.** $7\frac{1}{2}$. **6.** 26. **7.** $\frac{3}{8}$. **8.** $2\frac{1}{2}$. **9.** $\frac{2}{5}$.

Page 111. Test 8A. — **1.** 6399. **2.** 5976. **3.** 5993. **4.** 5818. **5.** 4588.

Test 8B. — **1.** 264,279. **2.** 255,757. **3.** 374,660. **4.** 194,330. **5.** 298,248. **6.** 493,268. **7.** 543,957. **8.** 684,691. **9.** 291,947. **10.** 347,567. **11.** 137,588. **12.** 113,585.

Test 8C. — **1.** 448,224. **2.** 318,934. **3.** 417,600. **4.** 447,034. **5.** 208,050.

Test 8D. — **1.** 259. **2.** 580. **3.** 825 *R* 345, or $825\frac{345}{688}$.

Page 129. Test 9A. — **1.** .86. **2.** .67. **3.** .97. **4.** .17. **5.** .56. **6.** .92. **7.** .87. **8.** .31.

Test 9B. — **1.** 218.43. **2.** 1.9665. **3.** .656. **4.** 65,940. **5.** 875. **6.** .4068. **7.** .9177. **8.** 51,000. **9.** 220. **10.** .000042. **11.** .01072. **12.** .99.

Test 9C. — **1.** 4.75. **2.** .625. **3.** 1.71.

Test 9D. — **1.** .54. **2.** 7.85. **3.** 4.76.

Test 9E. — **1.** 12.4. **2.** 3990. **3.** .0241.

Page 135. Test 10A. — **1.** 312,998. **2.** 210,123. **3.** 205,966. **4.** 232,519. **5.** 135,263. **6.** 245,607. **7.** 268,317. **8.** 208,250.

Test 10B. — **1.** 48,496. **2.** 40,968. **3.** 41,966. **4.** 50,233. **5.** 50,752.

Test 10C. — **1.** 5669. **2.** 2196. **3.** 3869. **4.** 4828. **5.** 5855. **6.** 1474. **7.** 7877. **8.** 2489. **9.** 4756. **10.** 1973. **11.** 6440. **12.** 1744. **13.** 2988. **14.** 1986. **15.** 3350.

Page 143. Test 11A. — **1.** 7297. **2.** 6437. **3.** 6576. **4.** 6361. **5.** 6059.

Test 11B. **1.** 4388. **2.** 8482. **3.** 13,660. **4.** 17,735. **5.** 15,950. **6.** 57,906. **7.** 53,738. **8.** 9620. **9.** 6678. **10.** 57,411. **11.** 8578. **12.** 2099.

Test 11C. **1.** 214,220. **2.** 173,540. **3.** 577,532. **4.** 405,158. **5.** 626,987. **6.** 288,269. **7.** 139,157. **8.** 8875. **9.** 109,680. **10.** 449,885. **11.** 346,378. **12.** 197,380.

Page 157. Test 12A. — **1.** 21,762. **2.** 12,629. **3.** 11,153. **4.** 75,768. **5.** 18,044. **6.** 7228. **7.** 19,234. **8.** 6574. **9.** 75,990.

Test 12B. — **1.** 194,557. **2.** 802,109. **3.** 433,329. **4.** 884,952. **5.** 369,234. **6.** 441,275.

Test 12C. — **1.** 7,795,722. **2.** 1,791,716. **3.** 5,524,770. **4.** 1,322,244.

Test 12D. — **1.** $317\frac{5}{36}$. **2.** $457\frac{15}{92}$. **3.** 709. **4.** 219. **5.** $845\frac{27}{38}$. **6.** $358\frac{1}{17}$.

Test 12E. — **1.** $647\frac{1}{2}$. **2.** $903\frac{11}{56}$. **3.** 523. **4.** $1110\frac{25}{31}$. **5.** 748. **6.** $573\frac{1}{2}$.

Page 165. Test 13A. — **1.** $15\frac{5}{8}$. **2.** $17\frac{4}{5}$. **3.** $16\frac{7}{8}$. **4.** $18\frac{11}{12}$. **5.** $18\frac{1}{4}$.

Test 13B. — **1.** $3\frac{1}{8}$. **2.** $\frac{1}{8}$. **3.** $2\frac{1}{6}$. **4.** $\frac{3}{5}$. **5.** $1\frac{3}{8}$. **6.** $4\frac{7}{12}$. **7.** $1\frac{1}{2}$. **8.** $2\frac{5}{12}$. **9.** $1\frac{9}{10}$. **10.** $3\frac{1}{3}$. **11.** $3\frac{1}{10}$. **12.** $1\frac{1}{16}$.

Test 13C. — **1.** 26. **2.** $5\frac{1}{4}$. **3.** $67\frac{1}{2}$. **4.** $4\frac{3}{4}$. **5.** 12. **6.** $1767\frac{1}{2}$. **7.** $17\frac{1}{2}$. **8.** 7. **9.** 1003.

Test 13D. — **1.** 16. **2.** $\frac{5}{8}$. **3.** $\frac{2}{3}$. **4.** $2\frac{6}{11}$. **5.** $7\frac{1}{3}$. **6.** $1\frac{1}{8}$. **7.** $\frac{1}{2}$. **8.** $4\frac{1}{2}$. **9.** $5\frac{5}{8}$.

Page 179. Test 14A. — **1.** $\frac{23}{50}$. **2.** $\frac{13}{20}$. **3.** $\frac{1}{6}$. **4.** $\frac{1}{8}$. **5.** $\frac{1}{125}$. **6.** $\frac{9}{10}$. **7.** $\frac{1}{4}$. **8.** $\frac{5}{8}$. **9.** $\frac{1}{20}$. **10.** $\frac{1}{15}$. **11.** .48. **12.** .875. **13.** .55. **14.** .74. **15.** .234.

Page 179 (*continued*). — **Test 14B.** — **1.** .43. **2.** .36. **3.** .93. **4.** .56. **5.** .56. **6.** .22. **7.** .08. **8.** .42.

Test 14C. — **1.** 157.17. **2.** 6.478. **3.** .000045. **4.** 53.07. **5.** 414. **6.** .02344. **7.** 1.2366. **8.** .0002095. **9.** 6370. **10.** .474. **11.** .6608. **12.** 982,800.

Test 14D. — **1.** .0827. **2.** 45,700. **3.** 7.923.

Page 201. Test 15A. — **1.** 728. **2.** 651. **3.** 785. **4.** 766. **5.** 714. **6.** 807.

Test 15B. — **1.** 625,578. **2.** 325,958. **3.** 333,748. **4.** 516,608. **5.** 729,175. **6.** 578,990. **7.** 131,789. **8.** 136,330. **9.** 449,189. **10.** 190,334. **11.** 224,427. **12.** 667,689.

Test 15C. — **1.** 211,475. **2.** 224,388. **3.** 518,805. **4.** 822,624. **5.** 234,144. **6.** 631,862.

Test 15D. — **1.** 63.1. **2.** 605. **3.** 59.1.

Page 221. Test 16A. — **1.** $17\frac{5}{8}$. **2.** $19\frac{1}{3}$. **3.** 22. **4.** $12\frac{7}{24}$. **5.** $16\frac{1}{4}$.

Test 16B. — **1.** $4\frac{1}{8}$. **2.** $1\frac{1}{2}$. **3.** $3\frac{1}{6}$. **4.** $3\frac{7}{10}$. **5.** $5\frac{7}{15}$. **6.** $2\frac{15}{16}$. **7.** $1\frac{1}{10}$. **8.** $1\frac{1}{6}$. **9.** $6\frac{7}{12}$. **10.** $4\frac{1}{2}$. **11.** $2\frac{1}{12}$. **12.** $3\frac{1}{2}$.

Test 16C. — **1.** 3420. **2.** $2062\frac{1}{2}$. **3.** 4824. **4.** $3\frac{1}{8}$. **5.** $31\frac{1}{3}$. **6.** $5\frac{5}{8}$. **7.** $\frac{7}{10}$. **8.** 70. **9.** $\frac{5}{12}$.

Test 16D. — **1.** 8. **2.** 14. **3.** $\frac{7}{10}$. **4.** $\frac{5}{32}$. **5.** $\frac{2}{3}$. **6.** $\frac{4}{5}$. **7.** $4\frac{2}{3}$. **8.** $1\frac{7}{8}$. **9.** $2\frac{3}{10}$.

Page 241. Test 17A. — **1.** 48,599. **2.** 44,899. **3.** 42,997. **4.** 40,859. **5.** 39,769.

Test 17B. — **1.** 34,278. **2.** 25,099. **3.** 29,890. **4.** 59,960. **5.** 19,485. **6.** 49,336. **7.** 24,668. **8.** 27,865. **9.** 43,631. **10.** 17,592. **11.** 47,874. **12.** 69,217. **13.** 55,938. **14.** 51,867. **15.** 15,451. **16.** 39,426.

Test 17C. — **1.** 166,222,872. **2.** 2,496,265,440. **3.** 397,321,816.

Test 17D. — **1.** 620.3. **2.** 577.5. **3.** 815. **4.** 656.3. **5.** 196.3.

Page 261. Test 18A. — **1.** $16\frac{1}{2}$. **2.** $14\frac{7}{12}$. **3.** 34. **4.** $22\frac{11}{30}$. **5.** 17.

Test 18B. — **1.** $1\frac{3}{8}$. **2.** $1\frac{15}{16}$. **3.** $\frac{4}{5}$. **4.** $5\frac{1}{6}$. **5.** $4\frac{1}{5}$. **6.** $3\frac{7}{8}$. **7.** $2\frac{1}{8}$. **8.** $\frac{1}{2}$. **9.** $6\frac{1}{2}$. **10.** $3\frac{1}{12}$. **11.** $1\frac{1}{10}$. **12.** $6\frac{3}{20}$.

Test 18C. — **1.** $6.88. **2.** $18. **3.** $36.33. **4.** $\frac{1}{3}$. **5.** $8\frac{1}{4}$. **6.** $18\frac{1}{3}$. **7.** $41\frac{2}{3}$. **8.** 175. **9.** 10.

Test 18D. — **1.** $\frac{1}{12}$. **2.** $\frac{14}{15}$. **3.** 15. **4.** 24. **5.** $12\frac{1}{3}$. **6.** $1\frac{11}{16}$. **7.** 108. **8.** $\frac{7}{8}$. **9.** $2\frac{4}{5}$.

Page 271. Test 19A. — **1.** .44. **2.** .73. **3.** .81. **4.** .92. **5.** .14. **6.** .31. **7.** .64. **8.** .66.

Test 19B. — **1.** 25.92. **2.** 2.8268. **3.** 24.024. **4.** 248.4. **5.** 7346.7. **6.** .27506. **7.** .3285. **8.** .00702. **9.** 340. **10.** .0004. **11.** .43508. **12.** 7290.

Test 19C. — **1.** .0581. **2.** 9180. **3.** 7.1903.

Test 19D. — **1.** 4.6. **2.** 997.3. **3.** 60.8.

Test 19E. — **1.** 7.25. **2.** 3.375. **3.** .00035.

Page 287. Test 20A. — **1.** 54,089. **2.** 50,951. **3.** 48,846. **4.** 51,255. **5.** 38,176.

Test 20B. — **1.** 1,268,677. **2.** 7,391,589. **3.** 1,143,376. **4.** 8,536,406. **5.** 1,434,880. **6.** 5,345,970. **7.** 4,302,248. **8.** 6,456,899. **9.** 3,872,871.

Test 20C. — **1.** 504,331,623. **2.** 203,911,670. **3.** 88,950,448.

Test 20D. — **1.** 903.3. **2.** 713.1. **3.** 5911. **4.** 608.1. **5.** 815.5.

Page 305. Test 21A. — **1.** .47. **2.** .06. **3.** 3.75. **4.** .285. **5.** .075. **6.** .85. **7.** .01. **8.** 4. **9.** 10. **10.** .0425. **11.** $\frac{4}{5}$. **12.** $\frac{7}{25}$. **13.** $\frac{7}{8}$. **14.** $\frac{16}{25}$. **15.** $\frac{13}{20}$. **16.** $66\frac{2}{3}\%$. **17.** 75%. **18.** 700%. **19.** 45%. **20.** $8\frac{1}{2}\%$. **21.** $37\frac{1}{2}\%$. **22.** 5%. **23.** 600%. **24.** 350%. **25.** $16\frac{3}{4}\%$.

Test 21B. — **1.** 50. **2.** 65. **3.** $8\frac{1}{2}$. **4.** 7. **5.** 46. **6.** $7\frac{1}{4}$.

Test 21C. — **1.** 28%. **2.** 25%. **3.** 180%.

Page 321. Test 22A. — **1.** 31,027. **2.** 60,987. **3.** 34,983. **4.** 40,344. **5.** 36,986.

Test 22B. — **1.** 775,824. **2.** 343,461. **3.** 577,189. **4.** 489,153. **5.** 144,957. **6.** 586,865. **7.** 169,478. **8.** 221,238. **9.** 133,329. **10.** 356,589. **11.** 239,696. **12.** 298,697.

Test 22C. — **1.** 373,596. **2.** 102,384. **3.** 393,737. **4.** 859,876. **5.** 194,775.

Test 22D. — **1.** 625. **2.** 709.1. **3.** 419.1. **4.** 589.

Page 337. Test 23A. — **1.** 19. **2.** $23\frac{1}{8}$. **3.** $6\frac{7}{20}$. **4.** $16\frac{23}{24}$. **5.** $24\frac{1}{3}$.

Test 23B. — **1.** $2\frac{1}{2}$. **2.** $1\frac{1}{4}$. **3.** $3\frac{11}{16}$. **4.** $4\frac{5}{12}$. **5.** $2\frac{1}{8}$. **6.** $4\frac{1}{6}$. **7.** $2\frac{9}{10}$. **8.** $3\frac{3}{8}$. **9.** $3\frac{3}{10}$. **10.** $3\frac{1}{15}$. **11.** $1\frac{23}{24}$. **12.** $1\frac{3}{16}$.

Test 23C. — **1.** 366. **2.** \$11.81. **3.** \$31.63. **4.** $21\frac{7}{8}$. **5.** $\frac{3}{4}$. **6.** $2\frac{1}{8}$. **7.** $53\frac{1}{3}$. **8.** $3\frac{5}{8}$. **9.** $16\frac{1}{3}$.

Test 23D. — **1.** $\frac{7}{16}$. **2.** 48. **3.** $\frac{2}{3}$. **4.** $1\frac{7}{20}$. **5.** $2\frac{1}{8}$. **6.** $\frac{1}{5}$. **7.** $\frac{5}{8}$. **8.** $3\frac{1}{3}$. **9.** $16\frac{1}{2}$. **10.** $1\frac{9}{16}$. **11.** $\frac{9}{20}$. **12.** $4\frac{1}{5}$.

Page 349. Test 24A. — **1.** 2268.21. **2.** 3541.67. **3.** 212.853. **4.** 357.496.

Page 349 (*continued*). — **Test 24B.** — **1.** 1551.67. **2.** 60.7984. **3.** 4.46309. **4.** 109.057. **5.** 724.848. **6.** 53.0175. **7.** 467.348. **8.** 6685.32. **9.** 17,784.8. **10.** 2387.34. **11.** 62.6256. **12.** 859.195.

Test 24C. — **1.** 354. **2.** .918. **3.** 3.282. **4.** 494.1. **5.** 845.74. **6.** .00063. **7.** .9035. **8.** 9. **9.** 46.58. **10.** 10.872. **11.** .062. **12.** 32,280.

Test 24D. — **1.** .67. **2.** 4.76. **3.** 91.95.

Page 361. Test 25A. — **1.** $19\frac{2}{3}$. **2.** $12\frac{15}{16}$. **3.** $17\frac{1}{6}$. **4.** $18\frac{1}{4}$. **5.** $23\frac{5}{6}$.

Test 25B. — **1.** $2\frac{2}{3}$. **2.** $8\frac{3}{16}$. **3.** $\frac{1}{10}$. **4.** $3\frac{1}{12}$. **5.** $1\frac{1}{10}$. **6.** $1\frac{2}{3}$. **7.** $1\frac{1}{4}$. **8.** $2\frac{7}{12}$. **9.** $6\frac{17}{24}$. **10.** $1\frac{7}{8}$. **11.** $\frac{1}{10}$. **12.** $4\frac{1}{15}$.

Test 25C. — **1.** $19{,}531\frac{1}{4}$. **2.** 3650. **3.** 8967. **4.** $9\frac{1}{3}$. **5.** $\frac{12}{25}$. **6.** $\frac{3}{4}$. **7.** $26\frac{2}{3}$. **8.** 1. **9.** $29\frac{9}{10}$.

Test 25D. — **1.** $6\frac{1}{2}$. **2.** $1\frac{9}{16}$. **3.** 32. **4.** $18\frac{2}{3}$. **5.** $\frac{3}{8}$. **6.** $2\frac{1}{5}$. **7.** $11\frac{3}{5}$. **8.** $\frac{5}{32}$. **9.** $5\frac{1}{2}$. **10.** $\frac{4}{5}$. **11.** $\frac{2}{15}$. **12.** $1\frac{11}{12}$.

Page 373. Test 26A. — **1.** .99. **2.** .03. **3.** 2. **4.** .001. **5.** .005. **6.** .1. **7.** .09. **8.** 1.06. **9.** .055. **10.** .0025. **11.** $\frac{3}{5}$. **12.** $\frac{1}{100}$. **13.** $\frac{1}{8}$. **14.** $\frac{4}{25}$. **15.** $\frac{11}{20}$. **16.** $83\frac{1}{3}\%$. **17.** 900%. **18.** 8%. **19.** 21.5%, or $21\frac{1}{2}\%$. **20.** 9.2%, or $9\frac{1}{5}\%$. **21.** 90%. **22.** 50%. **23.** $6\frac{1}{4}\%$. **24.** 0.6%, or $\frac{3}{5}\%$. **25.** 225%.

Test 26B. — **1.** $1.73. **2.** $1.18. **3.** $202.50. **4.** $330.80. **5.** $216.88. **6.** $1101.90. **7.** $116.74. **8.** $1.26.

Test 26C. — **1.** 6. **2.** $\frac{1}{2}$. **3.** 28. **4.** 120. **5.** $66\frac{2}{3}$. **6.** 205.

Test 26D. — **1.** 15%. **2.** 100%. **3.** 285%.

Page 387. Test 27A. — **1.** 8208. **2.** 6262. **3.** 7330. **4.** 6540. **5.** 5496.

Test 27B. — **1.** 5,477,359. **2.** 7,725,581. **3.** 4,156,287. **4.** 6,984,542. **5.** 3,365,860. **6.** 5,459,894. **7.** 4,129,479. **8.** 1,462,297. **9.** 3,537,819.

Test 27C. — **1.** 271,890. **2.** 115,231. **3.** 688,350. **4.** 455,323. **5.** 419,952.

Test 27D. — **1.** 519. **2.** 679. **3.** 357. **4.** 865. **5.** 784.

Page 397. Test 28A. — **1.** 72,977. **2.** 41,801. **3.** 68,400. **4.** 41,151. **5.** 53,639.

Test 28B. — **1.** 385,758. **2.** 175,669. **3.** 598,561. **4.** 197,588. **5.** 418,994. **6.** 245,329. **7.** 244,567. **8.** 795,480. **9.** 239,458. **10.** 113,476. **11.** 372,679. **12.** 386,239.

Test 28C. — **1.** 4,141,187,986. **2.** 115,143,589. **3.** 71,110,085.

Test 28D. — **1.** 327.21. **2.** 902.17. **3.** 311.42.

Page 421. Test 29A. — **1.** $17\frac{9}{16}$. **2.** $25\frac{7}{12}$. **3.** $20\frac{11}{12}$. **4.** $8\frac{2}{5}$. **5.** $27\frac{2}{3}$.

Test 29B. — **1.** $1\frac{3}{10}$. **2.** $2\frac{5}{8}$. **3.** $7\frac{1}{8}$. **4.** $3\frac{1}{3}$. **5.** $1\frac{1}{16}$. **6.** $\frac{5}{8}$. **7.** $6\frac{7}{15}$. **8.** $4\frac{9}{16}$. **9.** $4\frac{1}{20}$. **10.** $\frac{7}{12}$. **11.** $6\frac{11}{12}$. **12.** $3\frac{1}{24}$.

Test 29C. — **1.** $28.31. **2.** $32.43. **3.** $46.87. **4.** $\frac{11}{16}$. **5.** 15. **6.** $166\frac{1}{2}$. **7.** $\frac{3}{8}$. **8.** $110\frac{1}{4}$. **9.** $28\frac{3}{4}$.

Test 29D. — **1.** $\frac{7}{16}$. **2.** $\frac{3}{4}$. **3.** $34\frac{4}{5}$. **4.** 1. **5.** $2\frac{13}{16}$. **6.** $\frac{14}{17}$. **7.** $2\frac{1}{2}$. **8.** $\frac{7}{12}$. **9.** $6\frac{1}{2}$. **10.** $2\frac{2}{7}$. **11.** $\frac{5}{24}$. **12.** 7.

Page 451. Test 30A. — **1.** 430,537. **2.** 282,778. **3.** 471,450. **4.** 358,152.

Test 30B. — **1.** 5167. **2.** 6782. **3.** 7588. **4.** 6453.

Test 30C. — **1.** 67,513,715. **2.** 66,549,780. **3.** 29,387,268. **4.** 37,904,852. **5.** 14,297,443. **6.** 48,695,676. **7.** 35,393,549. **8.** 22,981,646. **9.** 41,891,558.

Page 467. Test 31A. — **1.** 317.782. **2.** 29.4516. **3.** 35,932.6. **4.** 3947.55.

Test 31B. — **1.** 28.2646. **2.** 65,444.8. **3.** 1738.75. **4.** 3.87654. **5.** 355.722. **6.** 66,695.3. **7.** 77.8819. **8.** 2.72931. **9.** 4962.86. **10.** 419.834. **11.** 58,174.9. **12.** 769.318.

Test 31C. — **1.** .625. **2.** .15. **3.** .725. **4.** .822. **5.** .929. **6.** .68. **7.** .938. **8.** .488.

Test 31D. — **1.** 158.928. **2.** 8922.96. **3.** .008464. **4.** .215589. **5.** 48.97.

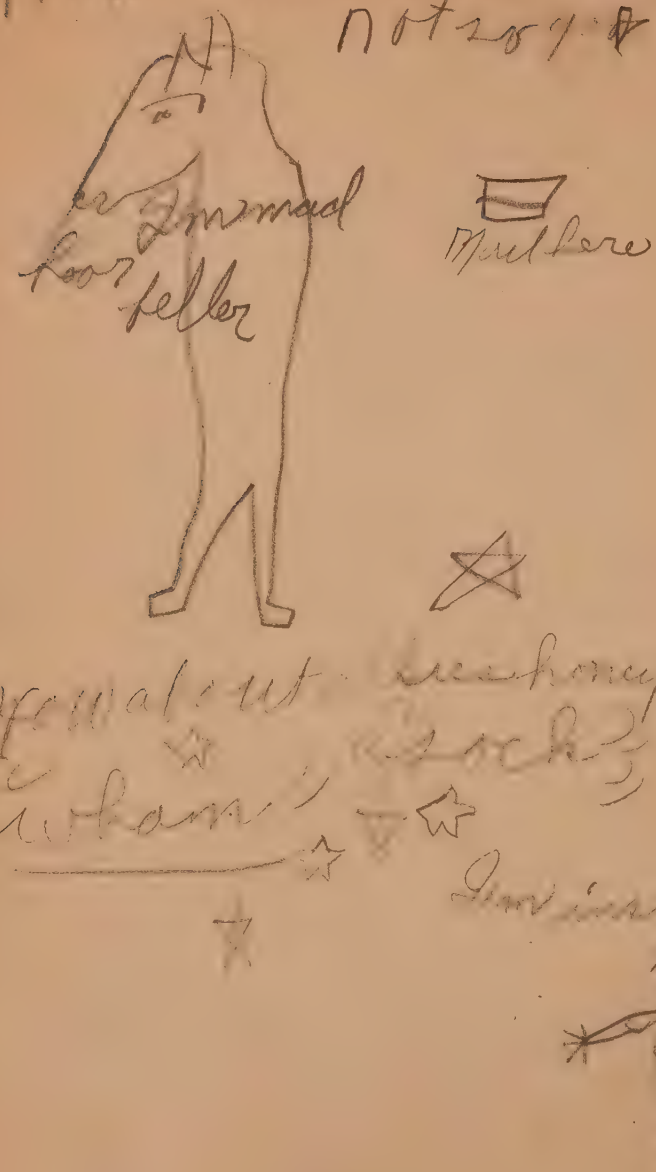
Page 479. Test 32A. — **1.** 23.8%. **2.** 117.8%.

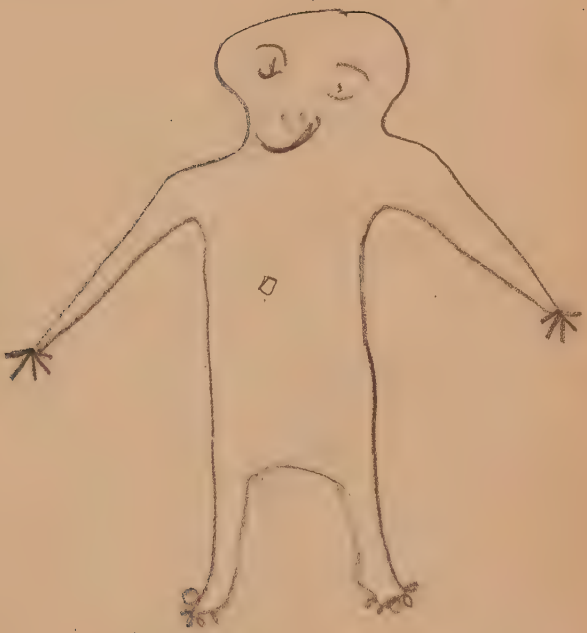
Test 32B. — **1.** 36.5%. **2.** 131.7%.

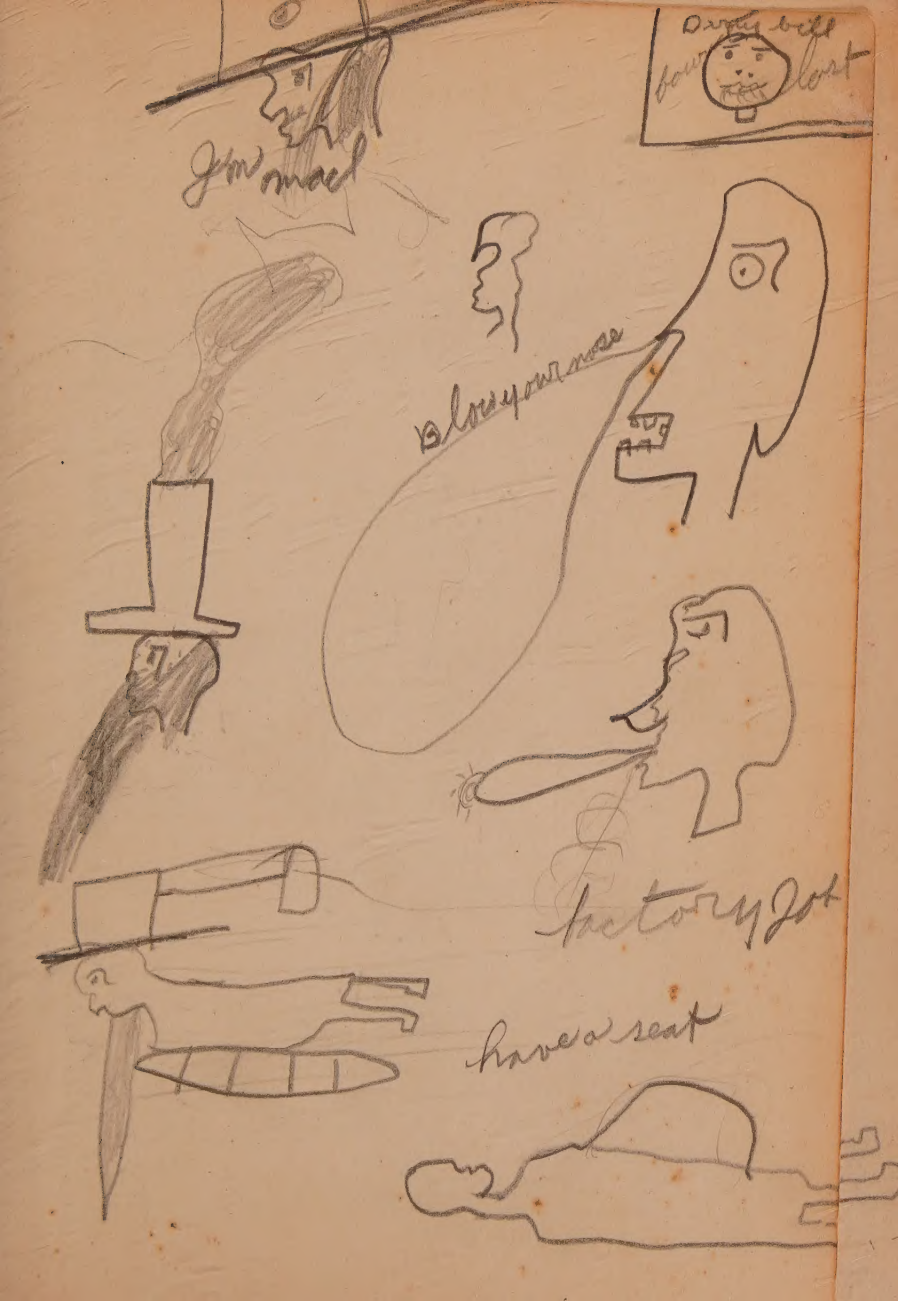
Test 32C. — **1.** 450. **2.** 4950. **3.** 640. **4.** 411. **5.** 1567.

Test 32D. — **1.** $5.33. **2.** $404.17. **3.** $499.80. **4.** $8.07. **5.** $658. **6.** $1617.96.

Test 32E. — **1.** $7.92. **2.** $18.13. **3.** $3.13.


Emerald
Rubber




I'm mad
Blow your nose
Factory Jot
have a seat

LIST OF IMPROVEMENT TESTS

Number of Test	Subject	Page
1	Whole numbers	3
2	Whole numbers	19
3	Fractions	41
4	Whole numbers	51
5	Fractions	69
6	Decimals	79
7	Fractions	99
8	Whole numbers	111
9	Decimals	129
10	Whole numbers	135
11	Whole numbers	143
12	Whole numbers	157
13	Fractions	165
14	Decimals	179
15	Whole numbers	201
16	Fractions	221
17	Whole numbers	241
18	Fractions	261
19	Decimals	271
20	Whole numbers	287
21	Percentage	305
22	Whole numbers	321
23	Fractions	337
24	Decimals	349
25	Fractions	361
26	Percentage	373
27	Whole numbers	387
28	Whole numbers	397
29	Fractions	421
30	Whole numbers	451
31	Decimals	467
32	Percentage	479

SCORING TABLE FOR IMPROVEMENT TESTS

NUMBER OF EXAMPLES IN TEST ↓	NUMBER OF EXAMPLES RIGHT															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
3	0	3	7	10													
4	0	3	5	8	10												
5	0	2	4	6	8	10											
6	0	2	3	5	7	8	10										
8	0	1	3	4	5	6	8	9	10								
9	0	1	2	3	4	6	7	8	9	10							
10	0	1	2	3	4	5	6	7	8	9	10						
12	0	1	2	3	3	4	5	6	7	8	8	9	10				
15	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10	
16	0	1	1	2	3	3	4	4	5	6	6	7	8	8	9	9	10

For directions for using this table, see page 4.